The crack modelling method and critical distance approaches

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ABSTRACT. This paper looks back over our work of the last fifteen years on two methods for the analysis of fatigue and fracture at stress concentration features: the Crack Modelling Method (CMM) and the Theory of Critical Distances (TCD). We describe the general principles and philosophy of this work, which was envisaged primarily as a solution to an engineering problem, using finite element analysis to estimate the behavior of bodies of arbitrary geometry and loading. The general principles of the two methods are described along with some discussion of their relative merits and applicability. The CMM is essentially a one-parameter method (equivalent K value) and so has some limitations which the TCD does not, since it uses at least two parameters (length L and equivalent stress) or three if multiaxial conditions are considered. We have extended the TCD into its most general form, including finite life fatigue under mutiaxial and variable amplitude conditions, and can now confidently advocate its use in industrial design.

KEYWORDS. Fatigue, fracture, crack, notch, stress-concentration, critical distance

THE PRESENT STATE OF FRACTURE MECHANICS

The subject of fracture mechanics made great advances in the 1960s and 1970s with the development of the key parameters of K, J and CTOD which allowed engineers to quantify the phenomena of brittle fracture (defined as failure by crack propagation, whatever the local mechanism of material separation) and fatigue crack propagation. Thanks to these theoretical advances it became possible to predict failure loads and times, at least when the conditions for linear elastic fracture mechanics (LEFM) were met, and to rank materials according to their toughness and fatigue crack growth resistance. The 1980s saw further advances, especially in our appreciation of the underlying mechanisms of crack growth at the microscopic scale. In particular, the discovery of crack closure revolutionised our thinking about near-threshold fatigue crack propagation, and the anomalous behaviour of short fatigue cracks was discovered and investigated. Since then, not much has happened. A researcher falling asleep in 1989 and waking up again today would be able to understand most of the published literature. Why is this? We suggest two reasons for the slow advance of the subject in recent years: (a) the engineering problem has largely been solved, and; (b) the barriers to further scientific advance are considerable. On the engineering side, we can predict failure when it occurs by cracking in most practical situations, at least to the point where our prediction errors are small compared to the errors caused by other limitations of engineering design and practice. The main limitations today are threefold: firstly, non-destructive inspection (NDI) puts a limit on the size of the smallest crack which can be reliably detected in a component. This limit was a few millimetres in the 1980s...and it's still a few millimetres. Whether this lack of progress in NDI is due to real physical limitations or to a lack of research funding is unclear. Secondly, when carrying out a stress analysis on a real structure or component there are significant uncertainties about the applied loads and other boundary conditions such as restraints, friction at contacts, etc. In practice this means that we are unlikely to know the exact stress at a given point in the body with an accuracy of better than 50%. Thirdly, there is often a lack of test data for the material being used. This last problem is simply an economic one: companies are reluctant to pay the few thousand Euro needed to measure the fatigue behaviour of a new material and so ask us to estimate this behaviour based on monotonic material properties, which is of course not possible with any reasonable accuracy.

On the scientific side, further progress requires us to delve into the physical processes which occur at the sub-micron to nano-scale. This requires large-scale computer modelling of failure from the atomic level upwards and the integration of failure processes at the various different length scales, supported by extensive experimental work. Such activities are now feasible – we possess the computing power and much of the basic physics to tackle the job – and some researchers have made significant progress in this area in recent years. However the amount of funding available for this kind of work is small; industry does not perceive the need to do it (and probably they are right, for the reasons given above) and national governments have other priorities for basic research funding.

OUR PHILOSOPHY

In the present contribution, we confine ourselves to the engineering problem. Here we look back over work which we carried out during the last fifteen years, aimed at improving predictions of fatigue and brittle fracture for engineering components containing stress concentrations. Our work has been guided by a number of general principles, as follows:

1) We addressed ourselves to engineering components rather than to test specimens. In practice this meant that we only considered predictive methods which could be applied to bodies of arbitrary shape and size, subjected to arbitrary loadings, containing stress concentration features of arbitrary geometry.

