Loading force reconstruction and impact location identification for unidirectional carbon/epoxy wide beam

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INTRODUCTION

Identification of impact force history and impact location is the first important task in structure health monitoring (SHM) processes. The task is critical in cases, when it is not possible to measure the impact force directly and the knowledge of intensity and position of loading is necessary for damage prediction of the structure. Number of methods for identification of impact event exists (see [1–5]). This paper shows method for reconstruction of impact force and location, which is based on decomposition of impact force and responses with the use of so-called base functions. Finite element (FE) analysis is successfully used for acceleration of the process. The method is tested on an orthotropic prismatic wide beam with rectangular cross-section. The beam is fixed on both sides and impact force is applied perpendicularly to the beam axis. Three piezoelectric patch transducers are bonded to the beam bottom-surface to measure deformations.

METHODOLOGY

Let us consider an impact event on a structure. Loading force caused by an impact, which is expressed as a time dependent vector, can be reconstructed as a sum of so-called base loadings Φ_m, each multiplied by a coefficient R_m [4, 5]

$$\mathbf{F} = \sum_{m=1}^{M} R_m \boldsymbol{\Phi}_m = \begin{bmatrix} \boldsymbol{\Phi}_1 & \boldsymbol{\Phi}_2 & \dots & \boldsymbol{\Phi}_M \end{bmatrix} \begin{bmatrix} R_1 & R_2 & \dots & R_M \end{bmatrix}^T = \boldsymbol{\Phi}\mathbf{R}. \tag{1}$$

A strain is used as the response quantity in this study. Strain εs measured in sensor s, expressed as time dependent vector, can be reconstructed as

$$\boldsymbol{\varepsilon}^s = \sum_{m=1}^{M} R_m \boldsymbol{\psi}_m^s = \begin{bmatrix} \boldsymbol{\psi}_1^s & \boldsymbol{\psi}_2^s & \cdots & \boldsymbol{\psi}_M^s \end{bmatrix} \begin{bmatrix} R_1 & R_2 & \cdots & R_M \end{bmatrix}^T = \boldsymbol{\Psi}^s \mathbf{R}, \quad s = 1, 2, 3, \tag{2}$$

where so-called base response $\boldsymbol{\psi}_m^s$ is the response in sensor s to the base loading Φ_m. Base loadings Φ_m are taken as ramp functions shifted in time by $m\Delta t$. The base responses can be calculated using finite element (FE) analyses. It is necessary to calculate base response only to one base loading, because all others are shifted in time. The base responses have to be calculated for all sensors and for all possible impact locations (it means for all nodes in FE mesh) in the process of impact location identification. One isolated loading force, which loads the structure in position xi is considered as

$$\mathbf{F} = \boldsymbol{\Phi}\mathbf{R}_i \tag{3}$$

The response for sensor s can be written as

$$\boldsymbol{\varepsilon}^s = \boldsymbol{\Psi}_i^s \mathbf{R}_i, \quad s = 1, 2, 3 \tag{4}$$

Coefficients R_m are calculated using the least square approximation

$$\begin{bmatrix} \mathbf{\Psi}_i^1 \\ \mathbf{\Psi}_i^2 \\ \mathbf{\Psi}_i^3 \end{bmatrix} \mathbf{R}_i = \begin{bmatrix} \boldsymbol{\varepsilon}^1 \\ \boldsymbol{\varepsilon}^2 \\ \boldsymbol{\varepsilon}^3 \end{bmatrix} \tag{5}$$

Finally, the impact location x_i, which corresponds to the minimum of the residuum (sum of squared errors) is the result of minimization problem

$$r_{\min} = \min_i \left[r(x_i) \right] = \min_i \left[\sum_{s=1}^{3} \left\{ \mathbf{u}^s - \mathbf{\Psi}_i^s \mathbf{R}_i \right\}^2 \right] \tag{6}$$

Afterwards, the impact force is reconstructed using Eq. (3).

APPLICATION

Wide unidirectional composite beam clamped on both sides is chosen to test the method. The beam is loaded using the impact hammer along the main axis of the beam and the responses (strains) are measured using the piezoelectric patch transducers. FE model is used for the calculation of the base responses. Dimensions of the beam and piezoelectric patches and FE mesh are shown in Fig. 1.

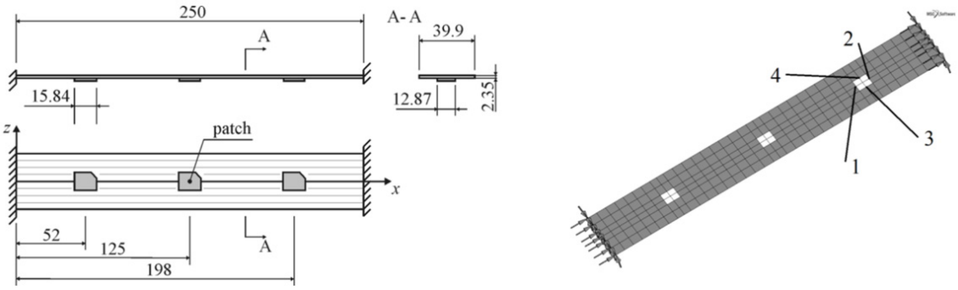

Figure 1: Scheme of the beam (left). FE mesh with adjusted parts, which take into account the piezoelectric patches – white elements. Important nodes are numbered (right).

