Structural integrity assessment of metallic components under multiaxial fatigue: the C-S criterion and its evolution

Verifica dell'integrità strutturale di componenti metallici soggetti a fatica multiassiale: il criterio C-S e la sua evoluzione

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ABSTRACT. The present paper aims at discussing the evolution of a multiaxial high-cycle fatigue criterion, known as the C-S criterion, based on the critical plane approach and initially proposed by the first two authors. By introducing appropriate devises and changes to the original formulation, this criterion is able to evaluate the structural integrity of smooth and notched metallic components, subjected to different fatigue loadings: multiaxial in- and out-of-phase synchronous cyclic loading, asynchronous cyclic loading, random loading. The results derived by applying it are compared with the experimental data collected from the relevant literature.

SOMMARIO. Il presente lavoro intende illustrare l'evoluzione di un criterio di fatica multiassiale per alto numero di cicli, noto come criterio C-S, basato sull'approccio del piano critico e inizialmente proposto dai primi due autori. Introducendo idonei accorgimenti e modifiche rispetto alla formulazione originaria, questo criterio è in grado di valutare l'integrità strutturale di componenti metallici lisci e intagliati soggetti a diversi tipi di carichi: ciclici multiassiali sincroni in fase e non, ciclici asincroni, variabili in modo random. I risultati ottenuti mediante l'applicazione di tale criterio vengono confrontati con i dati sperimentali reperiti in letteratura.

KEYWORDS. Multiaxial high-cycle fatigue; Critical plane approach; Constant-amplitude cyclic loading; Random loading.

INTRODUCTION

Metallic structural components of engineering interest can often be subjected to multiaxial time-varying loading in service. Several criteria have been proposed to evaluate the structural integrity of such components [1, 2]. Nevertheless, an approach recognized by the entire scientific community has still to be formulated.

For the high-cycle regime, characterised by elastic strains into the material, some of the criteria available in the literature are based on the so-called critical plane approach, according to which fatigue failure assessment is performed in a plane the position of which may be correlated with the amplitude or a combination of some stress components (acting on the critical plane) or with averaged principal stress directions (note that principal directions under fatigue loading are generally time-varying).

The present paper aims at discussing the evolution of a high-cycle multiaxial fatigue criterion, known as the C-S criterion [3], based on the critical plane approach and initially proposed by the first two authors. In particular, this evolution can be schematised by means of steps, where each step is characterised through devises and changes to the original formulation

developed for smooth metallic structural components subjected to constant-amplitude cyclic loading [3,4]. Such steps have simplified the weighting procedure of the principal stress axes and taken into account the detrimental effect of a tensile mean normal stress on fatigue limit [5,6], and have extended the applicability of the original criterion to notched metallic structural components under constant-amplitude cyclic loading (by using the so-called critical point method) [7] and to smooth components under random loading [8].
At present, the paper authors are going to develop a new formulation for smooth components subjected to random loading, by employing a frequency-domain approach (more time-effective than the above time-domain approach [8]) and a spectral fatigue damage law available in the literature, but further work is needed to complete such a formulation.

CRITICAL PLANE DETERMINATION

As was pointed out by Brown and Miller [9], the fatigue crack propagation process can be distinguished into two stages:
(1) Stage 1, in which a crack nucleates (near the external surface of a structural component) along a shear slip plane (Mode II, fatigue crack initiation plane);
(2) Stage 2, in which crack propagates in a plane normal to the direction of the maximum principal stress (Mode I, final fatigue fracture plane).
Since the aim of many high-cycle critical plane criteria is to predict the crack nucleation, several authors assume that the critical plane (i.e. the plane on which the fatigue failure assessment should be performed) corresponds to that of fatigue crack initiation. Nevertheless, in the determination of the critical plane orientation, both shear stress (Mode II) and normal stress (Mode I) mechanisms should be taken into account, because it has been experimental observed that some metallic materials exhibit fatigue crack propagation mechanisms in which Stage 2 is predominant over Stage 1 (e.g. metals at the threshold between hard and extremely hard materials).
The procedure here reported to determine the critical plane orientation allows us to take into account both the above Mode II and Mode I mechanisms, since the governing mechanism depends on the fatigue behavior of the material. According to the proposed procedure, the final fatigue fracture plane is firstly determined and then correlated, through an appropriate angle, to the critical plane.
In more detail, the position of the fatigue fracture plane may be connected with that of the principal stress directions, since it has long been recognised that the principal stresses are fundamental in determining fatigue life [10]. Since such directions under fatigue loading are generally time-varying, averaged principal stress directions are computed by using appropriate weight functions, which take into account the main factors influencing the fatigue fracture behavior.
At a given material point P, the direction cosines of the instantaneous principal stress directions 1, 2 and 3 (being $\sigma_1(t) \ge \sigma_2(t) \ge \sigma_3(t)$) with respect to a fixed PXYZ frame can be worked out from the time-varying stress tensor $\boldsymbol{\sigma}(t)$. The orthogonal coordinate system P123 with origin at point P and axes coincident with the principal directions can be defined through the 'principal Euler angles', ϕ, θ, ψ, which represent three counter-clockwise sequential rotations around given axes [11].
The averaged directions, $\hat{1}, \hat{2}$ and $\hat{3}$, of the principal stress axes are obtained from the averaged values $\hat{\phi}, \hat{\theta}, \hat{\psi}$ of the principal Euler angles. Such values are computed by independently averaging the instantaneous values $\phi(t), \theta(t), \psi(t)$, as follows [11]:

