Joined application of a multiaxial critical plane criterion and a strain energy density criterion in low-cycle fatigue

Andrea Carpinteri, Giovanni Fortese, Camilla Ronchei, Daniela Scorza, Sabrina Vantadori
Department of Engineering and Architecture, University of Parma - Parma, Italy

Filippo Berto
Department of Engineering Design and Materials, NTNU - Trondheim, Norway

ABSTRACT. In the present paper, the multiaxial fatigue life assessment of notched structural components is performed by employing a strain-based multiaxial fatigue criterion. Such a criterion, depending on the critical plane concept, is extended by implementing the control volume concept reated to the Strain Energy Density (SED) approach: a material point located at a certain distance from the notch tip is assumed to be the verification point where to perform the above assessment. Such a distance, measured along the notch bisector, is a function of both the biaxiality ratio (defined as the ratio between the applied shear stress amplitude and the normal stress amplitude) and the control volume radii under Mode I and Mode III. Once the position of the verification point is determined, the fatigue lifetime is assessed through an equivalent strain amplitude, acting on the critical plane, together with a unique material reference curve (i.e. the Manson-Coffin curve). Some uniaxial and multiaxial fatigue data related to V-notched round bars made of titanium grade 5 alloy (Ti-6Al-4V) are examined to validate the present criterion.

KEYWORDS. Critical plane approach; Control volume; Notched components; Multiaxial low-cycle fatigue; Strain-based criterion.

INTRODUCTION

I n situation of practical interest, metallic structural components often contain geometrical irregularities (such as notches, fillets and key-seats) because of the design requirements. The above irregularities result in stress/strain concentration phenomena which have to be properly take into account during the design process in order to avoid catastrophic in-service failures [1]. The presence of stress/strain concentrators gives rise to a multiaxial stress/strain state close to the geometrical irregularities even if the applied loading is uniaxial. Moreover, the complexity of the stress/strain state is further increased by the presence of multiaxial fatigue loadings as those experienced by structural components during in-service operation.

Since the pioneering work by Neuber [2], a big effort has been made by the scientific community in order to devise suitable criteria for accurately assessing the fatigue strength/life of structures weakened by either blunt or sharp notches, subjected to complex time-varying loading. Nowadays, a universally accepted criterion does not exist yet, and hence the multiaxial fatigue analysis of notched components is a research topic still open. Reviews of the most promising fatigue criteria for fatigue assessment of notched metallic components can be found in Refs [3,4].

Several multiaxial fatigue criteria available in the literature represent a reformulation of their counterparts for smooth components, by considering the detrimental effect of the stress/strain concentration phenomena on the material fatigue strength. Such criteria usually reduce the complex multiaxial stress/strain state to an equivalent uniaxial condition: they are stress-based in High-Cycle Fatigue (HCF), whereas they are strain-based in Low-Cycle Fatigue (LCF).

According to the above remarks, the present authors have recently proposed a strain-based multiaxial fatigue criterion in order to estimate the fatigue life of severely notched specimens under LCF [5]. Such a criterion is an extension of the critical plane-based multiaxial fatigue criterion proposed by Carpinteri et al. for smooth specimens [6,7] to the case of notched ones. The above extension is formulated by implementing the concept of the control volume, related to the Strain Energy Density (SED) criterion proposed by Lazzarin et al. [8,9]. More precisely, the fatigue life assessment is performed by taking into account the strain state at a material point (named verification point) located at a certain distance from the notch tip, depending such a distance on both the biaxiality ratio λ (remote shear stress amplitude over remote normal stress amplitude) and the control volume radii under loading conditions of Mode I and Mode III.

The goal of the present paper is to discuss the accuracy and reliability of the joined application of the strain-based criterion together with the control volume concept [5] in estimating multiaxial fatigue lifetime of structural components weakened by notches. In particular, the results obtained by employing the criterion proposed in Ref. [5] are compared with some experimental data recently published [10], related to circumferentially V-notched round bars made of titanium grade 5 alloy (Ti-6Al-4V) under both uniaxial and multiaxial fatigue loadings.

CRITERION FORMULATION FOR NOTCHED COMPONENTS

In order to check the accuracy of the strain-based multiaxial fatigue criterion together with the control volume concept (related to the SED criterion) in estimating fatigue life of notched components, we briefly outline the analytical basis of such a criterion (details may be found in Ref. [5]).

The fatigue life assessment is carried out at a verification point (point P), which is distant r from the notch tip. Such a distance r, measured along the notch bisector line, is a function of both the biaxiality ratio λ and the control volume radii provided by the SED criterion [8,9]:

$$r = -(0.221)^{\lambda - 1.484} \cdot R_m + 11.3 R_m \qquad (1)$$

where R_m is the mean control volume radius, computed by averaging the control volume radius related to Mode I, R_1, and that related to Mode III, R_3. According to the SED criterion, such radii depend on the mean values of Mode I and Mode III Notch Stress Intensity Factors (NSIFs) ranges, and on Mode I and Mode III HCF strengths of both smooth specimens and the notch geometry.

