Critical applied stresses for a crack initiation from a sharp V-notch

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ABSTRACT. The aim of the paper is to estimate a value of the critical applied stress for a crack initiation from a sharp V-notch tip. The classical approach of the linear elastic fracture mechanics (LELM) was generalized, because the stress singularity exponent differs from 0.5 in the studied case. The value of the stress singularity exponent depends on the V-notch opening angle. The finite element method was used for a determination of stress distribution in the vicinity of the sharp V-notch tip and for the estimation of the generalized stress intensity factor depending on the V-notch opening angle. Critical value of the generalized stress intensity factor was obtained using stability criteria based on the opening stress component averaged over a critical distance d from the V-notch tip and generalized strain energy density factor. Calculated values of the critical applied stresses were compared with experimental data from the literature and applicability of the LEFM concept is discussed.

KEYWORDS. Crack initiation; V-notch; critical stress; strain energy density factor; generalized linear elastic fracture mechanics; fracture criteria.

INTRODUCTION

Many of material discontinuities can be treated as notches which causes high stress and strain concentration near the notch root. Due to nature of notch, which represents stress concentrator, the crack can initiate in the notch root and consequently its existence can lead to the failure of the whole structure. Due to this reason, the notch behavior and crack initiation from the notch are still in the interest of researchers and engineers.

The problem of stress singularities at angular corners was firstly solved by Williams [1, 2] and others [3, 4]. Kotousov followed up Williams' works and studied the corner singularities for a sector plate within the first-order plate theory by using stress resultant and displacement functions [5-7] and adapting the eigenfunction expansion approach of Williams [1]. However, the specificity of the singular stress field in the vicinity of V-notch is studied from experimental side as well, see e.g. [8, 9]. In the last five years occur works pointing out on the complexity of the stress field around the notch tip and influence of out-of-plane singularity caused in the so-called vertex point [10-14]. The knowledge of the stress distribution near the V-notch tip is basic precondition for estimation of V-notch behaviour under specific loading conditions. The V-notch behaviour under static or quasi static loading was analyzed by many researchers (see, among the others [15-22]). The crack initiation from the sharp V-notch under conditions of fatigue loading is still in the focus of researchers, see e.g. [23-26].

Presented paper is focused on the estimation of critical value of applied stress for a crack propagation from sharp (radius in the notch tip is considered as zero) V-notch in the case of tensile loading. Different materials are considered in the presented study and applicability of the LEFM concept is discussed.

THEORETICAL BACKGROUND

Under the assumptions of linear elastic fracture mechanics the stress field near the crack tip in a homogenous material can be described by stress intensity factor [1, 2]. This stress distribution is characterized by stress intensity factor K [MPa√m] and the stress singularity exponent p = 0.5. In the case of a V-notch a classical approach based on the stress intensity factor cannot be used. The value of the stress singularity exponent p differs from 0.5 in this case and depends on V-notch opening angle. The stress distribution around the notch tip can be expressed as follows, e.g. [1]:

$$\sigma_{ij} = \frac{H_I}{\sqrt{2\pi} \cdot r^p} \cdot f_{ij}\left(\alpha, p, \theta\right) \tag{1}$$

where

$H_I\left[MPa \cdot m^p\right]$ is generalized stress intensity factor,

r, θ are polar coordinates with the origin at the V-notch tip,

p is the stress singularity exponent and

$f_{ij}\left(\alpha, p, \theta\right)$ are known functions.

The stress singularity exponent can be obtained analytically by solution of characteristic equation and H_I from numerical solution of the problem (e.g. finite element method can be used with an advantage).

Two different stability criteria are used in the paper for an estimation of beginning of crack propagation from the sharp V-notch. The first one is based on generalized Sih´s concept of strain energy density factor (SEDF). Generalized SEDF concept leads to the following expression for the critical value of the generalized stress intensity factor H_{IC}, see e.g. [27, 28]:

$$H_{IC} = K_{IC} d^{p-\frac{1}{2}} \sqrt{\frac{4k_n}{k_n U_1\left(\theta = 0\right) + V_1\left(\theta = 0\right)}} \tag{2}$$

where

K_{IC} is fracture toughness of the material,

k_n is function of the Poisson´s ratio ν,

U_1, V_1 are functions of the stress singularity exponent p (see [27,28] for details),

d is parameter corresponding to the mechanism of the body failure.