$$\hat{\phi} = \frac{1}{W}\int_0^T \phi(t)\,W(t)\,dt \qquad \hat{\theta} = \frac{1}{W}\int_0^T \theta(t)\,W(t)\,dt \qquad \hat{\psi} = \frac{1}{W}\int_0^T \psi(t)\,W(t)\,dt \qquad \text{with} \quad W = \int_0^T W(t)\,dt \tag{1}$$

where T is the observation time interval (corresponding to the period, in the case of constant-amplitude cyclic loading), and the weight function $W(t)$ is expressed by

$$W(t) = h\left[\sigma_1(t) - \frac{1}{2}\sigma_{af,-1}\right]\left(\frac{\sigma_1(t)}{\sigma_{af,-1}}\right)^{-\frac{1}{m}} \tag{2}$$

where h […] is the Heaviside function (h [x] = 1 for x > 0, h [x] = 0 for x ≤ 0), and m is the slope of the S-N curve for fully reversed normal stress. The adopted weight function is such that it includes into the averaging procedure those positions of the principal directions for which the maximum principal stress σ_1 is greater than half of the normal stress fatigue limit $\sigma_{af,-1}$ under fully reversed normal stress. The estimated final fatigue fracture plane (Stage 2), which is the one observed post mortem at the macro level, is assumed to be coincident with the averaged direction $\hat{1}$ of the maximum principal stress σ_1.

The procedure proposed has been applied to some data of experimental biaxial in- and out-of-phase synchronous sinusoidal fatigue tests [12,13] in order to analyse the correlation between the calculated and the experimental final fatigue fracture plane. The difference between such results is evaluated as $\Delta_\eta = \eta_{cal} - \eta_{exp}$, where η_{cal} and η_{exp} are the angles (calculated and experimentally measured, respectively) between the longitudinal axis of the specimen and the normal vector to the fracture plane. The comparison is quite satisfactory (see details in Ref. [11]): for example, Δ_η ranges from 0.02rad to 0.23rad for the specimens made of Swedish hard steel 982 FA [11,12], whereas for the specimens made of low carbon steel C=0.35% [11,13] Δ_η is equal to 0.00rad.

In order to implement the above procedure in a rather simple way, the following weight function has been proposed for constant-amplitude cyclic loading [5,6]:

$$W(t) = H\left[\sigma_1(t) - \sigma_{1,max}\right] \tag{3}$$

where $\sigma_{1,max}$ is the maximum value (in a loading cycle) of the maximum principal stress σ_1. The above weight function is such that no averaging procedure is actually required (this makes the implementation of the criterion rather simple), since the averaged principal stress axes coincide with the instantaneous principal stress directions corresponding to the time instant at which the maximum principal stress σ_1 attains its maximum value in the loading cycle.

