Effective stress parameters close to stress singularities

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ABSTRACT. The effective stress values have been defined, in order to find a measurement of the stress field directly related to the initiation of a failure mechanism, usually a fracture or fatigue crack initiation. This paper investigates several definitions of effective stress values and applies them to different stress parameters, as well as stress invariant or strain energy density components. In particular, this work examines the effective values not only at the tip of a stress raiser, but also in the surrounding material. The final aim is to understand the relationship between the actual distribution of effective values and the real failure initiation point, i.e. to verify if the location of maximum values of effective stress parameters is actually the position of the failure initiation.

KEYWORDS. Crack; Notch; Effective stress; Fatigue; Fracture; Stress parameters.

INTRODUCTION

Effective stress parameters are often used for the estimation of fracture strength or fatigue crack initiation at stress concentrations. Their aim is to make it possible to define a physical quantity which could be directly related to the strength of a structural detail. The problem is the well-known "notch sensibility", so that the mechanical behaviour of components is not directly related to the maximum values of stress or strain at the notch tips, but it is somehow dependent on the overall stress distribution, depending on the material and geometrical properties [1-7]. Therefore, effective values should at least consider: overall stress intensity, stress gradient effect, multiaxiality effect and, if necessary, size effect, material isotropy or homogeneity.

Effective stresses are defined in several different shapes:

- large effective stress values are defined as the maximal values, weighted by a coefficient lower than one, according to the gradient and material properties [1];
- another set of effective values is the critical distance values of stress field parameters, i.e. stress values are somehow considered at a given distance from the tip [3-4];
- related to previous definitions, the average values or the average quantities are evaluated in a large area close to the tip [2-7];
- finally, several effective values can be calculated by defining suitable sets of partial differential equations all over the components or close to the critical point [8-10].

Apart from the problem of the effective value description, the other critical aspect is the definition or the choice of the proper stress parameter to be considered.

In mainly uniaxial loading, strength is traditionally related to the principal stress intensity. However, under multiaxial loading, particularly in multiaxial fatigue, more parameters should be considered in order to take all variations of the stress tensor components into account; frequently chosen parameters are principal stresses or stress invariants, critical plane parameters (i.e. shear or normal stress on critical plane), or strain energy densities (total, deviatoric or hydrostatic component) [11-14].

One of the most positive applications of these approaches is the possibility to assess the fatigue strength in any kind of geometrical stress raiser and the opportunity to take advantage of a FE modelling by evaluating previously defined stress parameters all over the investigated structural component. Therefore, it could be possible to investigate the whole body of a structural component by evaluating the effective values of stress parameters all over the component. Under this assumption, it turns out that the critical point of an investigated component should be where effective values assume their maximal intensity [9]; for this reason, the failure location is not assumed or known "a priori", but turns out from the investigation [10].

In the authors' opinion, this is a very appealing possibility for real applications, but from a methodological point of view, several preliminary tests are necessary to verify if effective stress parameters are actually maximal at the failure location. A first feature that we would like to check is the relationship between the actual position of maxima of effective values and the expected critical position and failure path in a known geometrical stress raiser.

In these investigations, an important role is played by stress singularities. The linear elastic stress field at the cracks is generally used as a reference point for the computation of critical distances and material parameters. For this purpose, the stress parameters, evaluated at, or close to, the tip, are compared with parent material properties. For this reason, the most interesting stress raiser to be investigated is the sharp crack under simple loading modes; i.e. mode I or II in sharp cracks, under plane stress or plane strain conditions.

In other words, according to the authors, in order to generalize effective stress evaluations all over complex structural components, it is not sufficient to know effective stress parameters at the tip, but it is necessary to accurately examine their values in the whole field surrounding the tip.

This paper will discuss these properties by considering different effective stress definitions and stress field parameters at the tip and close to the tip of a crack under mode I and mode II loadings, both plane stress and plane condition are investigated.

