ON THE USE OF THE MODIFIED MANSON-COFFIN CURVES TO PREDICT FATIGUE LIFETIME IN THE LOW-CYCLE FATIGUE REGIME

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ABSTRACT
In the present study, a novel multiaxial strain based approach is proposed and validated using a number of data sets taken from the literature. The plane experiencing the maximum shear strain amplitude (critical plane) is assumed here to be coincident with the micro-crack initiation plane. The proposed technique requires the calculation both of the shear strain amplitude and of the maximum normal strain relative to the critical plane. Multiaxial fatigue life predictions are made by means of bi-parametric modified Manson-Coffin curves, which take into account the mean stress effect as well as the influence of non-zero out-of-phase angles.

1. INTRODUCTION
Real mechanical components are often subjected to external cyclic loads which result in multiaxial stress states at the component critical sites. Moreover, in many in field applications such components work in the low-cycle fatigue regime. It is evident that design engineers need sound methods to predict fatigue lifetime under this particular loading conditions.
In the recent years, many researchers have attempted to propose fatigue life estimation techniques which are suitable for addressing such a complex problem. Generally speaking, these methods are either strain based or energy based approaches.
In a pioneering work, Brown and Miller [1] observed that fatigue lifetime estimations could be done just by using the strain components normal and tangential to the crack initiation plane. In particular, they highlighted the fact that multiaxial fatigue damage depends on the crack propagation path, for this reason, different criteria have to be adopted, distinguishing between propagations occurring on the component surface and inside the material. When the fatigue crack grows on the surface, Brown and Miller proposed a relationship based on a combined use of a critical plane approach and a modified Manson-Coffin equation, and the critical plane is defined as the one experiencing the maximum shear strain amplitude. In a following work, Wang and Brown [2] reformulated this criterion in order to satisfactorily accounting for the presence of non-zero mean stresses by the mean stress normal to the critical plane.
Subsequently, Socie [3, 4] observed that Brown and Miller's idea could be better formalised by using the maximum stress normal to the critical plane instead of the normal strain component, because crack growth rates are strongly influenced by the stress component perpendicular to the crack path. Taking as starting point this assumption, he proposed two

different criteria according to the crack growth mechanism: when crack propagation is mainly Mode I dominated, then the critical plane is the one experiencing the maximum normal stress and fatigue lifetime have to be estimated by using the Smith-Watson-Topper parameter [3]; on the contrary, when propagation is mainly Mode II governed, the critical plane is that of maximum shear stress amplitude and the number of cycles to failure have to be estimated by using the torsional Manson-Coffin curve [4].
The criteria based on energetic parameters take as their starting point the idea that the energy density is the only quantity which is directly related to the fatigue damage amount. It is common idea that the use of energy parameters to predict fatigue lifetime should have a crucial advantage over the methodologies discussed above: theoretically speaking, the amount of energy required for the fatigue failure is independent from the complexity of the stress state present at critical points, therefore just a uniaxial fatigue curve should be enough to predict fatigue lives even in the presence of complex stress states.
Garud [5] suggested predicting multiaxial fatigue lifetime by considering only the energy due to the plastic deformation. Subsequently, Ellyin [6-9] observed that fatigue damage does not depend only on the plastic energy but also on the positive elastic energy. To be precise, his model takes in account the elastic energy due to the tensile stress components, because experimental investigations have clearly proved that fatigue failures can occur even when the plastic contribution to fatigue damage is negligible (for instance, in the high-cycle fatigue regime). Moreover, it is well known that a positive non-zero mean stress has a more detrimental effect on the fatigue endurance than a negative one. For these two reasons, Ellyin [6] formulated a criterion accounting for both the plastic and the positive elastic contribution.
The accuracy of all the criteria reviewed above have been systematically checked by considering plain specimens made of either metal or aluminium alloy.
The main problem in applying the above theories is that stress fields in the vicinity of crack initiation sites must be known both in terms of stresses and in terms of strains. It is well-know that modelling the plastic contribution to the material stress-strain behaviour during a fatigue cycle is a tricky problem. Apart from those methodologies based on numerical approaches, the technique proposed by Jiang and Sehitoglu [10, 11] is worth to be mentioned. This method is a powerful tool suitable for calculating stresses from strains (or vice versa) accounting for all the main physical phenomena influencing the shape of the hysteresis loop under multiaxial fatigue loading (ratchetting, softening, hardening, non-zero mean stresses, etc.) [12].
Finally, and theoretically speaking, the above criteria might be used even to predict fatigue lifetime of notched components under multiaxial fatigue loading. Again the main problem is the estimation of the elasto-plastic stress/strain field in the vicinity of notches. The problem of estimating actual stresses and strains can be faced by using techniques based on either FE analyses or analytical methods. The most sophisticated analytical approaches include those proposed by Hoffmann and Seeger [13], and based on the use of nominal stresses, and by Köttgen, Barkey and Socie [14], and based on the use of externally applied forces.
Unfortunately, just few systematic investigations have been carried out to check the accuracy of either the strain-based or the energy-based approaches in predicting fatigue lifetime of notched components.
In an initial work Fash, Socie and McDowell [15] observed that Brown and Miller's criterion performs non-conservative predictions in the low-cycle fatigue field and conservative in the high-cycle fatigue regime when applied to estimate fatigue lifetime of notched specimens made of SAE 1045. Subsequently, Yip and Jen proved, conducing an extensive experimental investigation, that fatigue life of specimens with transversal circular holes made of SAE 1045 can successfully be predicted by using Brown and Miller's criterion [16], whereas the best accuracy in predicting fatigue life of U-notched cylindrical specimens of AISI 316 is obtained by applying the criterion due to Fatemi and Socie [17].