The orientation of the critical plane has been proposed to be defined through the off-angle δ between the averaged direction $\hat{1}$ and the normal $\mathbf{w}$ to the critical plane (where $\mathbf{w}$ belongs to the principal plane $\hat{1}\hat{3}$). The empirical expression of δ is as follows [3]:

$$\delta = \frac{3\pi}{8}\left[1 - \left(\frac{\tau_{af,-1}}{\sigma_{af,-1}}\right)^2\right] \tag{4}$$

where $\tau_{af,-1}$ is the shear stress fatigue limit under fully reversed shear stress. Eq. (4) holds when $\tau_{af,-1}/\sigma_{af,-1}$ is ranging from $1/\sqrt{3}$ to 1. Therefore, when $\tau_{af,-1}/\sigma_{af,-1} > 1$, δ is assumed to be equal to 0 and, when $\tau_{af,-1}/\sigma_{af,-1} < 1/\sqrt{3}$, δ is assumed to be equal to $\pi/4$ [5,7]. Note that Eq. (4) is in line with the fact that, when Stage 2 is predominant, the critical plane is coincident with the final fatigue fracture plane and, when Stage 1 is predominant, the critical plane is coincident with the actual fatigue crack initiation plane.

THE C-S CRITERION

Original formulation for smooth specimens under multiaxial constant-amplitude cyclic loading

The stress vector $\mathbf{S_w}$, the normal stress vector $\mathbf{N}$ and the shear stress vector $\mathbf{C}$ acting on the critical plane are given by:

$$\mathbf{S_w} = \boldsymbol{\sigma} \cdot \mathbf{w} \qquad \mathbf{N} = (\mathbf{w} \cdot \mathbf{S_w})\mathbf{w} \qquad \mathbf{C} = \mathbf{S_w} - \mathbf{N} \tag{5}$$

with $\boldsymbol{\sigma}$ = stress tensor.

For multiaxial constant-amplitude cyclic loading, the direction of the normal stress vector $\mathbf{N}(t)$ is fixed with respect to time and, consequently, the mean value N_m and the amplitude N_a of the vector modulus $N(t)$ can readily be computed. As far as the shear stress vector $\mathbf{C}(t)$ is concerned, the definitions of the mean value C_m and amplitude C_a of the vector modulus $C(t)$ are not unique due to the time-varying direction of $\mathbf{C}(t)$, which describes a closed path during a loading cycle. The procedure proposed by Papadopoulos [14] to determine C_m and C_a can be adopted:

$$C_m = \min_{\mathbf{C}'} \left\{ \max_{0 \le t \le T} \left\| \mathbf{C}(t) - \mathbf{C}' \right\| \right\} \qquad C_a = \max_{0 \le t \le T} \left\| \mathbf{C}(t) - C_m \right\| \tag{6}$$

where the symbol $\| \cdot \|$ indicates the norm of a vector, and $\mathbf{C}'$ is a constant vector with respect to time.

As a multiaxial fatigue limit condition, the following nonlinear combination of the maximum normal stress ($N_{max} = N_m + N_a$) and the shear stress amplitude (C_a) acting on the critical plane has been proposed [3, 4]:

$$\sigma_{eq,a} = \sqrt{N_{max}^2 + \left(\frac{\sigma_{af,-1}}{\tau_{af,-1}} \right)^2 C_a^2} = \sigma_{af,-1} \tag{7}$$

For fatigue strength assessment at finite life, the fatigue limits $\sigma_{af,-1}$ and $\tau_{af,-1}$ appearing in Eq. (7) should be replaced by the corresponding fatigue strengths ($\sigma'_{af,-1}$ and $\tau'_{af,-1}$). Hence, using the Basquin-like relationships [15] for both fully reversed normal stress ($\sigma'_{af,-1} = \sigma_{af,-1} \left(N_f / N_0 \right)^m$, with $\sigma'_{af,-1} =$ fatigue strength for fully reversed normal stress at finite life N_f, and N_0=reference number of loading cycles, e.g. $N_0 = 2 \cdot 10^6$) and fully reversed shear stress ($\tau'_{af,-1} = \tau_{af,-1} \left(N_f / N_0 \right)^{m^*}$, with $\tau'_{af,-1} =$ fatigue strength for fully reversed shear stress at finite life N_f and m* = slope of S-N curve for fully reversed shear stress), Eq. (7) becomes:

$$\sqrt{\left(N_{max} \right)^2 + \left(\frac{\sigma_{af,-1}}{\tau_{af,-1}} \right)^2 \left(\frac{N_f}{N_0} \right)^{2m} \left(\frac{N_0}{N_f} \right)^{2m^*} C_a^2} = \sigma_{af,-1} \left(\frac{N_f}{N_0} \right)^m \tag{8}$$

The number N_f of loading cycles to failure can be determined by solving this non-linear equation.