GENERAL DEFINITIONS OF EFFECTIVE VALUES AT NOTCHES

In literature the problem of the effective stress definition at notches has been addressed by several authors [1-10]. In the following a short selection of the most important definitions are given, as well as the more recent outcomes from literature. This review mainly focuses on effective stress values defined for the fatigue strength assessment of notches.

Singular definitions at linear elastic singularities

Some definitions of effective stress values, and particularly the oldest in literature, are mainly a correction of the peak stress at the tip. Just to recall these definitions, they are usually defined as:

$$\sigma_{eff} = \sigma_{peak} \cdot a \qquad (1)$$

where σ_{peak} is the maximum stress at the tip, usually investigated under linear elastic assumption, and "a" is a weight, lower than one, dependent from the stress gradient and parent material properties. For instance, in the stress gradient approach [15] the effective value of stress is the peak value reduced according to the intensity of the stress gradient, i.e. the derivative of the main stress at the tip. Similarly another well-known definition is Highly-stressed-Volume definitions [16] where the weight depends on V_{90}, which is the volume of the material subjected to a stress value higher than 90% of the peak value. The main consequence of these definitions is that their application is not possible at the crack tip as it is the maximal value, under the linear elastic assumption, a singularity of the stress field, therefore, they will be not considered in the following.

Note that in these approaches the problem of a multi-parametrical investigation is not considered and the proposed effective values only take the main principal stress into account.

Average values and densities

According to Neuber's original insight [2], relating the fatigue behaviour of a component to an average value and not to the peak value, several definitions have been made of an effective value of stress based on the average integral in front of a notch tip. Starting from this statement, differences and difficulties arise from the parameter to be averaged and also from the integration field choice. Linear integrals are often used, but this paper aims to investigate methods applicable to complex three dimensional components by means of FE method, where, in the authors' opinion, the most interesting definitions are those considering area or volume integrals. Considering area-average, we define the effective stress $\sigma_{eff,av}$ at a point P, as:

$$\sigma_{eff,av} = \frac{1}{A}\int_A \sigma \, dA \tag{3}$$

where "σ" is a stress parameter and "A" is a generic geometrical domain; one of most simple domains, suitable along the surface or free edge on plane problems, is the semicircular one, centred in the investigated point and with a generic radius R.

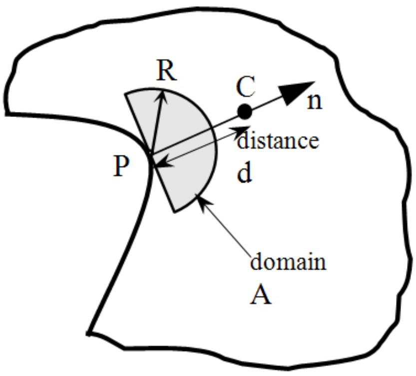

Figure 1: Definitions of integration domain "A" and critical distance "d" for the effective stress evaluation at a surface point P.

Critical distance definitions

Similarly to the previous effective stress definition, the critical distance effective stress definition relates the strength to the stress value close to the investigated point. We assume the following definition:

$$\sigma_{eff,d}(P) = \sigma(C) \tag{4}$$

where point "C" is at a distance "d" from the investigated point "P" along the perpendicular to the surface "n".

Implicit gradient effective value

More recently, specifically for numerical applications, effective stress has been defined by means of a partial differential equation; the used equation is the so-called "implicit gradient" definition and turns out in [9,10]:

$$\sigma_{eff,ig} - c^2 \nabla^2 \sigma_{eff,ig} = \sigma \tag{5}$$

where "c" is a material constant and the partial differential equation will be solved by assuming the Neumann type boundary condition: $\nabla \sigma_{eff,ig} \cdot n = 0$.

