# A FRACTURE MECHANICS APPROACH TO FAILURE MODE SCALING IN CONCRETE BEAMS

Alberto Carpinteri [1] and Giuseppe Ferro [1]

The first results of the Round Robin testing programme on "Scale effects and transitional failure phenomena of reinforced concrete beams in flexure" of the ESIS TC 9 on concrete are presented.

Carpinteri's model based on a transitional brittleness number, a zero crack opening compatibility condition ($\delta = 0$) and a fracture toughness accounting for slow crack growth, appears to be in good agreement with the observed failure mechanisms and peak loads in the case of ductile or brittle flexural failures.

Two additional theorical contributions, proposed by Karihaloo (the shear capacity due to diagonal failure with a two parametrical model that allows an analytical solution) and by Elices, Planas and Ruiz (the concrete fracture as a cohesive crack and the effect of reinforcement bond-slip) are also reported.

## INTRODUCTION

The use of new materials, as high-strength and fiber-reinforced concretes, and the ever-growing attention to economical and safety aspects, imposed in the recent years to consider theories ever more accurate for structural calculations. In this context, fracture mechanics represents a very suitable theory.

As regards structural safety, the evaluation of the minimum percentage of reinforcement represents one of the fundamental aspects, in order to prevent brittle or catastrophic failure. In the Standard Codes, the minimum reinforcement is defined on the basis of limit analysis, which determines constant minimum reinforcement ratios, independent of the beam dimensions. To consider the size effect, the analysis should be based on fracture mechanics concepts. In this context, the ESIS Technical Committee 9 on concrete proposed in 1992 [1] a Round Robin test programme on "Scale effects and transitional failure phenomena of reinforced concrete beams in flexure". The main goals of this Round Robin consist in: (a) the prediction of the load-carrying capacity of three-point bend, singly-reinforced concrete beams without shear reinforcement and (b) the determination of the minimum reinforcement ratio to prevent brittle failure. Six laboratories partecipated to this programme and the results will be reported in an ESIS TC9 final report. Only the results of three laboratories are available so far.

The model based on the *brittleness number*, proposed by Carpinteri [2], with a zero crack opening condition ($\delta = 0$) and a fracture thoughness accounting for slow crack growth appears to be in good agreement with the observed failure mechanisms

[1] Dipartimento di Ingegneria Strutturale, Politecnico di Torino, 10129 Torino, Italy.

obtained in experimental tests. The relationship established by the brittleness number provides an increment of the minimum reinforcement percentage with decreasing beam depths.

The failure mechanism changes completely when the beam depth is varied, the steel percentage remaining the same. Only when the steel percentage is inversely proportional to the square root of the beam depth, can the mechanical behaviour be reproduced. Numerical simulations on samples with different sizes and constant brittleness numbers were also performed, that permit to show that the structural behaviour is the same for analogous brittleness numbers.

Ruiz, Elices and Planas [3] fronted the effect of bond-slip on lightly RC beams and proposed the *effective slip-length model*, which is able to capture the experimental details, such a secondary peak load for a relatively large steel cover, and describes satisfactorily the transitional behaviour of lightly RC beams from brittleness to ductility. The analytical model was used to generate a closed-form expression for minimum reinforcement in bending.

Lange-Kornbak and Karihaloo [4] analysed their experimental results assuming a the diagonal tension failure and considering an approximate nonlinear fracture mechanics method, called the *superposition method* [5], that allows an analytical solution to evaluate the strength of three point bend RC beams.

From the mentioned contributions and comparing the proposed formulas with the recommendations of the Standard Codes, it can be evidenced that the latter could be improved to get safer or cheaper reinforcement able to prevent brittle behaviour.

## THEORETICAL MODELS

The simplest model to analyse the transitional failure of the RC beams was proposed by Carpinteri [2]. The steel action is equivalent to two closing forces acting on the crack faces at the point where the reinforcement crosses the crack. The reinforcement reaction is determined by the condition that the crack opening is zero at the point where the reinforcement crosses the crack. When the steel yields, the closing forces become constant and equal to the yielding force of the reinforcing bars. The model is briefly presented herein.

Let us consider a cracked beam element (Fig. 1.a), which undergoes simultaneously the bending moments $M$ and the closing forces $P$ applied onto the crack surfaces. In order to analyse the behaviour of a structure containing a cracked member, it is necessary to know the relation between the load and the deformation of the member. By linear superposition it is possible to write:

$$\Delta\delta = \Delta\delta_{PM} + \Delta\delta_{PP} = \lambda_{PM}M - \lambda_{PP}P, \tag{1}$$

where $\lambda_{MM}$, $\lambda_{PM}$, $\lambda_{MP}$, $\lambda_{PP}$, are the compliances of the member due to the existence of the crack. The factors $\lambda$ can be derived from energy methods and expressed by means of the function $Y_M(\xi)$ and $Y_P(c/h,\xi)$, where $h$ is the beam depth, $c$ the steel cover, $a$ the crack depth and $\xi = a/h$ the relative crack depth. To determine the reinforcement reaction $P = \sigma_s A_s$, the displacement discontinuity in the cracked cross-section at the level of reinforcement is assumed to be zero ($\Delta\delta = 0$), up to the moment of yielding or slippage of the reinforcement. This condition allows to obtain the unknown force $P$ as a function of the applied moment $M$:

$$\frac{Ph}{M} = \frac{1}{r''(c/h,\xi)} = \frac{\lambda_{PM}h}{\lambda_{PP}}. \tag{2}$$

Considering a rigid-perfectly plastic behaviour of the reinforcement, the moment of plastic flow or slippage is: $M_P = P_P h r''(c/h, \xi)$, where $P_P = f_y A_s$ indicates the yielding (or pulling-out) force, achieved when $\sigma_s = f_y$ (yielding stress of reinforcement).