2) A natural consequence of this first principle was that we developed methods which took as input only the stresses and strains in the body. Other parameters such as crack length, notch root radius etc were not used because they could not be defined unambiguously in the general case.

3) We assumed that cracks, notches and other stress concentration features could be considered in the same way, in particular that cracks are not a special case. This assumption is rather questionable, since it is known that different results can be obtained from, for example, a specimen containing a pre-crack induced by fatigue, and an otherwise identical specimen containing a sharp, machined notch. These differences arise due to history effects: the crack may have developed closure across its faces, the machining of the notch may induce residual stresses, etc. History effects have turned out to be so problematic that standards for fatigue crack threshold testing developed by ASTM and other national bodies have given rise to inaccurate results. In principle this problem is dealt with by point (2) above: these effects will alter the cyclic and mean stresses in the material ahead of the crack or notch. In practice we find that cracks are actually not an issue in the great majority of industries, where concern focuses on design features (holes, keyways etc) and defects (pores, inclusions). Apart from major exceptions such as the aircraft and offshore oil industries, which rely on NDI to maintain structural integrity, most engineers are not faced with components containing definable cracks whose effect has to be estimated.

4) As a result of point (3) above, we chose to measure material behaviour using test specimens containing notches rather than cracks. Thus we measure the fatigue threshold ΔK_{th} and toughness K_c using sharp notches rather than pre-cracks. This avoids the difficulties and uncertainties of carrying out standard fracture mechanics tests.

5) We assumed the existence of an accurate stress analysis of the body in question. Since we wished to investigate arbitrary geometries and loadings (point (1) above) this implied the use of finite element analysis (FEA) or other computer-based methods. Thus we always developed methods which could be interfaced to post-process FE data.

6) We invariably began by using linear elastic analyses. This was partly because we expected many problems to be solvable in this way, for the same reason that LEFM works despite the presence of plasticity at the crack tip. Also, whilst non-linear material modelling is now commonly used in FEA, linear models are still the norm in industry due to their convenience and speed of processing. Inevitably we found that we had to consider elastic/plastic behaviour in some cases, such as low cycle fatigue and monotonic fracture of tough materials, but this was done only after we had demonstrated that linear elastic analyses were insufficient.

7) Finally, we had to set ourselves a target as regards the accuracy of our predictions. We decided that we would be satisfied if our prediction errors were less than 20% on stress, reasoning that errors of the order of at least 10% arise due to uncertainties in material properties, stress analysis etc, so a method with less than 20% error would always be good enough in an engineering context. In cases where there was enough experimental data to accurately define the scatter, this was also used as a goal, since obviously one cannot predict with better accuracy than that.

We developed two methods, known as the Crack Modelling Method (CMM) and the Theory of Critical Distances (TCD). The central feature of both these methods is the comparison of stress fields in the region immediately ahead of the stress concentrating feature. In what follows we will use the term "notch" for convenience, but this should be taken to refer to any feature which gives rise to a local stress concentration, be it a notch, crack, point of contact between two bodies, etc.

The essential problem is that two stress fields, from two different features, will always be different. Consider the stress present on a line starting at the point of maximum stress, which will usually be on the surface (Fig.1). The stress σ varies with distance r, usually decreasing with increasing r (though not necessarily if plasticity occurs). If the feature has a zero root radius (a crack or infinitely sharp notch) then a linear elastic analysis will predict infinite stress at $r=0$. We can postulate that if the two stress fields are identical then the behaviour will be identical, giving the same static strength or the same fatigue life for the two cases. But of course in practice the two stress fields will never be exactly the same at all distances r. What this implies is that we must identify some properties of these stress fields, some parameters derived from them, which can be compared in order to compare behaviour. This approach (though not necessarily stated in these terms) is implicit in any fracture mechanics methodology. In the rest of this paper we will describe the two different forms of stress-field comparison which we developed.

