Multiaxial stress versus stress intensity factor based approaches to estimate short crack arrest in fretting fatigue

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ABSTRACT. The aim of this paper is to carry out a comparative analysis between a multiaxial stress and a stress intensity factor based criterion to estimate short crack arrest in fretting fatigue. The stress-based model involves the computation of a multiaxial parameter at a critical distance from the contact. The critical distance is identified by means of standard fatigue tests, namely two plain fatigue tests and the threshold test for long crack propagation. The fracture mechanics model compares the mode I stress intensity factor range at a critical distance from the contact with the threshold stress intensity for crack propagation. In this case, the critical distance is defined as the crack transition length between the short and long crack regimes, which is obtained from the Kitagawa-Takahashi diagram. In order to carry out a comparative analysis between the models, fretting fatigue tests were conducted. A cylinder on plane contact configuration, both made of an Al 7050-T7451 aeronautical alloy, was considered. Twelve tests were performed with the same experimental conditions except the mean bulk load, which varied from a tensile to a compressive value until a run out condition was achieved. Results showed that the multiaxial fatigue approach is more accurate than the stress intensity factor based one to estimate fretting fatigue limit for this alloy. The potentialities and limitations of both models in practical applications are discussed.

KEYWORDS. fretting fatigue, short crack arrest, multiaxial fatigue, stress intensity factor, stress gradient, distributed dislocation method.

INTRODUCTION

Fretting is a type of material damage developed under service operation. It takes place in mechanical couplings under minute relative displacement that generates a small amount of material loss, a strong stress concentration and a rapidly stress decay, such as in the dovetail connection between blade and disc in aero-engines. These characteristics of the fretting problem speed up the crack initiation stage. However, depending on the levels of the stress gradient and of the bulk fatigue load, crack arrest may occur in the short or long crack regime. Certainly, not only the stress/strain history plays an important role on the observed reduction of fatigue strength in the presence of fretting [1,2], but also the superficial damage caused by fretting wear [3]. In this setting, new modeling which try to incorporate such superficial modification in the fatigue analysis has been recently proposed [4,5]. Nevertheless, they are either computationally too expensive to be extended to the design of practical components [4] or difficult to calibrate [5]. Further, the computed fatigue strength estimates by such modeling of the fretting phenomenon do not seem to provide better results than others that discard the small amount of wear produced but incorporate the stress concentration/stress

gradient effects [6-10]. As a matter of fact, under the typical partial slip conditions under which fretting takes place, the amount of superficial damage is small, hence it should neither substantially alter the contact profile in the macroscopic continuum stress field context nor be enough to vanish small initiated cracks, as happens to contact problems under full sliding [11].
To design against High Cycle Fretting Fatigue (HCFF) the concept of a stress or stress intensity factor (*K*) threshold is hence useful. The use of non-destructive inspection (NDI) associated to damage tolerant approaches is discarded since cracks (or at least defects of any inspectable size) are not tolerated. In this scenario, two safe life design methodologies are usually considered: (i) non-local multiaxial stress based fatigue criteria [6-8] and (ii) short crack arrest approaches based on SIF evaluation [9-10]. The aim of this work is not only to carry out a comparative analysis between such design concepts but also to show that, due to difficulties associated to the characterization of the crack tip stress field in terms of *K* in the mechanically short crack regime, alternative strategies can be developed to estimate short crack arrest. The short crack models proposed in this work can, in principle, be considered to estimate fatigue threshold conditions for mechanical problems involving the presence of any type of stress concentration/stress gradient. It remains to respond whether they provide similar estimates and, if not, which one provides the most accurate estimates for the fretting fatigue tests here conducted on an aluminium aeronautical alloy.