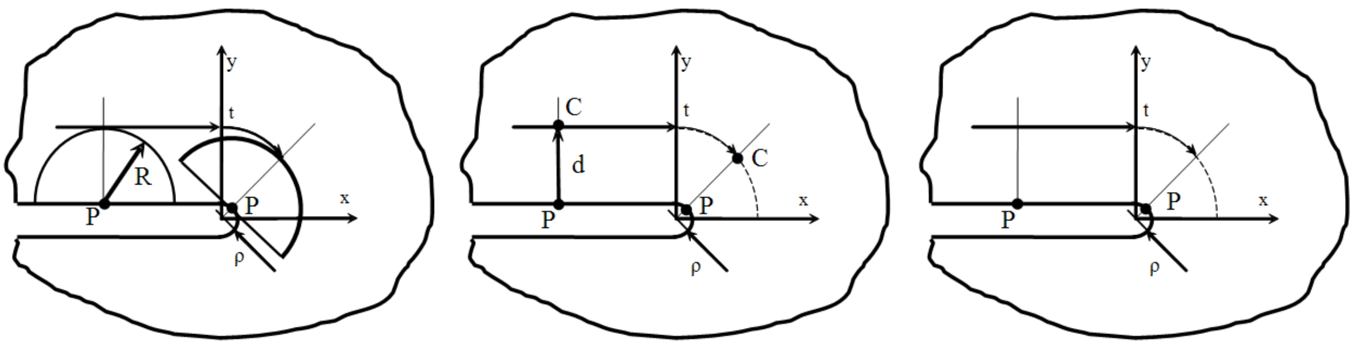

Figure 2: Notch frame of reference for effective stress evaluation along the free edge: average value on the left, critical distance in the middle and implicit gradient on the right.

VALUES OF EFFECTIVE STRESS PARAMETERS IN FRONT OF A SHARP NOTCH TIP

The main aim of this paper is the comparison of the properties and variability of the different effective stress definitions along the surface, or free edge, close to a sharp notch. Note that, if a sharp crack is considered, next to tip, there is a discontinuity in the definition of the surface and its normal. To overcome this problem a regular and continuous notch edge (i.e. a border that is a local function of class C^1) will be used; hence a simple assumption is taken: let us consider a rounded "U" notch, but with a notch tip radius much smaller than the material parameters used in the effective stress definition; i.e., referring to Fig. 2, notch tip radius "ρ" will be considered smaller than radius "R" and distance "d".

By so doing, it turns out that the effective stress evaluation is continuous along the edge and the obtained values are almost independent of the notch tip radius. Hence the values presented in this work are, in fact, suitable for stress singularity comprehension.

Considering the loading mode, when the following in-plane stress conditions are considered, the Mode I and Mode II loadings are separately evaluated. For the sake of simplicity, in numerical estimations, an external loading inducing unitary SIF is considered. Moreover, concerning the multiaxial effect, both plane stress and plane strain condition have been taken into account.

Considering the stress field parameters, this work will focus on stress components and strain energy components, by individually evaluating their global intensity and even by separating the hydrostatic and deviatoric components [17]. The following physical quantities are taken into account:

- W strain energy density
- W_H hydrostatic strain energy density
- W_D deviatoric strain energy density
- Σ maximum principal stress
- Σ_H hydrostatic stress component
- Σ_D maximum shear stress

This selection has been done according to the main part of uniaxial and multiaxial fatigue strength criteria, which is mainly based on these quantities [11, 13].