Once the position of the verification point P is determined according to Eq. (1), the strain tensor at such a point is obtained from a finite element analysis by examining a tridimensional model.

After the strain state at point P is deduced, the averaged directions of the principal strain axes can be determined on the basis of their instantaneous directions by means of the averaged values of the principal Euler angles. The orientation of the critical plane is linked to the above averaged directions through the off-angle δ:

$$\delta = \frac{3}{2} \left[1 - \left(\frac{1}{2(1 + v_{eff})} \frac{\gamma_a}{\varepsilon_a} \right)^2 \right] 45° \qquad (2)$$

where v_{eff} is the effective Poisson ratio, and ε_a and γ_a are defined by the well-known tensile and torsional Manson-Coffin equations, respectively.

Then, the fatigue strength is assessed by means of an equivalent strain amplitude together with a unique material reference curve (i.e. the tensile Manson-Coffin curve). More precisely, the above equivalent strain amplitude is expressed by a combination of the amplitudes of both the normal, $\eta_{N,a}$, and tangential, $\eta_{C,a}$, displacement vectors acting on the critical plane:

$$\varepsilon_{eq,a} = \sqrt{\left(\eta_{N,a}\right)^2 + \left(\frac{\varepsilon_a}{\gamma_a}\right)^2 \left(\eta_{C,a}\right)^2} \tag{3}$$

By equating Eq. (3) with the tensile Manson-Coffin equation, the number N_f of loading cycles to failure can be worked out through an iterative procedure.

FATIGUE EXPERIMENTAL CAMPAIGN

The strain-based multiaxial fatigue criterion formulated in conjunction with the control volume concept is here applied to a set of data recently published [10]. In particular, uniaxial and multiaxial fatigue tests have been carried out on circumferentially V-notched round bars made of Ti-6Al-4V titanium alloy.
Each specimen presented a V-notch depth equal to 6 mm, an opening angle equal to 90° and a notch root radius equal to about 0.1 mm.
The experimental fatigue tests have been performed by means of an MTS 809 servo-hydraulic biaxial machine. All tests have been conducted under load control at a frequency from 5 to 10 Hz, depending on the applied load. Details of the loading conditions being examined are reported in Ref. [10].
According to the notch geometry and the material properties (that is, the values of NSIFs ranges and HCF strengths), the control volume radius R_1 is equal to 0.051 mm, whereas the control volume radius R_3 is equal to 0.837 mm. The above difference in the values of control volume radii is not only due to the different behaviour in the crack propagation under tension or torsion loading, but also to the higher plasticity around the notch tip experienced under torsion loading with respect to tension loading.

CRITERION VALIDATION

All experimental data being examined (see the previous Section) are characterised by fatigue life between 10^3 and $6\cdot10^5$ loading cycles and nominal load ratio equal to -1. In particular, we consider six different fatigue test series on V-notched specimens:
(1) one series of tests under pure tension fatigue loading and one series under pure torsion fatigue loading;
(2) two series of tests under combined in- ($\Phi=0°$) and out-of-phase ($\Phi=90°$) tension and torsion loading, with a constant biaxiality ratio, λ, equal to 0.6;
(3) two series of tests under combined in- ($\Phi=0°$) and out-of-phase ($\Phi=90°$) tension and torsion loading, with a constant biaxiality ratio, λ, equal to 2.0.
Different values of the distance r to determine the verification point position are computed according to Eq. (1):
(a) for pure tension loading, i.e. $\lambda=0$, r is equal to $1.9\cdot R_m$;
(b) for pure torsion loading, i.e. $\lambda=\infty$, r is equal to $11.3\cdot R_m$;
(c) for combined in- and out-of-phase tension and torsion loading characterised by $\lambda=0.6$, r is equal to $7.5\cdot R_m$;
(d) for combined in- and out-of-phase tension and torsion loading characterised by $\lambda=2.0$, r is equal to $10.8\cdot R_m$.
Note that Eq. (1) has been obtained through a best-fit procedure by taking into account some values of λ related to the experimental data reported in Refs [5,10].
The values of the material parameters related to the Manson-Coffin curves, required for the application of the criterion, are reported in Ref. [11]. The effective Poisson ratio, ν_{eff}, is assumed to be equal to the elastic Poisson ratio (that is, $\nu=0.3$).

The comparison between experimental, $N_{f,\text{exp}}$, and theoretical, $N_{f,cal}$, fatigue life for both uniaxial and multiaxial loadings is shown in Figs 1(a) and 1(b), respectively, where the solid line indicates $N_{f,cal} = N_{f,\text{exp}}$, the dashed lines correspond to $N_{f,cal}/N_{f,\text{exp}}$ equal to 0.5 and 2.0 (scatter band 2), and the dash-dot lines correspond to $N_{f,cal}/N_{f,\text{exp}}$ equal to 0.3 and 3.0 (scatter band 3).