EXPERIMENTAL PROGRAM

The material used in this investigation was a 7050-T7451 aluminum alloy provided by EMBRAER-LIEBHERR (ELEB). The mechanical properties are given in Tab. 1. Fretting fatigue tests were carried with a pair of cylindrical pads pressed against a flat dog-bone specimen. Fig. 1a shows the fretting apparatus, which is mounted on a MTS 810 servo-hydraulic machine and works as a "spring" that reacts to the motion of the pads pressed against a specimen subjected to a cyclic bulk load. The reaction of the "spring" results in the cyclic tangential load. Full details about the fretting device can be found in [1]. Schemes of the contact configuration and the loading program are given in Figs. 1b and 1c. First, a mean bulk load B_m was applied to the specimen. The pads were clamped producing a static normal load P. A sinusoidal bulk loading of amplitude B_a was then applied to the specimen which, due to the stiffness of the fretting device, also experienced a sinusoidal tangential loading whose amplitude was termed Q_a. Tab. 2 reports the experimental parameters and observed lives. The following nomenclature is adopted: p_o is the peak pressure, a is the contact semi-width, ω is the bulk loading frequency and σ_a and σ_m are the bulk stress amplitude and its mean value, respectivelly. All tests were carried out in the partial slip regime ($Q_a < fP$ where f is the friction coefficient) under the same loading conditions, except the mean bulk stress, which varied from a compressive to a tensile value. Fatigue failure was defined as the complete fracture of the specimen, and run out was set at 10^7 cycles.

Yield strength, σ_Y [MPa]	454
Elastic modulus, E [GPa]	73
Poisson ratio, υ [MPa]	0.3
Plain fatigue strength, σ_{-1} [MPa] (R = -1; 10^7 cycles)	161
Plain fatigue strength, σ_0 [MPa] (R = 0; 10^7 cycles)	120
Threshold stress intensity range, ΔK_{th}, [MPa.m$^{0.5}$] (R = 0.1)	2.5

Table 1: Mechanical properties of Al 7050-T7451.

Test	1	2	3	4	5	6	7	8	9	10	11	12
σ_m	50	50	50	30	30	30	0	0	0	0	-30	-30
Life (×10^6cycles)	2.66	1.50	0.94	2.24	1.24	1.18	3.77	10	4.62	10	10	10

Table 2: Parameters of fretting fatigue tests and observed lives for: $p_0 = 175\text{MPa}$, $Q_a / fP = 0.62$, $a = 0.61$ mm $\sigma_a = 35\text{MPa}$.

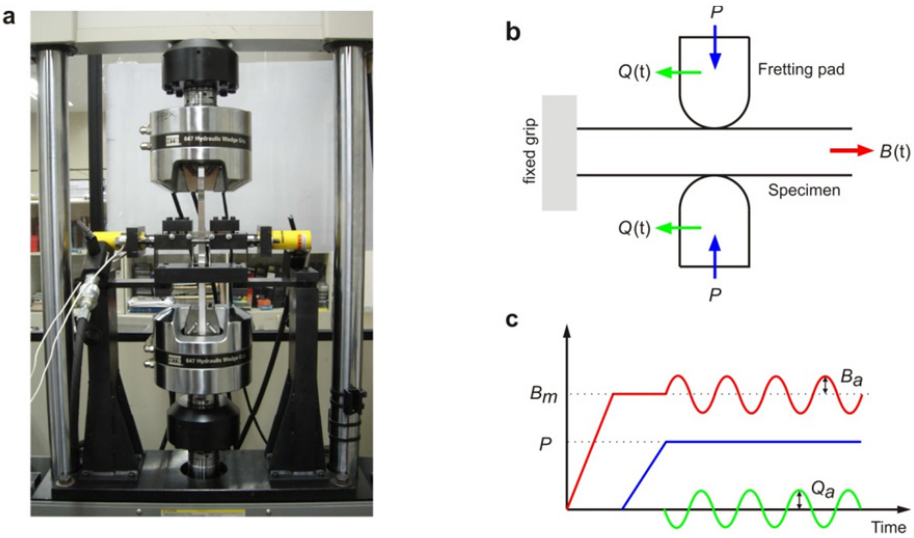

Figure 1: (a) fretting fatigue apparatus, (b) contact model and (c) loading program.

