Optical measuring system for equibiaxial test of hyperelastic rubber-like materials

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ABSTRACT. In some field of engineering (e.g. in the precision mechanics, medical instrumentation) elastic constructions are frequently applied using hyperelastic rubber-like materials which geometry is optimised by FEM software. To define the parameters of the material model applied in the FEM software it is necessary to use real experimental data getting from the uniaxial or/and biaxial strain tests of the rubber.
An equibiaxial measuring system using inflating method with image processing was developed for the characterization of hyperelastic rubber-like materials (in our case silicone rubber).
The paper shows this optical measuring system and the result of the biaxial test of MED-4930 silicone rubber. We analysed the resulting error of the measuring system.
The benefit of this system constructed by our research group is that the measuring procedure can be automatized. By this visual monitoring method the test time could be significantly reduced. In addition the accuracy of the measurement could be increased due to curve fitted mathematically on the contour.

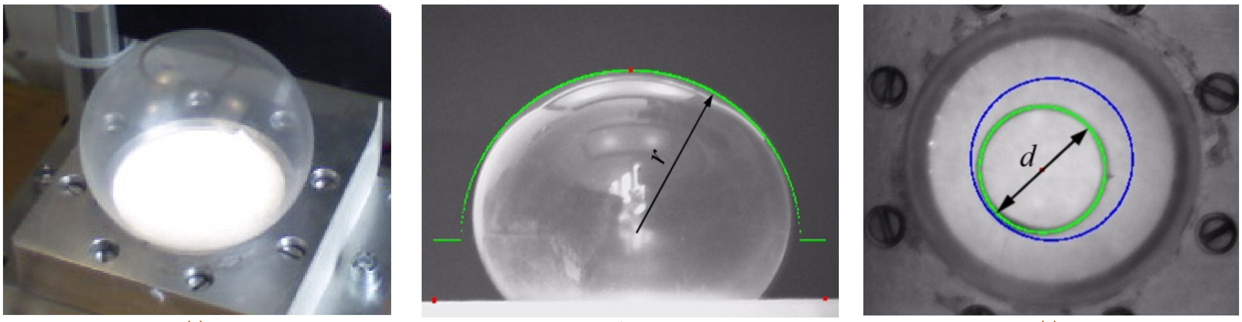
KEYWORDS. Biaxial test; Equibiaxial test; Bubble inflation; Hyperalastic material characterization.

THE OVERVIEW OF THE INFLATING METHODS

The short summary of the equibiaxial test methods [7-10] can be found in our previous paper in this topic [1]. In case of the inflating test the test specimen is a thin membrane which is fixed by a ring shaped element (circular fitting). The air supply on the inner side of the membrane and fitting device results the swelling out of the rubber specimen through the circular hole of the fitting like a "bubble" [1-4].
In the little area around the pole of the hemisphere of the inflated membrane the stress state can be considered for equibiaxial stretch (Fig. 1a) [5, 6]. At this area the stretch ratio of the rubber can be defined by the diameter changing of a marker circle (Fig. 1c).
The stretch ratio is the quotient of the initial l_0 and the current l length of arc of the surface element of the pole ($\lambda = l / l_0$). l and l_0 can be calculated by the diameter of the circle d detected by the upper camera (Fig. 1c) and the radius of curvature of the bubble r at the pole point (Fig. 1b) [1-3, 7]. Assuming that centre of the circle remains near to the pole point during inflation, the stretch ratio can be calculated in the following way:

$$\lambda = \frac{2r \cdot arc\sin\frac{d}{2r}}{d_0} \qquad (1)$$

Where: λ – is the stretch ratio, r – is the radius of curvature of the bubble on the pole point, d – is the diameter of the marker circle, d_0 – is the initial diameter of the marker circle.



(a) (b) (c)

Figure 1: The inflated membrane (a). A recorded image of the front (b) and the top (c) camera, with the measured dimension.