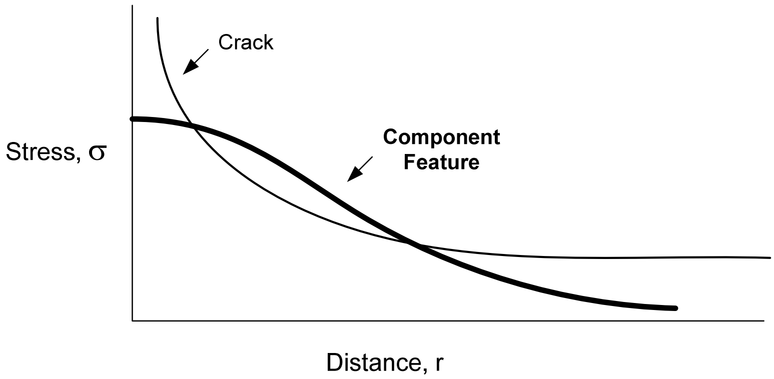

Figure 1: Stress as a function of distance from the stress-concentration point, for a typical crack and a typical component feature. The stress fields are not identical but the behavior (in monotonic loading or in fatigue) may nevertheless be identical. So what aspects of these two stress fields are important?

THE CRACK MODELLING METHOD

The CMM takes as its starting point some work published by Smith and Miller in 1978 [1], who made a very useful discovery about the behaviour of notched specimens in high-cycle fatigue. The value of this work has not, in our opinion, been sufficiently recognised even today. By plotting the fatigue limit as a function of the stress concentration factor K_t, they showed that all notches could be divided into one of two categories, which we can call "blunt notches" and "sharp notches". Blunt notches have low K_t values: they reduce the fatigue limit by a factor equal to K_t. Sharp notches behave like cracks and so can be analysed by calculating the value of ΔK for a crack of the same length. An example is given in Fig. 2. The prediction line labelled "Stress-life prediction" simply divides the plain-specimen fatigue limit by K_t, whilst the "LEFM prediction" here is a horizontal line because all notches tested had the same length and therefore corresponded to the same equivalent crack. The data points fall on the highest of these two prediction lines. We realised that this approach is extremely useful but has a major limitation when applied to engineering components: how can one determine the stress intensity of the equivalent crack if the feature is not a simple notch of known depth? Our solution to this problem is illustrated in fig.3. We compared the actual stress distribution for the component to the stress distribution for an ideal crack. Arbitrarily, we chose as our model a centre crack in plate under uniform tension. One can then vary the parameters of the model (applied stress and crack length) until the two stress distributions are as similar as possible. The stress intensity of the model crack is then used as the equivalent K value for the component feature.

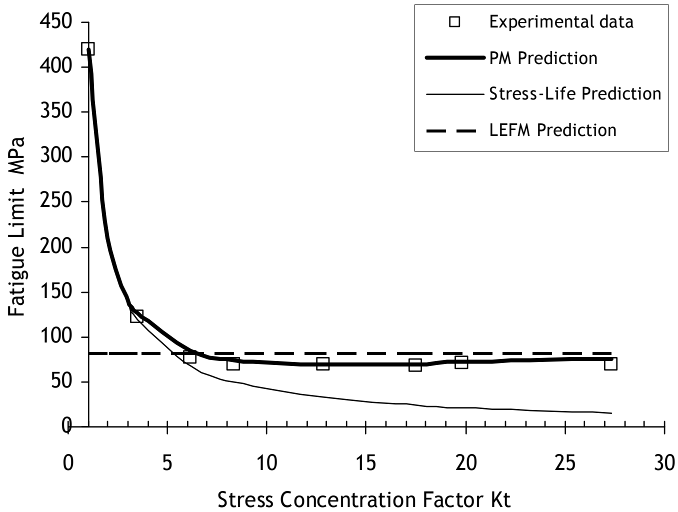

Figure 2: Data and predictions on the fatigue limit of notched steel specimens, showing predictions using Smith & Millers's approach and also using the TCD (Point Method). Experimental data from Frost *et al* 1974 [2].