Modified formulation for smooth specimens under multiaxial constant-amplitude cyclic loading

As is well-known, the effect of a tensile mean normal stress superimposed upon an alternating normal stress strongly reduces the fatigue resistance of metals. Therefore, the above multiaxial fatigue limit condition in Eq. (7) is replaced by the following expression [5,6]:

$$\sigma_{a,eq} = \sqrt{N_{a,eq}^2 + \left(\frac{\sigma_{af,-1}}{\tau_{af,-1}} \right)^2 C_a^2} = \sigma_{af,-1} \tag{9}$$

where the effect of the mean normal stress is accounted for through the parameter $N_{a,eq}$

$$N_{a,eq} = N_a + \sigma_{af,-1} \left(\frac{N_m}{\sigma_u} \right) \tag{10}$$

with σ_u = ultimate tensile strength. Eq. (10) is based on the well-known linear interaction between normal stress amplitude and normal stress mean value (diagram of Goodman [16]), which can be written through the components of the stress $N(t)$ acting on the critical plane, as is reported in Eq. (10).

For fatigue strength assessment at finite life, Eq. (8) can be applied by replacing N_{max} with:

$$N'_{a,eq} = N_a + \sigma_{af,-1} \left(\frac{N_f}{N_0} \right)^m \left(\frac{N_m}{\sigma_u} \right) \tag{11}$$

The original C-S criterion (with the weight function in Eq. (2)) and the modified one (with the weight function in Eq. (3)) have been applied to some data of experimental biaxial in- and out-of-phase synchronous sinusoidal fatigue tests [12,13]. The quality of the criterion results under fatigue limit conditions can be evaluated through an error index, $I_\sigma = \left(\sigma_{a,eq} - \sigma_{af,-1} / \sigma_{af,-1} \right) \cdot 100\%$. The comparison is quite satisfactory (see details in Refs [4,6]): for example, for specimens made of Swedish hard steel 982 FA [4,6,12], the error index ranges from -12% to 13% and from -7% to 13% by applying the original and the modified criterion, respectively. For specimens made of low carbon steel C=0.35%

[4,6,13], the error index ranges from -1% to 16% and from to -10% to 12% by applying the original and the modified criterion, respectively.

Extended formulation for notch specimens under multiaxial constant-amplitude cyclic loading by using the critical-point method

An extension of the criterion presented in Refs [3,4] has been proposed in Ref. [7] to estimate the fatigue limit of notched structural components under multiaxial constant-amplitude cyclic loading, by using the critical point method [17,18].

Consider a traction-free notch surface in a body submitted to a periodic fatigue loading. At any point on the notch surface, one of the principal stresses is always null and its direction is normal to the notch surface. The point H (the so-called hot spot) of crack initiation on the notch surface is assumed to be that point experiencing the maximum value of $\sigma_{eq,a}$ (Eq.(7)).

According to the weight function in Eq.(2), only the time instants related to $\sigma_1 > \sigma_{af,-1}/2$ are taken into account in the averaging procedure and, therefore, the averaged principal stress directions at any point on the notch surface are represented by the axis $\hat{1}$, tangent to the notch surface, and the axis $\hat{3}$ normal to such a surface. The orientation of the critical plane at point H is computed by considering the off-angle δ (Eq.(4)). Then, it is assumed that such an orientation does not change up to point P (critical point) which is at a distance L/2 from the notch surface, where L is the material characteristic length [19] given by:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\sigma_{af,-1}}\right)^2 \qquad (12)$$

ΔK_{th} being the threshold range of the stress intensity factor for long cracks.

The equivalent normal stress $\sigma_{eq,a}$ (Eq.(7)) is computed from the stress tensor (obtained from a linear-elastic analysis) at point P, bearing in mind that it is assumed that the critical plane orientation does not change moving from H to P.

The procedure proposed, by employing the original C-S criterion and the weight function in Eq. (2), is applied to some experimental data related to round bars with artificially drilled surface holes, subjected to fully reversed bending or torsion [20]. The quality of the criterion results under fatigue limit conditions can be evaluated through the above error index, I_σ. The comparison, by varying the value of the hole diameter, is quite satisfactory (see details in Refs [7,20]): for specimens made of 0.46%C annealed steel under bending, the error index ranges from -3% to 20%, whereas it ranges from -12% to 10% under torsion.