This parameter is usually called critical distance. The stability criterion has the form:

$$H_I = H_{IC} \tag{3}$$

The crack starts to propagate from the V-notch tip if the value of generalized strass intensity factor reaches its critical value H_{IC}. The critical value is determined from (2) and value H_I can be obtained from numerical solution of the stress distribution in front of the tip of the stress concentrator.

The second criterion, used in the presented study, is based on an average value of concentrator opening stress ahead of the notch tip. This criterion assumes that the crack behaviour is controlled by the value of the opening stress ahead of the notch tip. If the average stress calculated over the distance d ahead of the notch tip reaches its critical value a failure occurs. The critical value is related to the average stress ahead of the crack calculated over the distance d during the remote tensile load on the level of K_{IC}, see [22] for details. The critical value of the generalized stress intensity factor can be expressed as [22]:

$$H_{IC} = K_{IC} \frac{2d^{p-\frac{1}{2}}}{(2-p)(1+q)} \tag{4}$$

where q is a known function and other quantities are defined above.
An advantage of used criteria is that for their application it is necessary only knowledge of fracture toughness of the material and its elastic constants. No other experimental measurements are necessary.
From the practical point of view the value of critical tensile applied stress can be defined as follows:

$$\sigma_{appl,crit} = \frac{H_{IC}}{H_C} \sigma_{appl} \tag{5}$$

where
σ_{appl} is a remote applied stress on the body with V-notch (value H_I corresponds to this load)
$\sigma_{appl,crit}$ is critical value of remote applied tensile stress when the crack starts to propagate from V-notch tip.

NUMERICAL CALCULATIONS

In the following the behaviour of double edge notch specimen loaded by tension is studied, see Fig. 1. Stability criteria (3) and (4) were applied after numerical calculations. Values of the critical applied stress $\sigma_{appl,crit}$ necessary for the estimation of V-notch behaviour were determined. The geometry of the specimen, loading and material characteristics were considered according to reference [29]. Dimensions of the specimens were:

- length $L = 192\,mm$
- width $2w = 109\,mm$
- notch depth $a = 27\,mm$
- thickness $t = 4\,mm$
- V-notch opening angle $\alpha = 0° \div 70°$ (varies with step of 10°).

The specimens were made of polymethyl methacrylate (PMMA). The material was considered as linear-elastic and isotropic with following material properties:

- Young´s modulus $E = 2.3\,\text{GPa}$
- Poisson´s ratio $\nu = 0.36$
- Fracture toughness $K_{IC} = 1.9\,\text{MPa}\sqrt{\text{m}}$
- Tensile strength of the material $\sigma_c = 70\,\text{MPa}$.

Finite element system Ansys was used for the modelling. The eighth of the specimen was modelled due to the symmetry in geometry and loading conditions. Considerable mesh refinement around the V-notch tip was used to obtain accurate stress distribution around of the V-notch tip.

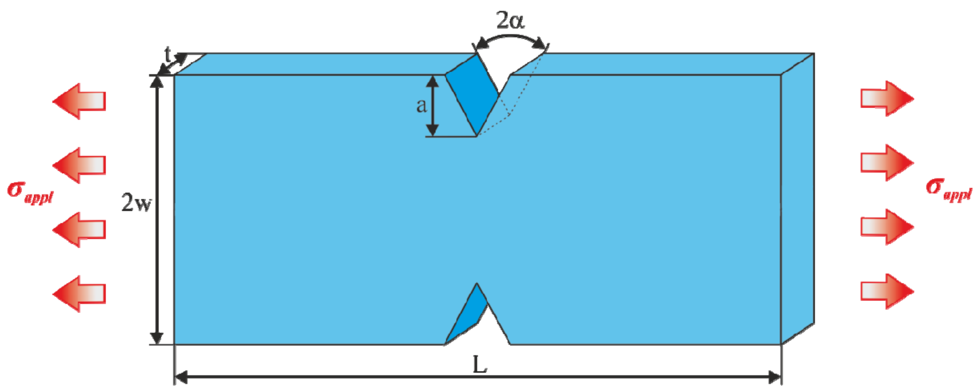

Figure 1: Double edge notch specimen under tensile loading.