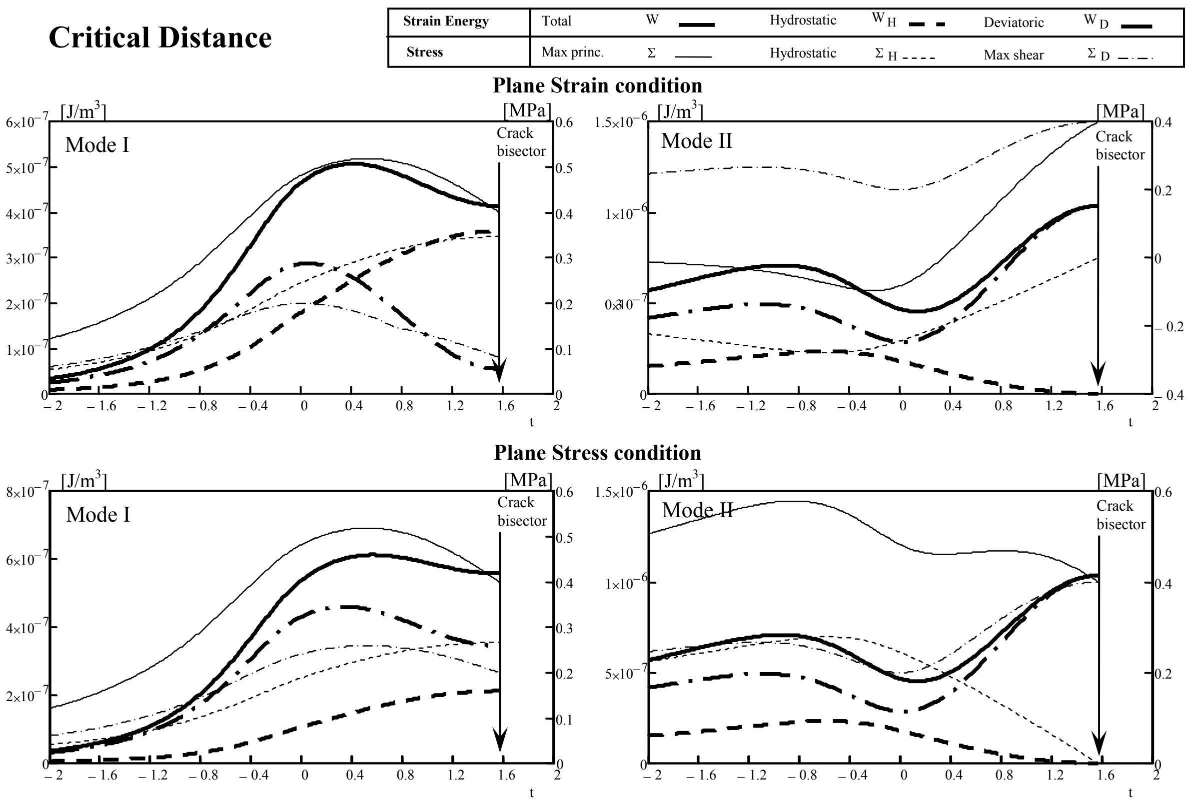

Figure 3: Effective values of stress parameters: free edge, according to critical distance approach.