MODELLING SHORT CRACK ARREST IN FRETTING FATIGUE

Cyclic contact stress field

The cyclic stress field developed under the contact in the experimental configuration may be evaluated using a well-established technique and only an overview will be presented here. The first step towards a solution for the subsurface stress field is to solve the contact problem itself, i.e. to find the magnitude and distribution of the surface tractions. The shear tractions will not cause any disturbance in the Hertzian pressure distribution for the elastically similar surfaces considered in the fretting tests. Thus, since the normal load is constant, the normal traction will also be independent of time. The cyclic shear load, on the other hand, will give rise to history-dependent tractions as described by Cattaneo [12] and Mindlin [13]. Once the normal and shear tractions have been found, the cyclic stress field can be obtained at each load step by using Muskhelishvili's potential theory [14].

Stress Intensity Factor Point Based Model

Short crack models have shown to be a useful approach in fretting fatigue [9, 10]. The starting point in such models is the diagram introduced by Kitagawa and Takahashi [15], which represents the threshold condition for crack propagation as a function of crack length. Accordingly, for long cracks the threshold condition propagation is observed to be independent of the crack length, i.e:

$$\Delta K = \Delta K_{th} \tag{1}$$

where ΔK and ΔK_{th} are, respectively, the applied and threshold stress intensity factor ranges under mode I loading. On the other hand, the threshold condition for short crack propagation is given by:

$$\Delta \sigma = \Delta \sigma_{-1} \tag{2}$$

where $\Delta \sigma$ is the applied stress range and $\Delta \sigma_{-1}$ is the plain specimen fatigue limit range. Alternatively, expression (2) can be written in terms of stress intensity factor range as:

$$\Delta K = Y \Delta \sigma_{-1} \sqrt{\pi b} \tag{3}$$

where Y is a geometrical correction factor and b is the crack length. The crack transition length b_0, which divides the short and long crack regimes, can be obtained by equating expressions (1) and (3):

$$b_0 = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{Y\Delta\sigma_{-1}}\right)^2 \tag{4}$$

To estimate crack arrest in fretting fatigue, the stress intensity factor range of a crack growing inwards the contact is compared with the threshold stress intensity for crack propagation. Although in fretting fatigue tests of some metal alloys slant cracks have also been observed usually at the trailing edge of the contact or within the slip zone, we shall consider an approximated model where a straight crack of length *b* initiates from the trailing edge of the contact (x = -*a*) and is driven by the mode I stress intensity factor range (Fig. 2a). In Fig. 2b some possibilities of stress intensity factor range variation with crack size are illustrated within the framework of a Kitagawa–Takahashi: in curve A a short crack becomes non-propagating, curve B represents the threshold condition for crack propagation, whereas in curve C the crack achieves the long crack regime. Curves E and D will be commented later on.

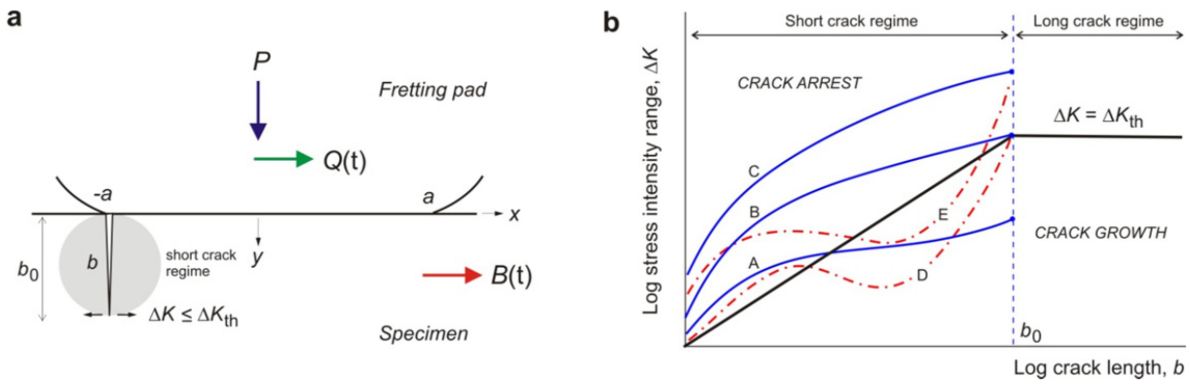

Figure 2: (a) Scheme of short crack arrest model based on stress intensity factor. (b) Cracking behaviors: short crack arrest (A), threshold condition (B), long crack initiation (C), and inadmissible behaviors (D and E).