This approach turned out to be accurate in many cases [3] and was applied to some component problems with success [4, 5]. We have found that, though the number of citations for these papers is modest, the method has been adopted by many engineers in industry, faced with FE data showing very high local stresses resulting from features with sharp radii. These stresses may in some cases actually occur, if very sharp corners etc exist on the component, but in many cases they arise due to simplifications in the FE model, which has been made without inserting fillet radii. We showed that the approach could be usefully employed in both of these situations, though in the latter case some checks are needed to make sure that the actual feature still falls within the "sharp notch" category.
In the example shown in Fig. 2 the transition from blunt to sharp notch occurs at a K_t value of about 5. Depending on material properties and notch geometry this can occur at even lower values, sometimes less than 3. This means that there are many notches which visually appear quite blunt and which have stress fields very different from those of cracks, but which nevertheless behave exactly the same as cracks in high-cycle fatigue situations. (The same will be true for brittle fracture provided LEFM conditions prevail, though admittedly we have not applied this method to brittle fracture problems.) This information is extremely useful for industrial designers contemplating changes to component features. The lesson to be learnt is that increasing root radius is not always going to be beneficial.
The CMM is essentially a one-parameter method, since its output is an equivalent K value. It thus suffers from the same limitations that arise in traditional LEFM. For example it cannot predict the size effect (short fatigue crack effect) or the effect of the T-stress. However it can be easily extended to consider multiaxial loadings using standard LEFM approaches.

THE THEORY OF CRITICAL DISTANCES

In 1999 one of the authors (David Taylor) published a paper [6] describing an approach to high-cycle fatigue of notches which we would later come to call the Theory of Critical Distances (TCD). At the time, he believed the analysis described in that paper to be original (and presumably the reviewers did also) but it soon emerged that the work had all been done before by others. This paper proposed two things:

(a) That the fatigue limit of a notched specimen could be predicted using the stress at a single point, located a certain distance from the notch root, or alternatively using the average stress over a certain distance, both of these distances being constant for a given material. These two approaches, which we call the point method (PM) and line method (LM) are illustrated in Fig.4.

(b) That these critical distances could be calculated from first principles, knowing the material's plain-specimen fatigue limit and its crack propagation threshold. This is achieved by noting that the approach, if it applies to all notches, must also apply to a crack, for which the stress field is already known. The result is that the critical point lies at a distance L/2 from the notch, and the critical line has a length 2L, where L is a function of the material's threshold ΔK_{th} and fatigue limit $\Delta\sigma_o$, as follows:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta \sigma_o}\right)^2 \tag{1}$$

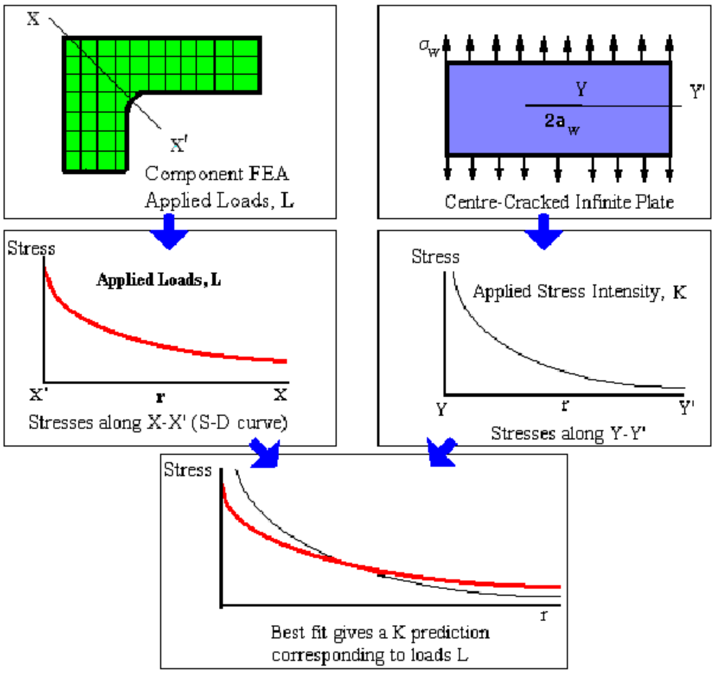

Figure 3: A summary of the Crack Modelling Method: stress/distance curves are compared for the actual component (on the left) and a model crack geometry (on the right). Minimising the difference between the curves gives the equivalent stress intensity for the component feature.