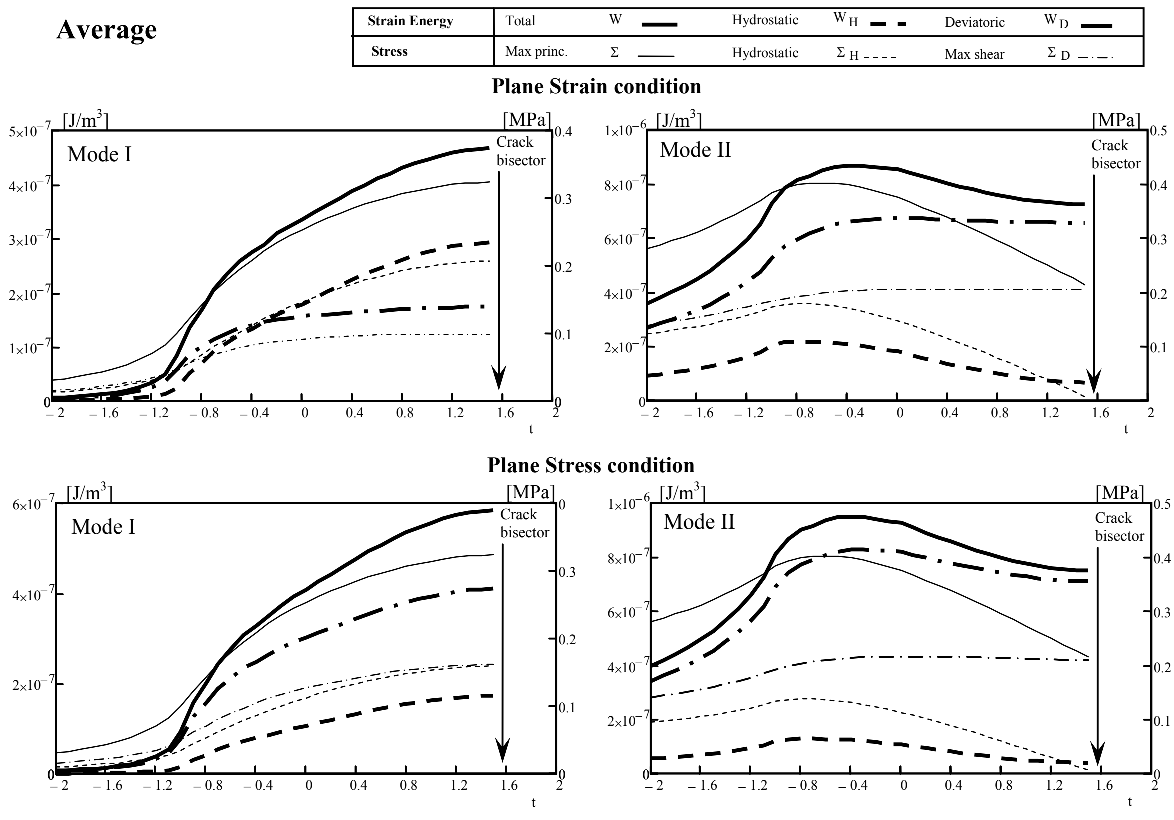

Figure 4: Effective values of stress parameters: free edge, according to average approach.