Stability criteria (3) and (4) were applied after numerical calculations. Values of the critical applied stress $\sigma_{appl,crit}$ necessary for the estimation of V-notch behaviour were determined. The value of critical distance *d* was chosen according to reference [29]:

$$d = \frac{1}{\pi}\left(\frac{K_{IC}}{1.122 \cdot \sigma_c}\right)^2 \tag{6}$$

Note, that the expression (6) was derived for brittle materials on the base of strain energy release rate.
Results obtained are summarized in Fig. 2. Good agreement between calculated and experimental data is evident for range of α from 10° to 60°. Only for the highest considered angle $\alpha = 70°$ the generalized SEDF criterion exhibits difference of 10% and MS criterion exhibits difference about 15% between calculated and experimental data.

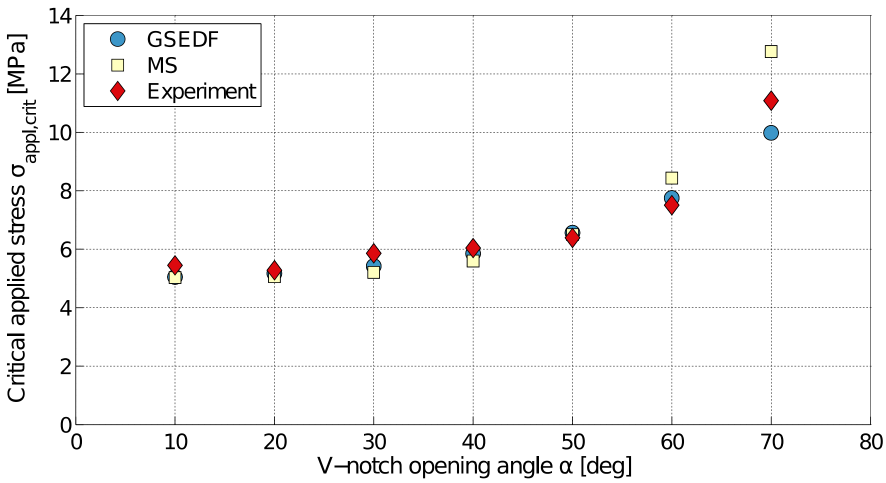

Figure 2: Estimated values of the critical applied stress $\sigma_{appl,crit}$ for different V-notch opening angle α and both stability criteria: based on generalized strain energy density factor (GSEDF) and on the mean stress ahead of the V-notch (MS). Material PMMA.

In the following the same specimen made of duraluminum was considered. The geometry, loading and boundary conditions were the same as in the case of PMMA specimen except of thickness, which was *t* = 5 mm in this case. Material properties used in analytical and numerical calculations were:

- Young´s modulus $E = 72\,\text{GPa}$
- Poisson´s ratio $\nu = 0.33$
- Fracture toughness $K_{IC} = 54.3\,\text{MPa}\sqrt{\text{m}}$
- Tensile strength of the duraluminum $\sigma_c = 454\,\text{MPa}$
- Yield strength $\sigma_y = 260\,\text{MPa}$.

Comparison of estimated critical applied stresses and experimental values is shown in the Fig. 3.
It should be noticed that the results obtained (shown in Fig. 3) are influenced by good plastic properties of duraluminum. Therefore, elastic-plastic numerical calculations (bilinear material curve was considered) were performed to obtain reliable estimation of the plastic zone size ahead of the V-notch tip. The results show an important plastic zone size around the V-notch tip, see Fig. 4 (the plastic zone size is marked by grey colour). For the estimation of the plastic zone size the von Mises stress was used and value of the yield stress was considered as 260 MPa. The plastic zone size increases with V-notch opening angle *a*. It is evident that conditions of small scale yielding are not fulfilled in this study. In spite of big plastic zone size a relatively good agreement between numerically predicted values of critical applied stress and the one experimentally obtained was reached. Especially in the case of GSEDF criterion the difference between calculated and measured date was smaller than 15%. These results surprisingly show good applicability of the GSEDF criterion even if the yielding conditions are quite far from the small scale.