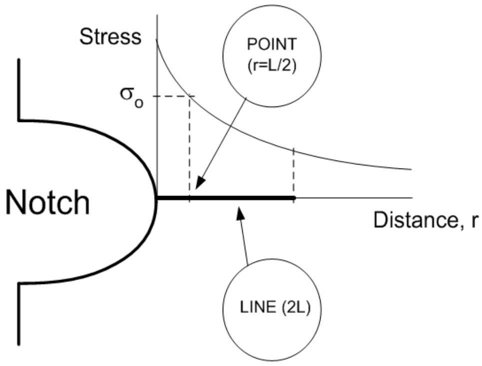

Figure 4: Schematic illustration of the principles of the TCD, point and line methods.

Proposition (a) was originally made in the 1930s-1950s by Neuber [7] (who discovered the LM) and Peterson (who modified it to the PM [8]). Proposition (b) requires one to make a connection between Neuber's method and LEFM; this was first reported for fatigue in a short theoretical paper by Tanaka in 1983 [9] though in fact the same theory had already been developed for brittle fracture of composite materials as early as 1974 by Whitney and Nuismer [10]. However Taylor's 1999 paper can at least claim to be the first to demonstrate that this approach works by comparing it with experimental fatigue data. Despite not being as original as it first seemed, this paper has become one of the most highly cited papers in the *International Journal of Fatigue*.

We went on to carry out some more extensive comparisons using all the available data on notches (Fig. 2 shows one example, in which the PM accurately predicts the data), and to try the methods out on various engineering components. In doing this we were surprised that these approaches, despite their long history, were not being actively researched, and were almost unknown in industry. The reason for this seems to be that, before the widespread use of FEA, these methods relied on approximate estimates of the elastic stress field near the feature of interest. Good accuracy is needed because the critical distances are small, typically in the range 0.1-1mm. Until recently FEA was not capable of this kind of resolution, at least not for most real components. Even now this remains a limitation in some cases, and explains why the CMM, which does not require such high resolution of near-notch stresses, has been adopted more readily in industry despite its relative complexity of implementation.

Our work up to 2006, summarised in a book [11], brought together previous investigations in high-cycle fatigue of metals and brittle fracture of fibre composite materials (in which the approach has seen extensive use, the relatively large values of the critical distances in those materials making the approach more feasible for component analysis) and extended the approach to include polymers and ceramics. The theoretical basis for the approach was also advanced, especially through the concept of finite fracture mechanics [12] and the approach's relationship to the underlying physical mechanisms of crack growth was investigated, though much still remains to do in this regard.

In recent years our emphasis has been on addressing the well-known problems associated with predicting fatigue in real-life situations, especially mixed-mode loading, variable amplitude loading and failure in the medium and low cycle fatigue regimes. We approached these issues from the starting point that existing methodologies could be combined with the TCD. Thus, for example, we used an existing approach to multiaxial loading, which defined an equivalent stress amplitude on a critical plane. This could be combined with the TCD simply by implementing the approach at the critical point (i.e at a distance L/2 from the notch root) or along a line (of length 2L) from the notch. Some problems arise regarding the location of this point or line, but these are exactly equivalent to the problem of finding the critical plane, which is already well known.

Investigating medium and low-cycle fatigue, along with monotonic failure, we again used existing approaches, adopting criteria based on plastic strain range rather than elastic stress range where appropriate. A new problem arose however when using these methods with the TCD: the critical distance turned out to be a function of the number of cycles to failure. In retrospect this is hardly surprising: one would expect the value of L to be related to the scale over which failure processes are occurring, therefore it can be expected to increase considerably when applied stresses increase and lifetime decreases. For cases of constant-amplitude loading this could be handled relatively easily by finding an empirical law linking L and the number of cycles to failure. For variable amplitude loading we were faced with the problem that L would now be different for every different cycle of stress; we dealt with this by defining a constant value of L based on a weighted average of individual-cycle values. The work in this field up to 2008 has been summarised in a book [13]; Fig. 5 shows examples of some of our more recent predictions covering multiaxial and variable-amplitude fatigue over a wide range of materials and lifetimes.