Aim of the present study is to formalise a novel approach suitable for predicting fatigue lifetime of plain components under multiaxial fatigue loading: this criterion represents a reformulation in terms of strains of the high-cycle fatigue criterion recently proposed by Susmel and Lazzarin [18, 19].

2. FORMALISATION OF THE METHOD

Recently, Susmel and Lazzarin [18, 19] proposed a new stress based criterion suitable for predicting the fatigue strength of components subjected to multiaxial fatigue loading. This method postulates that fatigue life predictions must be performed by using non-conventional bi-parametric Wöhler curves. In particular, it takes as its starting point the idea that fatigue can be summarised in diagrams having in the ordinate the shear stress amplitude, τ_a, relative to the plane experiencing the maximum shear stress amplitude (critical plane) and in the abscissa the number of cycle to failure, N_f (Fig. 1a). By a systematic investigation Susmel and Lazzarin [18, 19] proved that, by changing the ρ values, different fatigue curves are generated in the modified Wöhler diagram. The stress parameter ρ is the crack initiation plane stress ratio, which is defined as:

$$\rho = \frac{\sigma_{n,max}}{\tau_a} \tag{1}$$

being $\sigma_{n,max}$ the maximum stress perpendicular to the critical plane.
This approach was seen to be successful in predicting the multiaxial fatigue life of both smooth and notched components in the medium as well as in the high-cycle fatigue regime [18-20].
In the present study the same idea is re-applied to plain components in terms of strains. In particular, the hypothesis is formed that, in the low/medium cycle fatigue regime, fatigue damage depends on both the maximum shear strain amplitude, γ_a^*, and the maximum strain normal to the plain experiencing the maximum shear strain amplitude, $\varepsilon^*_{n,max}$. The combined effect of these two strain components can simultaneously be accounted for by the following strain ratio:

$$\rho_\varepsilon = 2\frac{\varepsilon^*_{n,max}}{\gamma^*_a} \tag{2}$$

Following a procedure similar to the one proposed by Susmel and Lazzarin in Refs [18, 19], it is possible to easily build a modified Manson-Coffin diagram summarising fatigue damage in terms of strains (Fig. 1b). This diagram has in the abscissa the number of reversals to failure, whereas in the ordinate the maximum shear strain amplitude relative to the critical plane, γ^*_a. According to Socie's fatigue damage model [21], as the ρ_ε value increases, modified Manson-Coffin curves move downward in the diagram (Fig. 1b). This is a consequence of the fact that a positive strain component normal to the critical plane, that is, the plane on which crack initiation is supposed to occur, increases fatigue damage because it favours the crack growth phenomenon (Fig. 2).
The equation of a generic modified Manson-Coffin curve can be expressed as:

$$\gamma^*_a = Q \cdot (2 \cdot N_f)^\beta + Z \cdot (2 \cdot N_f)^\chi \tag{3}$$

where Q, Z, β and χ are constants to be determined which depend on the strain ratio ρ_ε. To calibrate these constants the fully-reversed uniaxial and torsional Manson-Coffin curve can be used. In particular, these calibration curves rewritten in terms of the critical plane approach turn out to be:

$$\gamma_a^* = \varepsilon_{x,a} = \frac{\sigma'_f}{E}\cdot(2\cdot N_f)^b + \varepsilon'_f\cdot(2\cdot N_f)^c \qquad \text{(Push-Pull, } \rho_\varepsilon=1) \tag{4}$$

$$\gamma_a^* = \gamma_{xy,a} = \frac{\tau'_f}{G}\cdot(2\cdot N_f)^{b_0} + \gamma'_f\cdot(2\cdot N_f)^{c_0} \qquad \text{(Torsion, } \rho_\varepsilon=0) \tag{5}$$

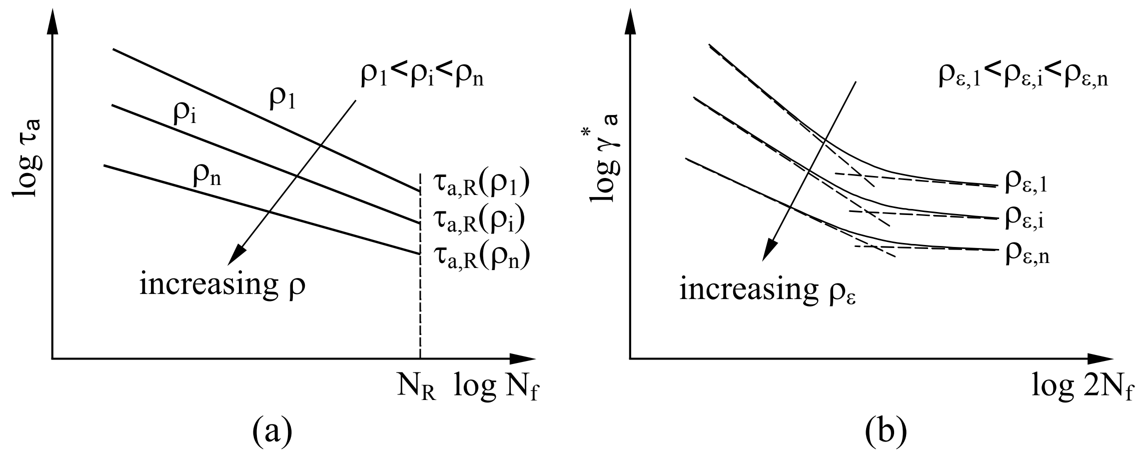

Figure 1: *Modified Wöhler curves (a) and modified Manson-Coffin curves (b).*

In order to have always positive values of γ^*_a independently of the ρ_ε value, the constants Q, Z, β and χ in equation (3) have been expressed as:

$$Q = \frac{L_1}{\rho_\varepsilon^2 + M_1};Z = \frac{L_2}{\rho_\varepsilon^2 + M_2};\beta = \frac{L_3}{\rho_\varepsilon^2 + M_3};\chi = \frac{L_4}{\rho_\varepsilon^2 + M_4} \tag{6}$$

where the constants L_i and M_i (i=1, 2, 3, 4) can be determined using equations (4) and (5) as calibration curves. In particular, remembering that due to the definition of ρ_ε given by equation (2), the strain ratio ρ_ε is equal to 1 and 0 under fully-reversed uniaxial and torsional fatigue loading, respectively, it is trivial to obtain the following expressions:

$$Q = \frac{\frac{\tau'_f}{G}\cdot\frac{\sigma'_f}{E}}{\rho_\varepsilon^2\cdot\left(\frac{\tau'_f}{G} - \frac{\sigma'_f}{E}\right) + \frac{\sigma'_f}{E}} \tag{7}$$

$$Z = \frac{\gamma'_f\cdot\varepsilon'_f}{\rho_\varepsilon^2\cdot(\gamma'_f - \varepsilon'_f) + \varepsilon'_f} \tag{8}$$