An alternative, simple and mechanically consistent method to estimate short crack arrest in fretting fatigue is now proposed:

$$\Delta K\left(b_0\right) \le \Delta K_{th} \quad \Leftrightarrow \quad \text{short crack arrest} \tag{5}$$

Hence, short crack arrest occurs if the inequality is satisfied being the threshold condition achieved when the equality holds true. A short crack will evolve to a long crack whenever the inequality is not satisfied. It should be noted that the proposed criterion is intended to be applied in components where the presence of non-propagating short cracks is tolerated. When arrested long cracks are allowed to exist, the long crack stress intensity variation has also to be computed for cracks following the behaviour given by curve C in Fig. 2b. Eventually, due to a decrease in the far field stress the level of ΔK can drop below the threshold condition for a crack $b > b_0$.

The model is motivated by the examination of the cracking behaviours depicted in Fig. 2b by curves A, B, and C. Clearly, in such cases the proposed model is valid. On the other hand, it could cease to function in the presence of abrupt changes of ΔK in the short crack regime, as also shown in Fig. 2b. For instance, in curve D, a threshold condition is estimated instead of short crack arrest, whereas long crack growth is estimated instead of short crack arrest in curve D. However, we shall assume that abrupt ΔK variations with a strong decay immediately followed by rapid recovery within such small crack length are spurious situations. Indeed, the normal stress component along the line of the crack — produced by the normal and tangential contact loads and the bulk load — is not observed to vary in such an irregular manner.

It is claimed here that the proposed method improves similar approaches presented in the literature [9]. The questionable use of Linear Elastic Fracture Mechanics parameters to model short crack propagation [16, 17] is now circumvented. On the other hand, the computation of the stress intensity only at the crack transition length (eq. 4) is still not an exact procedure, because of the large uncertainty such a dimension is subjected to [18].

Non-local multiaxial stress-based approach

Since the works of Neuber and Peterson [19, 20], non-local fatigue models have been used to estimate the fatigue strength of notched components [21, 22]. In these models, the effective stress which produces fatiguedamage is an average value of the stresses around a stress concentrator. Although the effective stress can be defined on volume or line elements, the

most common procedure is to consider the stress at a critical distance from the notch (Point Method). For the sake of simplicity, we only deal with the Point Method in this paper.
Taylor [21] proposed that non-local models can estimate the fatigue threshold of both notches and cracks. In the case of a model based on the maximum principal stress amplitude, one can show that the critical distance can be identified from the threshold test for long crack propagation as

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_{-1}}\right)^2 \tag{6}$$

where ΔK_{th} is the mode I threshold stress intensity range and $\Delta\sigma_{-1}$ is the plain fatigue limit range, both under fully reversed loading. In order to extend Taylor's method to notches under complex multiaxial loading, Susmel [22] applied the Modified Wöhler Curve Method [25] in terms of the Point Method. In this case, it can be shown that the same critical distance given by Eq. (6) is obtained. Additionally, Susmel [22] introduced the structural volume concept as the process zone where micro/meso-cracks are arrested in fatigue threshold conditions, and estimated that its size is correlated to L.
Estimation of fretting fatigue thresholds with notch methodologies has been investigated in the literature [8]. This is motivated by the similarities between fretting and notch fatigue, since both present stress distributions characterized by multiaxial stresses and stress gradients. However, fretting fatigue encourages premature crack initiation and formation of material debris, due to the surface damage caused by the relative displacement between the contacting surfaces. Based on the previous remarks, notch methodologies could, in principle, be applied to fretting fatigue if the surface damage could be regarded as mild. Some researchers have sustained such a viewpoint when the fretting damage occurs in the partial slip regime [6, 9, 18]. Hereafter, the notch analogy is assumed to hold true. In this context, a short crack arrest model based on the Point Method and a critical plane parameter is proposed as follows:

$$F\left(\tau_{ac},\sigma_{n\max}\right) = \tau_{ac} + \kappa\frac{\sigma_{n\max}}{\tau_{ac}} - \lambda \le 0 \quad at \quad \left(x,y\right) = \left(-a,L\right) \tag{7}$$