FE mesh takes into account dimensions of active parts and material properties of the patches. Active parts are squares $l_0 \times l_0$, where $l_0 = 10$ mm. Piezoelectric patches do not capture the strain in just one direction. The measured response signal from one sensor is the sum of strains calculated as

$$\boldsymbol{\varepsilon}^s = \frac{t}{2l_0} \left[\left(\boldsymbol{\varphi}_z^2 - \boldsymbol{\varphi}_z^1 \right) + \left(\boldsymbol{\varphi}_x^3 - \boldsymbol{\varphi}_x^4 \right) \right] \tag{7}$$

where t is thickness of the beam, $\boldsymbol{\varphi}_z$ and $\boldsymbol{\varphi}_x$ are time dependent vectors of rotations around corresponding axis and upper indices denotes node on the edge of the piezoelectric patch (see Fig. 1). Eq. (7) is used for calculation of responses in FE model.

Material parameters of orthotropic material model for both parts of the beam are shown in Tab. 1. Fig. 2 shows the experimental setup.

Pure composite			Composite and piezoelectric patches		
E_L^c	[GPa]	130	E_L^p	[GPa]	118.2
E_T^c	[GPa]	10.5	E_T^p	[GPa]	15.1
G_{LT}^c	[GPa]	3.81	G_{LT}^p	[GPa]	4.5
ν_{LT}^c	[-]	0.3	ν_{LT}^p	[-]	0.3
ρ^c	[kg·m⁻³]	1500	ρ^p	[kg·m⁻³]	2465

Table1: Material parameters.

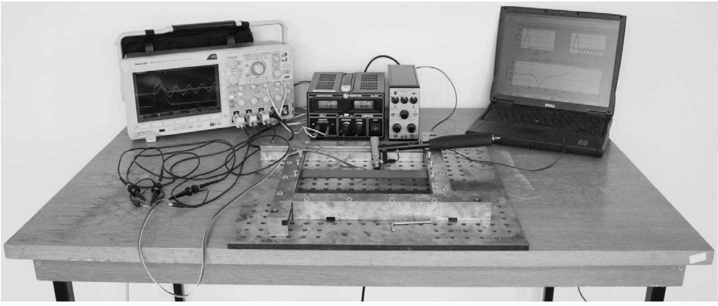

Figure 2: Experimental setup.

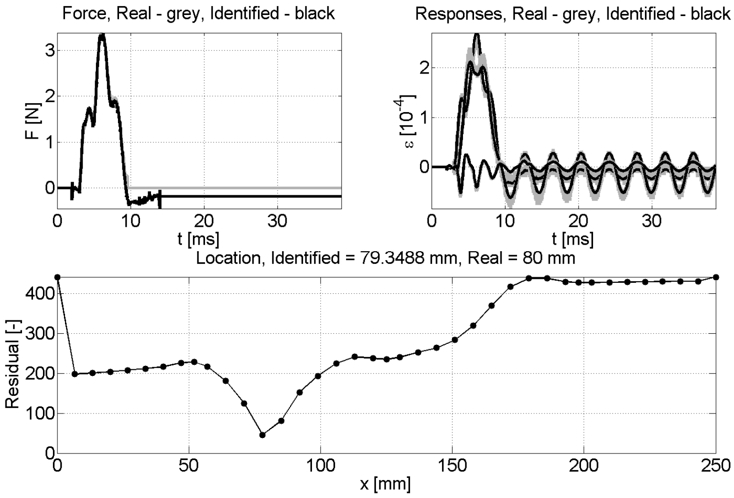

Figure 3: Results of the identification. Dots on the dependence of residual designate positions of the nodes in FE mesh.

Results of the identification process and reconstructed impact force for impact in position 80 mm from left end of the beam are shown in Fig.3. The dependence of the residual on x coordinate shows similar shape to the one in Fig. 3 for every impact position except positions close to both outer sensors. The dependence of residual on x coordinate has always one global minimum in the node closest to the real impact position. The error between real impact position and identified one is smaller than half of the edge length of the finite element in positions distant from both outer sensors.
Methodology was previously tested without using real experimental data [6]. This study was aimed at determination of the minimum number of sensors required for the case of thin prismatic isotropic beam. Results from [6] and results from the presented paper indicate that three sensors are the minimum number of sensors for reconstruction of the impact force with acceptable accuracy. In fact, the number should be greater for real situations. This helps to suppress the oscillations of time dependence of the identified impact force at the beginning and at the end of identified time interval (see Fig. 3).

CONCLUSION

The methodology of the force reconstruction and impact location identification was successfully tested for one dimensional problem using real experimental data measured by three piezoelectric patches on unidirectional wide composite beam. Further work will be aimed at more complex structures and determination of the sufficient number of sensors and the appropriate spatial distribution of the sensors.

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