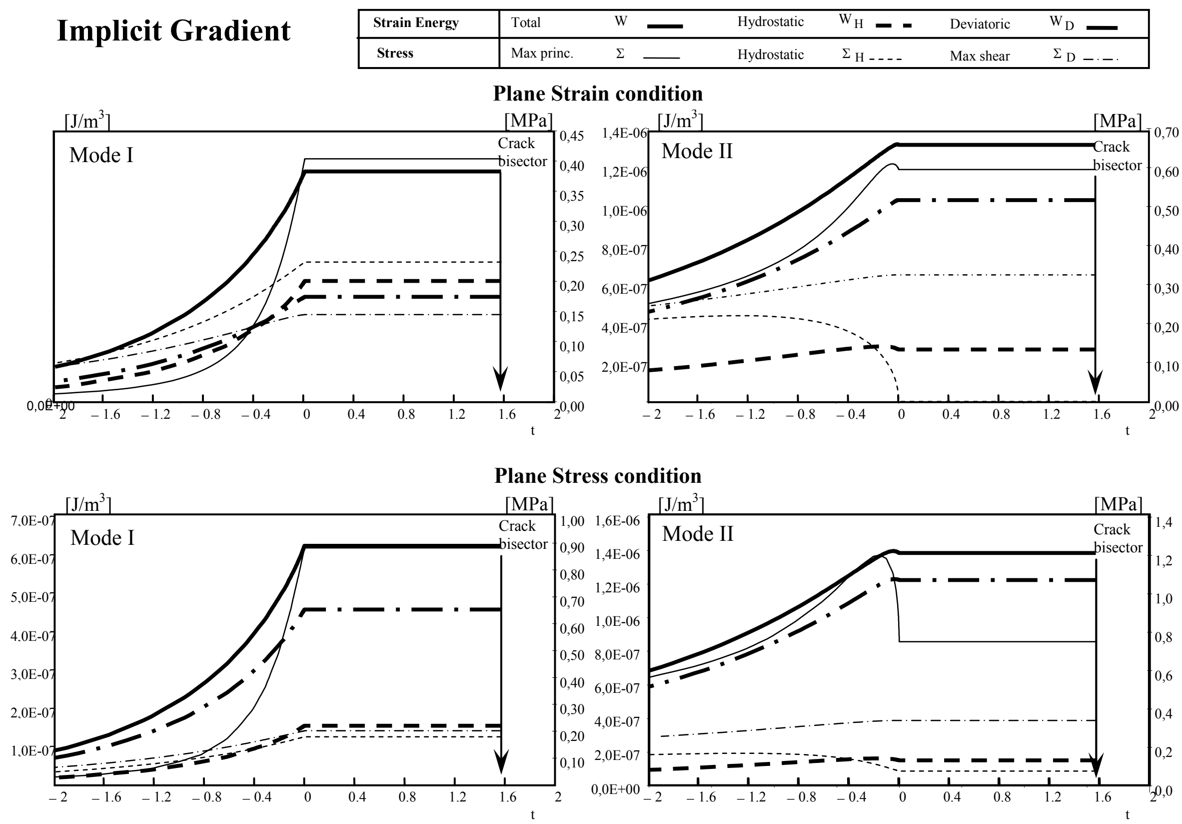

Figure 5: Effective values of stress parameters: free edge, according to implicit gradient approach.

A numerical estimation has been calculated by considering the material constants equal to one, i.e. “R” in average quantities, “d” in critical distance methods and “c” in the implicit gradient approach. In the mechanics of materials, relationships among these quantities, providing appropriate estimations, have been proved different and dependent on specific criterion and parameters; nevertheless, in this paper the material parameters have been arbitrarily assumed equal. The result is that the following estimations will not be considered for a quantitative comparison of strength assessments, but simply for a qualitative investigation on relative trends.

Free edge investigation

In Fig. 3, the effective values of the critical distance approach are given for mode I and II loadings, under plane stress and plane strain assumptions. In the same way, in Fig. 4, corresponding values are given for the semicircular average approach. Note that the abscissa of these graphs is the curvilinear coordinate “t” (t will be dimensionless because it is relative to distances “R” and “d” which are unitary) depicted in Fig. 2. Such a coordinate is a straight distance for the negative value, and it is an arc for positive values, hence the tip of the notch is at $+\pi/2$. Unlike previous cases, for the implicit gradient approach, the effective stress value is constant at the notch root, since the effective value is independent from the tip radius, when the radius is sufficiently smaller than “c”. Moreover, the obtained value is exactly the same when obtained at the tip of a corresponding crack, i.e. the zero radius notch. Obtained results are given in Fig. 5.

Inner points investigation

In the authors’ opinion, a further possible investigation is to apply the effective stress definitions, not only at the free surface, but also inside the component. In this case the critical distance approach cannot be used, being strictly related to a free surface relative distance. The implicit gradient is defined all over the body, therofre, there is no problem to evaluate related effective stress anywhere; the average stress definition will be modified for internal evaluation because the perpendicular to the surface is no longer available and the semicircular domain is not suitable.

In the following, the circular area definition will be used for the average stress definition, see Fig. 6. Note that even in this case the radius is reported in the figure, but it has been considered very small and is in fact negligible.

The analysis of the effective stress in the internal position is particularly interesting along the bisector of the notch. Figs. 7 and 8 show the average values of proposed parameters and the implicit gradient values respectively, both are evaluated along the bisector under mode I and II in the plane stress and plane strain condition (in Figs. 7 and 8, x is a dimensionless distance).

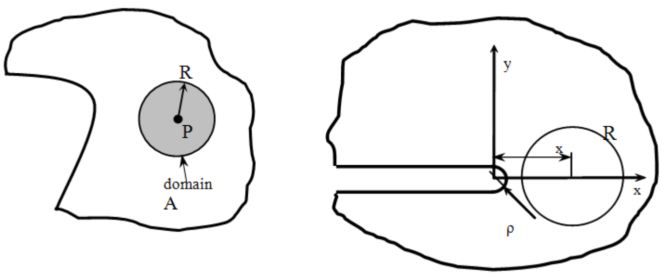

Figure 6: Definitions of integration domain “A” and critical distance “d” for effective stress evaluation at surface point P.