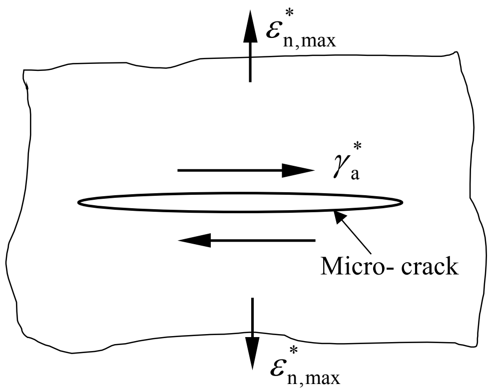

Figure 2: *Fatigue Damage Model.*

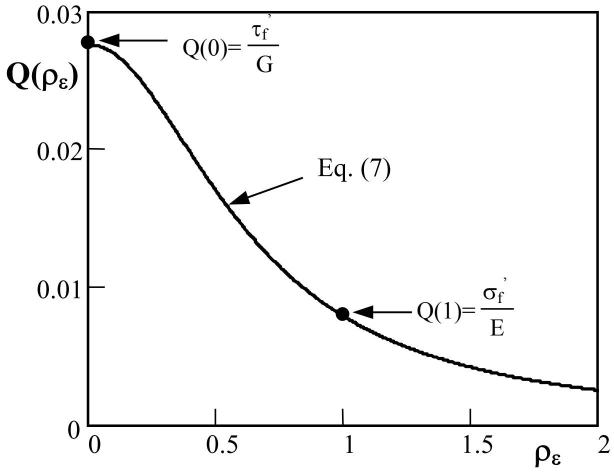

Figure 3: Assumed variation of parameter Q as a function of ρ_ε.

$$\beta = \frac{b_0 \cdot b}{\rho_\varepsilon^2 \cdot (b_0 - b) + b} \tag{9}$$

$$\chi = \frac{c_0 \cdot c}{\rho_\varepsilon^2 \cdot (c_0 - c) + c} \tag{10}$$

As example, in figure 3 the Q vs. ρ_ε relationship has been plotted. This diagram makes it evident that, when ρ_ε tends toward to infinity, Q tends to zero. Finally, in figure 4 it has been summarised the procedure for the in field application of the proposed method to assess real components.

3. METHOD VALIDATION BY EXPERIMENTAL DATA

In order to check the accuracy of the proposed method in predicting fatigue lifetime under multiaxial fatigue loading, 190 experimental tests have been selected from the technical literature. These results were generated testing five different materials under tension-compression and torsion. To be precise, 34 results were generated under tension-compression, 51 under torsion, 73 under in-phase tension/torsion and 32 under out-of-phase tension/torsion. The summary of the collected fatigue tests and the material mechanical properties are reported in Table 1.