The critical plane is defined as the material plane where the equivalent shear stress amplitude τ_a attains its maximum value τ_{ac} . Formally, $\tau_{ac} = \max_{\theta,\phi}\left\{\tau_{ac}\right\}$, where θ and ϕ are the spherical angles which describe the vector normal to an arbitrary material plane. In order to account for the mean stress effect, a non-linear relationship between τ_{ac} and the maximum normal stress $\sigma_{n\max}$ on the critical plane is adopted [23]. The material parameters are represented by κ,λ and L. The model is illustrated in Fig. 3a and can interpreted as follows: if the stress history at the critical distance L is such that the multiaxial criterion (7) is satisfied, it means that either there will be no crack initiation or short crack arrest within the critical distance. When the multiaxial criterion is exactly satisfied, it corresponds to the threshold condition for short crack arrest, whereas long crack growth is estimated when it is not satisfied. The crucial aspect in expression (7) is how to properly define the equivalent shear stress amplitude. A review of the most common equivalent shear stress amplitudes in fatigue problems can be found in [22]. In the present paper, the equivalent shear stress amplitude is given by the Maximum Rectangular Hull bounding the shear stress vector path Ψ in a material plane, as proposed by Araújo et al. [24]. Next, the main steps of the method are described. For each φ-oriented rectangular hull, described with respect to a basis $\left\{\tau_1\left(\varphi\right),\tau_2\left(\varphi\right)\right\}$, one can define its amplitude as

$$\tau_a\left(\varphi\right) = \sqrt{a_1^2\left(\varphi\right) + a_2^2\left(\varphi\right)} \tag{8}$$

where

$$a_i\left(\varphi\right) = \frac{1}{2}\left[\max_t \tau_i\left(\varphi,t\right) - \min_t \tau_i\left(\varphi,t\right)\right],\ i = 1,2 \tag{9}$$

Are halves of the sides of the φ-oriented rectangular hull bounding the shear stress path Ψ . Then, the equivalent shear stress amplitude is the one which maximizes Eq. (7), as is illustrated in Fig. 3(b):

$$\tau_a = \max_{\varphi} \sqrt{a_1^2(\varphi) + a_2^2(\varphi)} \tag{10}$$

As highlighted by Araújo et al. [24], the Maximum Prismatic Hull method is very simple to implement, and more accurate than traditional approaches. The procedure to identify the model parameters is now described. The parameters κ, λ were obtained from an exact fit between the model and the fatigue strengths σ_{-1} and σ_0 of plain specimens subjected to load ratios R = -1 and R = 0. The results are:

$$\kappa = 0.5(\sigma_{-1} - \sigma_0), \;\; \lambda = \sigma_{-1} - 0.5\sigma_0 \tag{11}$$

The parameter L was identified so as to obtain an exact fit between estimated and observed threshold stress intensity ranges. Such a strategy is fully described by Castro et al. [25].

$$L = \frac{1}{32\pi}\left(\lambda - \frac{2\kappa}{1-R}\right)^2 \Delta K_{th,R}^2 \tag{12}$$

where $\Delta K_{th,R}$ is the threshold stress intensity range at a load ratio R. In the present work, the parameter L was computed with expression (12) for R = 0.1.