DISCUSSION

Several thoughts could arise from an accurate investigation of the trends shown in the given pictures. We would like to give a brief and simple description of a few aspects:

- under mode I, the critical distance approach shows several local maxima far from the tip, i.e. far from the expected initiation point; this is particularly valuable for deviatoric components, shear and principal stress;
- under mode II; each proposed effective stress definition shows complex distributions, several parameters present different local maxima, even along the straight edge; it is worth noting that under this loading mode even the actual failure initiation point is not always clearly predictable;
- the criteria suitable for a definition inside the material, not only the free edge, show local maxima just behind the actual tip; this aspect is more evident in the average approach compared to the implicit gradient;
- in any case, apparently different parameters have similar trends; for instance the deviatoric strain energy density and the max shear value are similarly distributed in almost every combination.

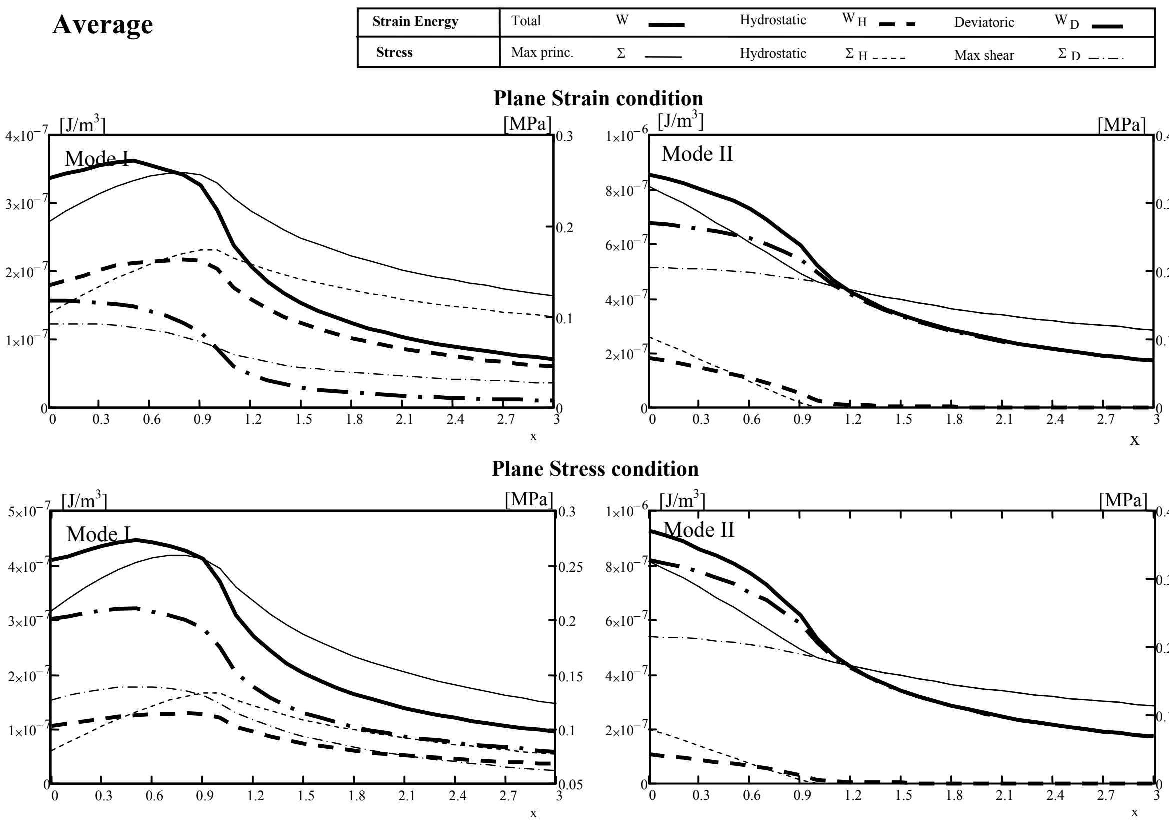

Figure 7: Effective values of stress parameters: notch bisector, according to average approach.