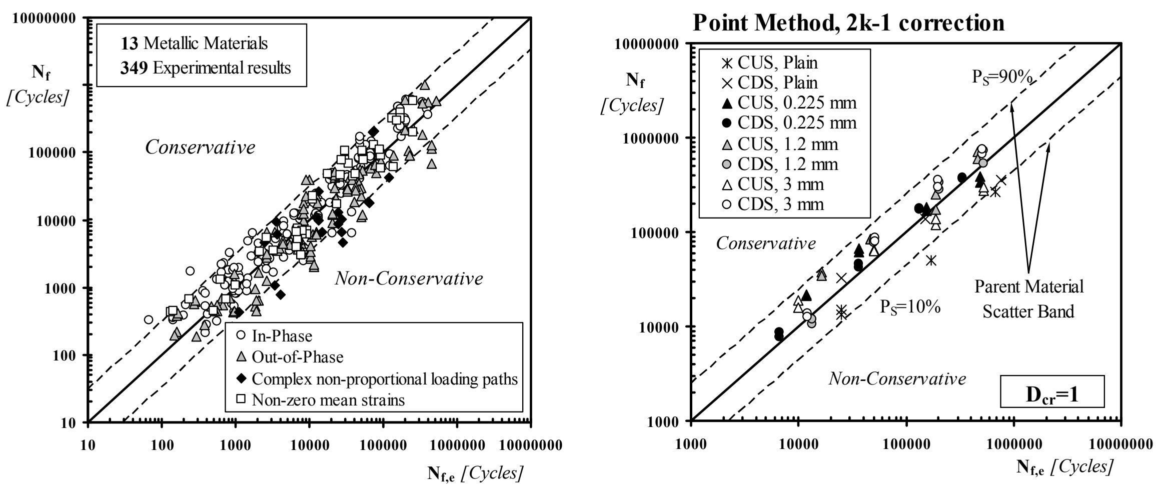

Figure 5: Examples of recent predictions of experimental data for notched specimens subjected to multiaxial loading (on the left) and variable amplitude loading (on the right). In each case the predicted number of cycles to failure is plotted on the horizontal axis, against the experimental value on the vertical axis. The dashed lines indicate the scatter band of the original data and thus represent the best possible accuracy for our predictions.

Thus our overall, general approach is essentially a three-parameter approach, in that we use a length constant, L, and two stress parameters: tensile and shear stress on the critical plane. Some of the issues which cause difficulty in fracture mechanics are not problems for us; for example the existence of the T-stress and other higher-order terms, and how they affect crack propagation, does not arise in the TCD because we automatically use a full description of the whole stress field, therefore these other terms are already present. This is essential in the correct prediction of short crack effects, for example, when the critical distance becomes similar in magnitude to the crack length.

Our investigations into multiaxial conditions have evolved to the point where we advocate a quite different approach, which recognises the fact that the stress field close to a crack or notch is *always* multiaxial, even when the applied load is uniaxial. Thus, in our view, any proposed multiaxial failure criterion should also be able to predict the simpler cases, and a robust, general methodology should be inherently multiaxial in nature.

In all these recent investigations our emphasis has been on developing systems of analysis which adhere to our original principles, as stated at the start of this paper, being based solely on analysis of the stress/strain field, as simple to implement as possible, and ultimately tested by their robustness in predicting experimental data. Ultimately any theoretical model is only valid if it is able to predict experimental data. We have shown that these methods can maintain predictive accuracy, even in complex loading situations. Much of this recent work has, by its nature, concentrated on prediction of data from test specimens, using all available data from the published literature. By this means we have developed the confidence to now offer these approaches for the analysis of industrial components. With the increasing use, and availability of FEA and other computer-aided design methods, we hope that these methods can become thoroughly integrated into the industrial design process.

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