Extended formulation for smooth specimens under random loading

In the case of random loading, the scalar value of the vector $\mathbf{N}(t)$ is taken as the cycle counting variable, since the direction of such a vector is fixed with respect to time, while $\mathbf{C}(t)$ describes an open path. For the sake of simplicity, $\mathbf{N}(t)$ and $\mathbf{C}(t)$ can be treated as discrete variables if the sampling frequency is greater than the highest frequency component of the applied loading history [8]. Firstly, the sequence N_i is reduced by eliminating the time instants corresponding to non-extreme values, and a peak/valley sequence N_j^* is obtained.

The same time instants just eliminated in the sequence N_i are also eliminated in the sequence $\mathbf{C}_i$, and a new sequence $\mathbf{C}_j^*$ is determined by carrying out a reduction procedure which preserves the maximum values of the shear stress amplitude, as is now described. The sequence $\mathbf{C}_i$ is treated as follows. Let i and $i + K$ be the generic time instants corresponding to two successive extreme values of N_i. For a two-value discrete sequence, i and $i + k$ with $k = 1, 2, \dots, K$, the mean value $C_m^{(i,i+k)}$ and the amplitude $C_a^{(i,i+k)}$ of the shear stress are computed for the two vectors $\mathbf{C}_i$ and $\mathbf{C}_{i+k}$ (details are reported in Ref. [8]).

Then, the vector $\mathbf{C}_{i+\bar{k}}$ is retained in the new sequence $\mathbf{C}_j^*$, where $\bar{k}$ is the value of k (with $k = 1, 2, \dots, K$) for which $C_a^{(i,i+k)}$ attains its maximum (i.e. the maximum value between $C_a^{(i,i+1)}, C_a^{(i,i+2)}, \dots, C_a^{(i,i+K)}$).

At the end of the above reduction procedure, we can determine the maximum value $N^*_{max,z}$ for the z-th resolved reversal through the cycle counting of the variable N^*_j, by using the rainflow method [21]. Moreover, the amplitude $C^*_{a,z}$ is obtained from the sequence $\mathbf{C}^*_j$, where now i and i + k are related to the time instants defining the range of the z-th reversal. Then, the z-th amplitude of the equivalent normal stress $\sigma_{eq,a,z}$ is expressed by Eq.(7) by replacing N_{max} with $N^*_{max,z}$ and C_a with $C^*_{a,z}$.

Using a nonlinear cumulative damage rule for $\sigma_{eq,a,z}$ [22], the total damage at time instant T_0 is computed as follows :

$$D(T_0) = \sum_{z=1}^{Z} (D_z)^q \quad \text{with } D_z = \begin{cases} \dfrac{1}{2N_0\left(\dfrac{\sigma_{af,-1}}{\sigma_{eq,a,z}}\right)^{-\frac{1}{m}}} & \sigma_{eq,a,z} \geq 0.5\sigma_{af,-1} \\ 0 & \sigma_{eq,a,z} \leq 0.5\sigma_{af,-1} \end{cases} \qquad q = 1 + \frac{0.25}{0.5\sigma_{af,-1}}\left(\sigma_{eq,a,z} - \sigma_{af,-1}\right) \tag{13}$$

The symbol Z refers to the total number of reversals (of N^*_j) determined through the rainflow method at time interval T_0. Hence, the calculated fatigue life of the structural component is given by:

$$T_{cal} = T_0 / D(T_0) \tag{14}$$

The procedure proposed is applied to some data of fatigue tests on round specimens made of 10HNAP steel, subjected to a combination of random proportional bending and torsion [23]. The comparison between the calculated and the experimental fatigue life is quite satisfactory, as is detailed in Ref. [8].

CONCLUSIONS

In the present paper, the evolution of a high-cycle multiaxial fatigue criterion based on the critical plane approach, known as the C-S criterion, has been discussed. It has been shown as the criterion, originally formulated for smooth specimens under constant-amplitude cyclic loading, it is able to assess the structural integrity even in the case of notched specimens under constant-amplitude cyclic loading and smooth specimens under random loading, by introducing appropriate devises and changes to the original formulation. The results obtained by employing such a criterion are generally in good agreement with the experimental data collected from the relevant literature, for different specimen geometries and fatigue loadings.

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