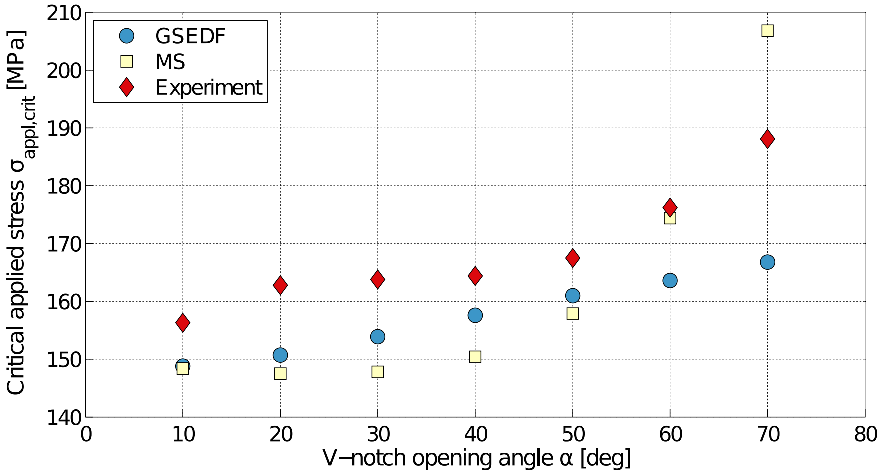

Figure 3: Estimated values of the critical applied stress $\sigma_{appl,crit}$ for different V-notch opening angle α and both stability criteria: based on generalized strain energy density factor (GSEDF) and on the mean stress ahead of the V-notch (MS). Material duraluminum.

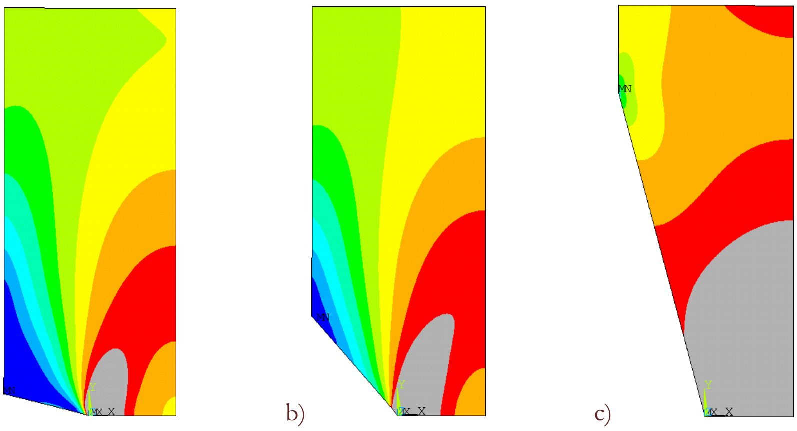

Figure 4: Distribution of von Mises stress ahead of V-notch tip in duraluminum sample. The plastic zone is marked by grey colour. Numerical calculations were performed for V-notch opening angle: a) *a*=10°, b) *a*=40° and c) *a*=70°. The remote applied tensile load corresponded to the critical one.

CONCLUSIONS

The work is devoted to the estimation of critical applied stresses for a crack initiation from a sharp V-notch tip. The estimations are done under assumptions of linear elastic fracture mechanics. Due to stress singularity exponent different from 0.5 in the case of V-notch, generalized form of LEFM was used. Two different stability criteria based on different physical bases were applied. The first one is based on generalized Sih´s strain energy density factor and the second one is based on the average value of opening stress ahead of the tip of the stress concentrator calculated over the critical distance *d*. Chosen specimens were loaded by tensile loading. Comparison of calculated critical applied stresses and experimentally measured data took from the literature was performed. Very good agreement was found for both applied stability criteria in the case of specimen made of PMMA. Following analysis was performed for specimen made of duraliminum. Very important plastic zone size ahead of the stress concentrator tip was determined by elastic-plastic numerical calculations. Both considered stability criteria were applied in spite of important plasticity. Good agreement between calculated and experimentally obtained data was surprisingly found in the studied case of duraluminum specimens.

The paper shows applicability of two stability criteria in the case of tensile loaded bodies with V-notches. While both are derived under assumptions of LEFM, they can be applied with care too in the cases, where the plastic zone size exceeds small scale.

ACKNOWLEDGEMENT

This work was supported through the grant No. CZ.1.07/2.3.00/30.0063 (K. Štegnerová), No. FSI-S-14-2311 (specific academic research grant provided to Faculty of Mechanical Engineering, Brno University of Technology) and No. CZ.1.07/2.3.00/20.0214 of the Ministry of Education, Youth and Sports of the Czech Republic.

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