In particular, the following remarks can be made:
- for uniaxial fatigue loading (that is, pure tension or pure torsion loading, Fig. 1(a)), 71% of fatigue life calculation results are included into the scatter band 3;
- for multiaxial fatigue loading (that is, combined tension and torsion loading with different values of biaxiality ratio, Fig. 1(b)), 76% of fatigue life calculation results are included into scatter band 3. Moreover, better estimations are obtained by examining multiaxial fatigue data characterised by $\lambda = 0.6$, since all the theoretical fatigue lives fall within the scatter band 3.
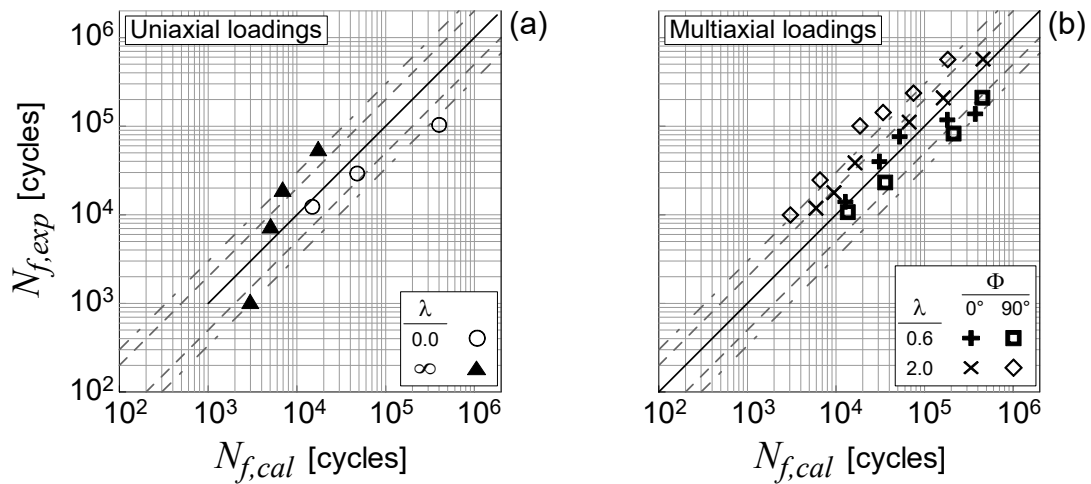

Figure 1: Comparison between theoretical and experimental fatigue life: (a) uniaxial loadings; (b) multiaxial loadings.

Figures 2(a) and 2(b) show the experimental fatigue life, $N_{f,\text{exp}}$, plotted against the equivalent strain amplitude $\varepsilon_{eq,a}$ (see Eq. (3)) for uniaxial and multiaxial fatigue loading, respectively. The solid curves are related to the experimental tensile Manson-Coffin equation. A good agreement is in general observed since the experimental results lie very close to the theoretical curves, and this holds true independent of the applied loading conditions (i.e. for both uniaxial and multiaxial fatigue tests).
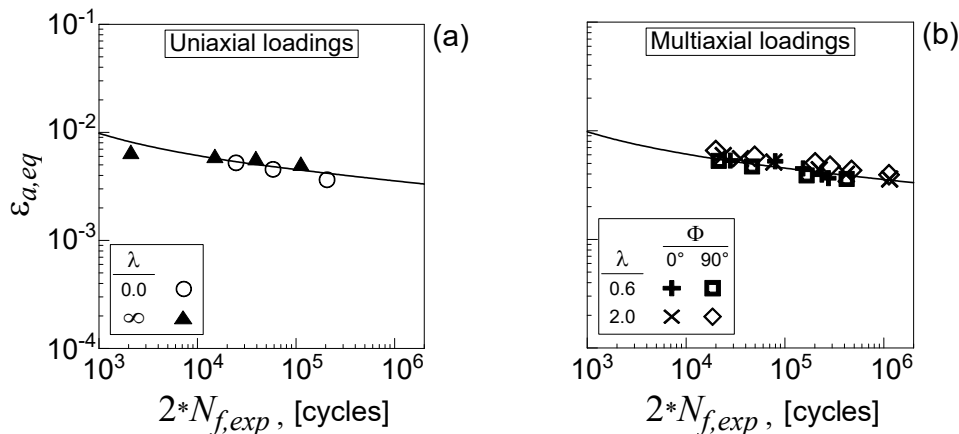

Figure 2: Fatigue life experimental data plotted in terms of the equivalent normal strain amplitude (Eq. (3)): (a) uniaxial loadings; (b) multiaxial loadings. The solid lines refer to the tensile Manson-Coffin equation.

CONCLUSIONS

In the present paper, the multiaxial fatigue life assessment of notched structural components has been performed by employing a strain-based multiaxial fatigue criterion in conjunction with the concept of control volume, related to the SED approach proposed by Lazzarin and co-workers. In particular, the above assessment is carried out at a verification point, which is located at a certain distance from the notch tip, depending such a distance on both the biaxiality ratio and the control volume radii under Mode I and Mode III.

Some uniaxial and multiaxial fatigue data, recently published in the literature, have been analysed to evaluate the effectiveness of the present criterion. The agreement between experimental and theoretical fatigue lives is quite satisfactory.

In conclusion, the present criterion appears to be a promising tool to estimate fatigue lifetime of notched structures, although further investigations are needed by performing experimental tests characterised by fatigue ratio values different from −1 and different notch geometries.

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