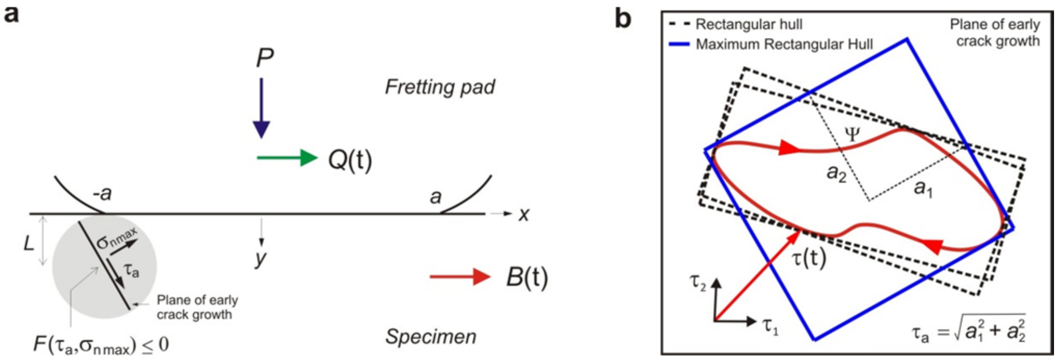

Figure 3: (a) Scheme of short crack arrest model based on multiaxial parameter. (b) Computation of the equivalent shear stress amplitude in terms of the Maximum Rectangular Hull.

MODELS ASSESSMENT

To implement the short crack arrest method based on Fracture Mechanics, a straight crack of length b was assumed to have originated at x = $-a$. The mode I ΔK was numerically calculated by the Distributed Dislocation Method [9,10]. It was assumed that negative stress intensities do not contribute to crack propagation. The crack transition length (eq. 4) was estimated as b_0 = 62 μm, based on the following assumptions: (i) Y = 1, and (ii) at a load ratio R = -1, ΔK_{th} = 4.5MPa.m$^{0.5}$, which is nearly twice the threshold value for R = 0.1 [26]. Fig. 4a shows the ΔK variation as a function of crack length for each experimental condition. For mean bulk stresses σ_m = -30 and σ_m =50 MPa, infinite life due to short crack arrest and fracture of the specimen are estimated, respectively. The threshold condition is estimated to be near σ_m = 30 MPa, whereas the tests suggest that it occurs between 0 and 30 MPa. The results for the crack arrest model based on multiaxial parameters are shown in Fig. 4b. The parameters given by expressions (11) and (12) to adjust the threshold line are: λ= 20.8 MPa, λ = 101.5 MPa and L = 20μm. Note that then experimental conditions were such that the critical plane always experienced the same equivalent shear stress amplitude, but different maximum normal stresses. A sound agreement between the estimates and the experimental data is observed. Long crack growth in the specimens subjected to positive mean stresses was correctly estimated, whereas a safe condition characterized by micro/short crack arrest was estimated for the run out tests with σ_m = -30 MPa. The data dispersion for m = 0 (two run outs and two fractures) may be caused by the proximity of the experimental conditions to the fretting fatigue threshold.