In case of the first inflation the initial length l_0 is equal to the diameter of the marker circle d_0, on the plain membrane. After the first inflation the initial length is must be calculated by the $2r \cdot arcsin(d/2r)$ equation (using the initial d_0 and r_0 parameters), because the residual stretch the membrane is not perfectly plain at zero pressure.
The stress in the membrane can be determined using the inner air pressure and the radius of curvature of the bubble. The current thickness can be calculated from the initial thickness assuming the volume invariance ($t = t_0 / \lambda$).
The stress in the pole point of the axial symmetric bubble can be calculated according to the shell theory from the following equations [1, 2, 7].

$$\sigma = \frac{p \cdot r \cdot \lambda^2}{2t_0} \tag{2}$$

Where: σ – is the arisen principal stress in the observed point, p – is the inner air pressure of the bubble, r – is the radius of curvature of the bubble on the pole point, λ – is the strech ratio, t_0 – is the initial thickness of membrane.
Inserting λ to Eq. 2, the whole result function of the stress can be calculated in the following way:

$$\sigma = \frac{2p \cdot r^3 \cdot \arcsin^2\left(\frac{d}{2r}\right)}{d_0^2 \cdot t_0} \tag{3}$$

It is shown in the equations that in order to define the stress and stretch it is necessary to measure the initial thickness t_0, and the diameter l_0 of the marker circle on the membrane surface. During the measurement the pressure of the inflating air p, the radius of curvature r at the pole and the diameter of the marker circle d had to be recorded.
The pressure p was measured by the integrated sensor of the proportional pressure regulator. The diameter d of the marker circle, the radius r and the height h of the bubble were detected by two CCD cameras.
Both cameras were calibrated in order to avoid the errors coming from the image processing. The calibration of the upper (Eq. 4) and the side camera (Eq. 5) have resulted the functions see below:

$$d = \frac{R_{px}}{3.2455 + 0.0201 \cdot h + 0.00035 \cdot h^2} \tag{4}$$

$$h = 0.1691 \cdot h_{px} + 0.4081 \tag{5}$$

Where: d – The real diameter of the circle plate in mm, R_{px} – The measured radius of the circle plate in pixel, h – The real height of the etalon in mm, h_{px} – The measured height of the etalon in pixel.
The resolution of the cameras is *2.52 lp/mm (line pair / mm) => 0.397 mm*. It was determined using the USAF 1951 method.

THE RESULT OF THE MEASUREMENT

During the tests the inflation of a flat rubber membrane was performed by incrementally increased air pressure. The air pressure was controlled by an electro-pneumatic proportional air valve. The operation of the valve and the analysis of the signal from the pressure sensor were made by a NI data acquisition unit operated by LabView. The change of geometry of the inflated rubber membrane was detected by a CCD camera. The image processing and the analysis of the measured points was realised by Matlab.

MED-4930 siliconerubber (30 Shore hardness) was examined. A result of the measurement is shown in the Fig 3. The examination was performed 7 times per one specimen.

The error of the measurement was defined by the summarised standard deviations of the calculated variants (σ, λ) according to the ISO result type "A". The standard deviations of the Stretch Ratio and the stress were calculated using the partial derivation (Eq. 6, 7) of the result functions (Eq. 1, 3).

$$s_\lambda = \sqrt{\left(\frac{\partial\lambda}{\partial r}s_r\right)^2 + \left(\frac{\partial\lambda}{\partial d}s_d\right)^2 + \left(\frac{\partial\lambda}{\partial d_0}s_{d_0}\right)^2} \qquad (6)$$

$$s_\lambda^2 = \left(\frac{2\mathrm{ArcSin}\left(\frac{d}{2r}\right)}{d_0} - \frac{d}{d_0\cdot r\cdot\sqrt{1-\frac{d^2}{4r^2}}}\right)^2 s_r^2 + \left(\frac{1}{d_0\sqrt{1-\frac{d^2}{4r^2}}}\right)^2 s_d^2 + \left(-\frac{2\mathrm{ArcSin}\left(\frac{d}{2r}\right)}{d_0^2}\right)^2 s_{d_0}^2$$

$$s_\sigma = \sqrt{\left(\frac{\partial\sigma}{\partial p}s_p\right)^2 + \left(\frac{\partial\sigma}{\partial r}s_r\right)^2 + \left(\frac{\partial\sigma}{\partial d}s_d\right)^2 + \left(\frac{\partial\sigma}{\partial d_0}s_{d_0}\right)^2 + \left(\frac{\partial\sigma}{\partial t_0}s_{t_0}\right)^2} \qquad (7)$$