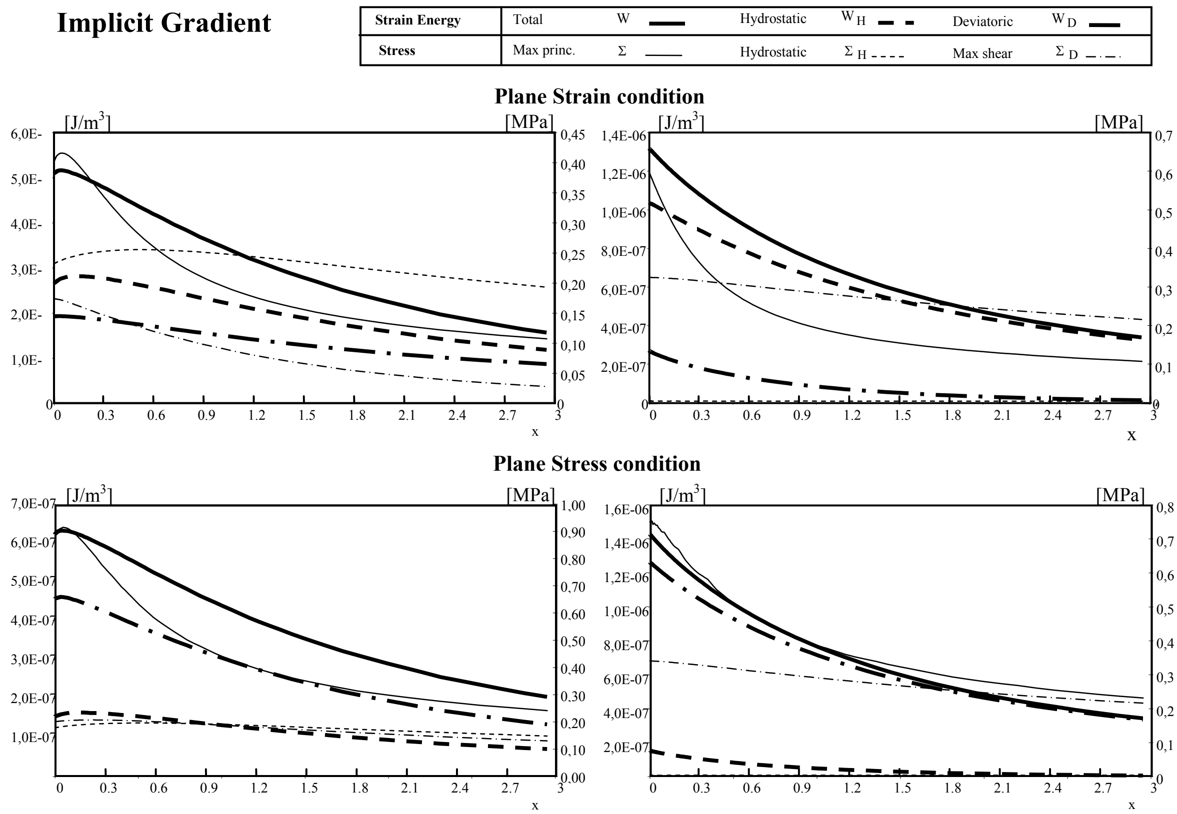

Figure 8: Effective values of stress parameters: notch bisector, according to the implicit gradient approach.

CONCLUSION

This paper has dealt with the problem of the effective value distribution at sharp notches of several stress and energy parameters. The main problem addressed is the possibility of using the effective stress value not only to assess the strength of a known crack initiation point, but also to estimate the crack initiation position in a geometrically complex component. For this investigation, as a starting point, a simple crack-like sharp notch has been considered.

The outcome is as follows: there are several combinations of the effective value definition and stress parameters, often efficiently used, which have higher values far from the actual initiation point. In other cases, when the position of the actual initiation point could be questionable, the effective value trends are often complex. In the authors' opinion, these aspects should be carefully considered in all extensive applications of the effective stress definition, particularly in Finite Element analysis.

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