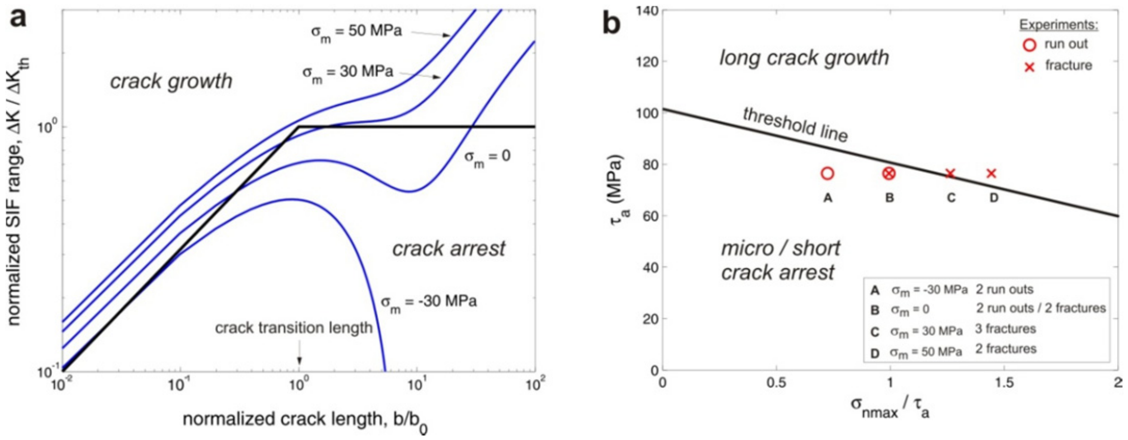

Figure 4: Comparison between estimated and observed cracking behavior for (a) K-based and (b) stress-based models.

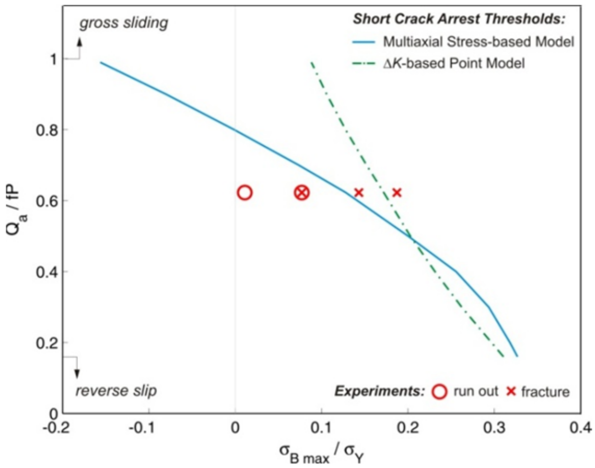

Figure 5: Short crack arrest thresholds estimated by the stress and ΔK based models.

DISCUSSION AND CONCLUSIONS

Two models to estimate short crack arrest in fretting fatigue have been proposed. The models are based on the mode I ΔK and on a multiaxial stress based parameter, both computed at a critical distance. In order to assess the models, new fretting fatigue tests were carried out on a cylinder on plane contact configuration made of an Al 7050-T-7451 aeronautical alloy. Experiments were conducted so that all salient test parameters remained constant but the mean bulk load. The threshold level of mean bulk load dividing run-out and complete fracture of the specimens was observed to be within 0 and -30 MPa. The ΔK based Point Model (Eq. 5) allows the estimation of the threshold condition for short crack arrest by considering only the evaluation of ΔK at the crack transition length, b_0, dividing the short and long crack regimes. Therefore, the model does not require the introduction of any type of correction in order to try to reproduce the so called anomalous short crack behaviour when described in term of ΔK. For design purposes, our ΔK based Point Model could be regarded as a conservative approach. Indeed, at the estimated threshold condition, the real non-propagating crack length could be less than b_0 due to, for instance, microstructural barriers such as grain boundaries [17]. As a matter of fact, both models presented have been elaborated to estimate thresholds loadings (or design allowables) for short crack arrest. However, no information is given about the lengths of these cracks or about the number of cycles after which they become non-propagating. In designs where such information is important, modelling of the short crack regime has to be considered. For instance, short crack growth laws based on a multiparameter characterization of the crack tip stress/strain field (e.g. incorporating the T-stress) could be considered for this purpose. See [27] and references therein. In the threshold fretting fatigue diagram depicted in Fig. 5 it becomes clear that both predictive methodologies are not equivalent, i.e, they provide different safe load domains for the same contact configuration. For the limited amount of experimental data here produced the non-local multiaxial stress based model provided slightly better estimates of fatigue resistance than the ΔK based Point Model. More experimental data considering different contact configurations and other alloys are necessary to further challenge the accuracy of these models.

For two-dimensional short crack arrest analysis, the computation of both models is relatively straightforward. However, for real three-dimensional geometries, the calculation of ΔK is not well-established and usually demands special techniques. On the other hand, the results of non-local stress-based models can, in principle, be visualized in a post-processing procedure of an elastic finite element analysis. Hence, it seems that stress-based models can potentially provide more rapid and simple design solutions than a fracture mechanics model.

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