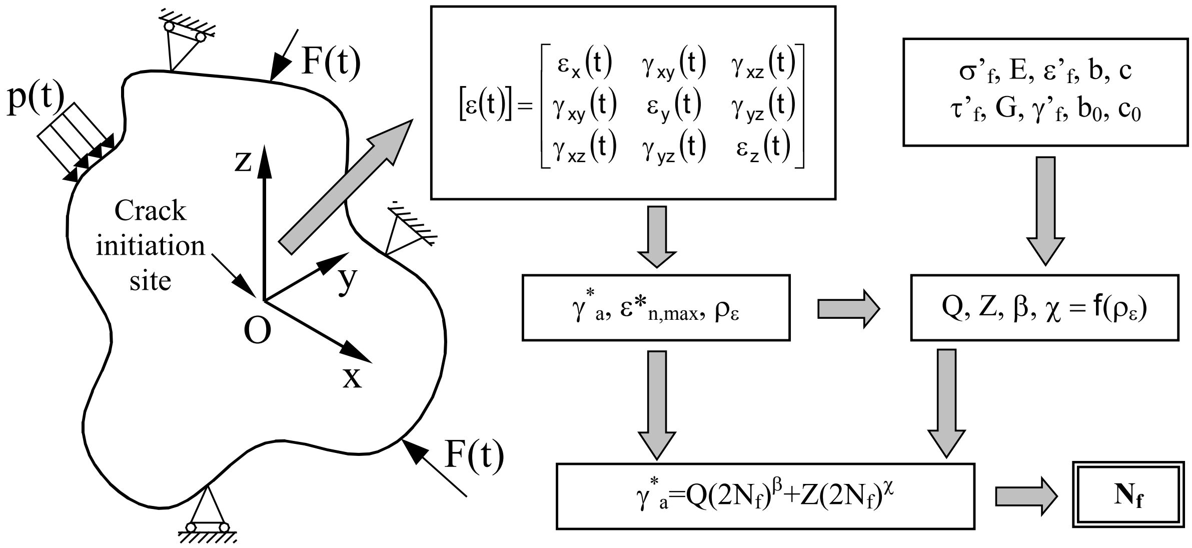

Figure 4: Procedure to apply the Modified Manson-Coffin curve method.

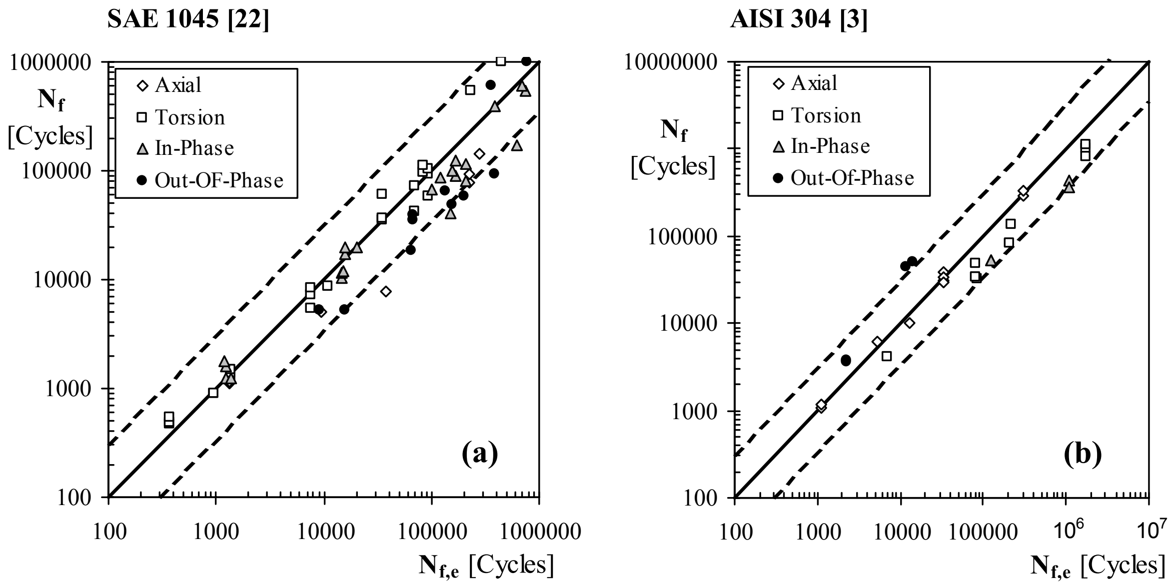

Figure 5: *Estimated, $N_{f,e}$, vs. experimental, N_f, fatigue life diagrams for specimens made of SAE 1045 [22] (a) and AISI 304 [3] (b).*