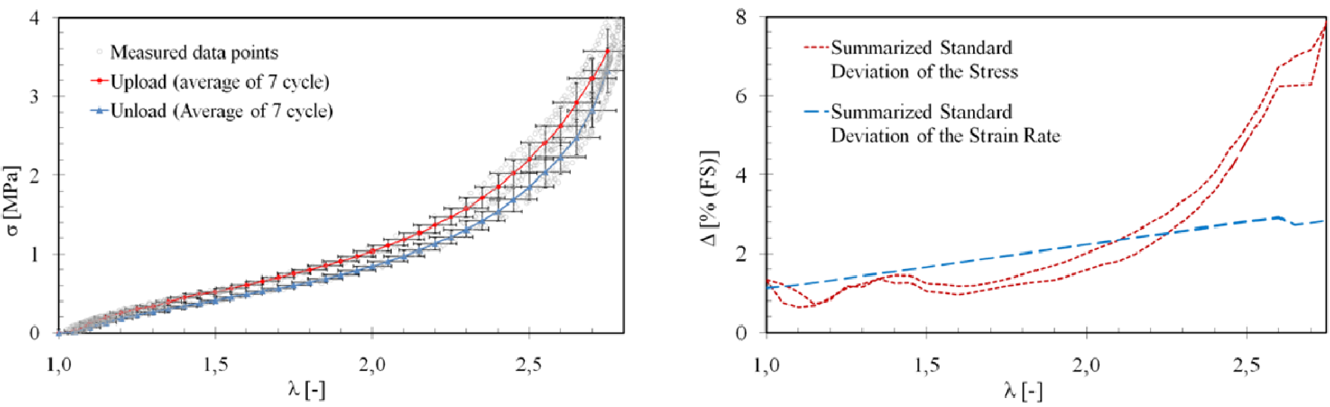
$$s_\sigma^2 = \left(\frac{2r^3\cdot\mathrm{ArcSin}^2\left(\frac{d}{2r}\right)}{d_0^2\cdot t_0}\right)^2 s_p^2 + \left(\frac{6p\cdot r^2\cdot\mathrm{ArcSin}^2\left(\frac{d}{2r}\right)}{d_0^2\cdot t_0} - \frac{2d\cdot p\cdot r\cdot\mathrm{ArcSin}^2\left(\frac{d}{2r}\right)}{d_0^2\cdot t_0\cdot\sqrt{1-\frac{d^2}{4r^2}}}\right)^2 s_r^2 + \left(\frac{2p\cdot r^2\cdot\mathrm{ArcSin}\left(\frac{d}{2r}\right)}{d_0^2\cdot t_0\cdot\sqrt{1-\frac{d^2}{4r^2}}}\right)^2 s_d^2 + \left(-\frac{4p\cdot r^3\cdot\mathrm{ArcSin}^2\left(\frac{d}{2r}\right)}{d_0^3\cdot t_0}\right)^2 s_{d_0}^2 + \left(-\frac{2p\cdot r^3\cdot\mathrm{ArcSin}^2\left(\frac{d}{2r}\right)}{d_0^2\cdot t_0^2}\right)^2 s_{t_0}^2$$

Where: s_σ – is the summarised standard deviation of the stress, s_λ – is the summarised standard deviation of the stretch ratio.

The calculated quantity:	Stretch Ratio			Stress				
The measured quantity:	r [mm]	d [mm]	d_0 [mm]	p [MPa]	r [mm]	d [mm]	d_0 [mm]	t_0 [mm]
The factor (at maximum):	0.002	0.126	0.329	156.11	0.089	0.327	0.856	3.940
Factor · Standard Deviation:	-0.0013	0.050	-0.060	0.181	0.049	0.131	-0.155	-0.032

Table 1

The Tab. 1, shows which parameters cause the dominant errors (in this case *p, d* and d_0). The measurement error could be decreased by improving the measurement of these parameters, increasing the number of the measurements and reducing the number of the indirect quantities. This last term would only be realisable if the stress state and the stretch ratio could only be calculated from a geometrical parameter such as the heights of the bubble or its radius.



(a) (b)

Figure 3: a) The biaxial stress as a function of the stretch ratio in case of the MED-4930 silicone rubber.
b) The standard deviation of the measurement of the stress and the strain rate (full scale).

SUMMARY

The benefit of this procedure is that the measuring procedure can be controlled (made automatically). By this visual monitoring method the test time can be significantly reduced. In addition the accuracy of the measurement could be increased due to the curve fitted on the contour.

In all cases when the camera was used it is necessary curve fitting on to the contour and the marker circle. To avoid the aberration of the optical system it is necessary to use an objective with long focus and it have to calibrate as a function of the distance from the camera.

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