All the constants needed to apply the proposed method were calibrated using the tension-compression and the torsional fatigue curve. As example, in figure 5 the estimated vs. experimental fatigue life diagrams have been reported for two different materials: SAE 1045 (fig. 5a) and AISI 304 (fig. 5b). The first one was a commercial carbon steel characterised by failures which were mainly Mode II dominated [22]. On the contrary, the second material was a stainless steel showing failures mainly Mode I dominated both under tension-compression and under torsion [3]. Focusing attention on figure 5, it can initially be observed that predictions fall within an error band of factor 3, independently of the material cracking behaviour. Moreover, figure 5 proves the fact that the proposed criterion is highly sensitive to the calibration curves. For instance, the medium cycle fatigue behaviour of SAE 1045 specimens under uniaxial fatigue loading is not well predicted. This is a consequence of the fact that the constants of the uniaxial Manson-Coffin curve given in Ref. [22] were not capable of correctly describing the material fatigue behaviour under tension-compression. This inaccuracy influenced the predictions of multiaxial fatigue data, which became non-conservative when N_f was larger than about $8 \cdot 10^4$ cycles to failure.

Table 1: *Summary of the collected multiaxial fatigue tests and material static and fatigue properties (K' and n' are the cyclic stress-strain curve parameters).*

Material	**Inconel 718**	**AISI 304**	**SAE 1045**	**6061 T6**	**S45C**
Ref. Test	[21]	[3]	[22]	[23]	[24]
N. of tests	55	31	64	16	24
Ref. mat. Constants	[21]	[3]	[22]	[25]	[25]
E [MPa]	208500	183000	204000	72700	210000
G [MPa]	77800	82800	80300	27330	80000
ν_e	0.34	0.3	0.27	0.33	0.3
σ_y [MPa]	1160	325	380	313	375
ε'_f	2.67	0.171	0.26	0.22	0.295
σ'_f **[MPa]**	1640	1000	948	645	1013
b	-0.06	-0.114	-0.092	-0.097	-0.105
c	-0.82	-0.402	-0.445	-0.6	-0.488
γ'_f	18	0.413	0.413	0.381	0.51
τ'_f [MPa]	2146	709	505	373	585
b_0	-0.148	-0.121	-0.097	-0.097	-0.07
c_0	-0.922	-0.353	-0.445	-0.6	-0.488
K' [MPa]	1530	1660	1258	445	1280
n'	0.07	0.287	0.208	0.088	0.209

Finally, in order to have an overview on the accuracy of the proposed criterion, in figure 6a the estimated vs. experimental fatigue life diagram built using all the collected experimental data has been reported. This diagram shows that our method is capable of predictions falling within an error band of about 3, and it holds true independently of material, loading path and out-of-phase angle value. This diagram can directly be compared to the one built predicting fatigue lifetime by using Fatemi-Socie's criterion [4, 26]. This well-known critical plane approach takes as its starting point the assumption that crack initiation occurs on the plane of maximum shear strain amplitude, and fatigue damage depends also on the maximum stress, $\sigma_{n,max}$, perpendicular to the critical plane. This criterion is formalised by the following equation [26]:

$$\gamma_a^* \left(1 + S \frac{\sigma_{n,max}}{\sigma_y} \right) = \frac{\tau'_f}{G} (2N_f)^{b_0} + \gamma'_f (2N_f)^{c_0} \tag{11}$$

where σ_y is the yield stress and S is a constant to be determined using some calibration tests. The procedure applied to determine S is widely discussed in Ref. [27].
In order to quantitatively compare the proposed method accuracy to the one obtainable by applying Fatemi-Socie's criterion, figure 7 was built assuming a normal distribution for the error, which was defined as:

$$E_N = \mathrm{Log}_{10} \left(\frac{N_{f,e}}{N_f} \right) \tag{12}$$

This figure shows that both criteria are slightly non-conservative, but the method proposed in the present paper is much more accurate than the classical approach by Fatemi and Socie.

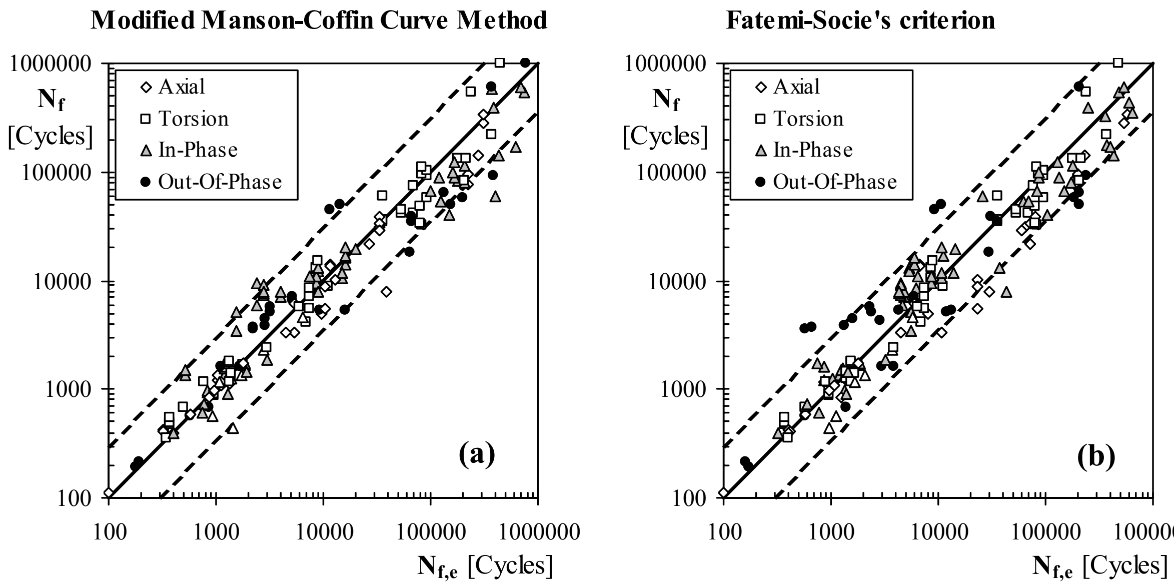

Figure 6: Estimated, $N_{f,e}$, vs. experimental, N_f, fatigue life diagrams obtained applying both the Modified Manson-Coffin curve method (a) and Fatemi-Socie's criterion [26] (b).

4. CONCLUSIONS

The proposed method is seen to be more accurate than Fatemi-Socie's approach: both require two calibration curves to be applied, but our method has the advantage over the other one that for its application only the strain state at the critical site must be determined. Further work also needs to be done in this area to more deeply investigate the applicability of our approach in the presence of notches subjected to multiaxial fatigue loading.

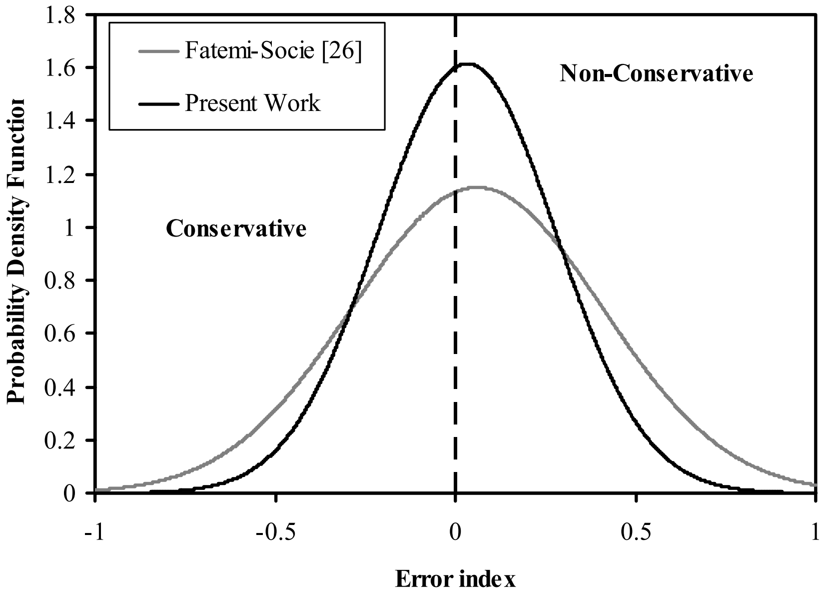

Figure 7: Probability Density Function vs. Error diagram for the two compared methods.

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