The stress intensity factor at the crack tip can be expressed as:

$$K_I = \frac{M}{h^{3/2}b} Y_M(\xi) - \frac{P}{h^{1/2}b} Y_P(c/h, \xi), \tag{3}$$

where $P = P_P$ for $M > M_P$. Assuming that $K_I$ is equal to the matrix fracture toughness $K_{IC}$, from eq.(3) it is possible to write:

$$\frac{M_F}{K_{IC} h^{3/2} b} = \frac{1}{Y_M(\xi)} + \frac{P}{K_{IC} h^{1/2} b} \frac{Y_P(c/h, \xi)}{Y_M(\xi)}. \tag{4}$$

If the force $P$ transmitted by the reinforcement is equal to $P_P = f_y A_s$ or, in other words, if the reinforcement yielding limit has been reached ($M = M_F \geq M_P$), eq.(4) becomes:

$$\frac{M_F}{K_{IC} h^{3/2} b} = \frac{1}{Y_M(\xi)} + N_P \frac{Y_P(c/h, \xi)}{Y_M(\xi)}, \tag{5}$$

where the *brittleness number* :

$$N_P = \frac{f_y h^{1/2}}{K_{IC}} \frac{A_s}{A}, \tag{6}$$

is introduced and $A = bh$ is the total cross-sectional area.

The transitional value $N_{PC}$ can be determined from eq.(5) if we impose $M_F = P_P(h - c) \approx P_P h$, i.e. absence of discontinuity in the diagram $M$ versus $\Delta\phi$:

$$\frac{M_F}{K_{IC} h^{3/2} b} = \frac{1}{Y_M(\xi)} + N_P \frac{Y_P(c/h, \xi)}{Y_M(\xi)} \approx \frac{P_P h}{K_{IC} h^{3/2} b}, \tag{7}$$

and then, recalling eq.(6):

$$N_{PC} = \frac{1}{(Y_M(\xi) - Y_P(c/h, \xi))}. \tag{8}$$

It is then possible, simply on the basis of the cross-sectional geometrical characteristics, to distinguish the cases of unstable fracture ($N_P < N_{PC}$) from those of stable fracture ($N_P > N_{PC}$).

The model proposed by Ruiz, Elices and Planas [3] presents three fundamental variations with respect to the previous model: (a) the crack is cohesive, i.e. stresses are transferred between the crack faces; (b) the steel is considered as elastic-perfectly plastic and (c) the bond between steel and concrete is modeled by means of a rigid-perfectly plastic shear stress-slip law.

In the model, the Authors repleace the actual bond shear stress distribution by a pair of concentrated forces acting inside the concrete. The length of the slip zone $L_p$ is evaluated by means of an equilibrium condition. The reduction of the action of the reinforcement to a pair of closing forces has the virtue of being amenable by analytical tretment.

Five nondimensional parameters are introduced in the model. One in particular, $\eta = (\frac{E_s}{E_c} \frac{\tau_c}{f_t} p l_{ch} A_s)^{1/2}$, is a nondimensional bond parameter that depends on the bond strength $\tau_c$, on the rebar perimeter $p$ and on the relative stiffness $E_s/E_c$, while the remaining parameters, the characteristic length $l_{ch}$, the steel cross-section area $A_s$ and the tensile strength of concrete $f_t$, are introduced to make it nondimensional.

From the numerical simulations performed by the Authors, it follows that the model is able to capture experimental details, such as a secondary peak in the load-deflection diagram for relatively large steel covers. The model, however, requires better estimates of the bond between steel and concrete than those obtained from pull-out tests.

Lange-Korbak and Karihaloo [3] considered two different failure modes of the three-point bend RC beams. For the diagonal tension failure, they evaluated the shear strength $P_u$ by the superposition method, in which $P_u$ is regarded as being the sum of contributions from the reinforcement $(P_s)_u$ and concrete $(P_c)_u$, i.e. $P_u = (P_s)_u+(P_c)_u$. In this model, a parameter that depends on bond-slip relationship is introduced for the reinforcement contribution.

The contribution of concrete to the shear strength is obtained by an incremental procedure proposed by Karihaloo [5]. The location of the incipient diagonal tension crack and the direction of its growth are varied with a view of delineating the critical diagonal tension crack. The instant of growth and direction of propagation of the crack are determined by using a proper mixed mode fracture criterion. When the diagonal tension crack propagates almost in a straight line, the procedure converges in just one step. For the tested beams, the calculated ultimate load is lower than the measured ultimate load, as the interlocking of the crack surfaces with coarse aggregates and the dowel action of the reinforcement have been neglected.

For the flexural failure, the Authors used the Carpinteri's model, previously presented. As suggested by the model, the beams with a brittleness number $N_P$ less than the critical value $N_{PC}$ failed in a brittle manner, while the beams with $N_P > N_{PC}$ failed in a ductile one. The Authors proposed to use the exact theoretical relation for $N_{PC}$ reported in eq.(8), rather than the approximate expression, $N_{PC} = 0.1+0.0023 f_c$, with $f_c$ being the compresive strength of concrete, proposed by Bosco and Carpinteri [6]. Even the measured peak loads are in good agreement with the predicted ones. The prediction, however, becomes less accurate as the percentage of steel $\rho$ is reduced. The Authors attributed this phenomenon to the increase in crack opening $\delta$ with decreasing $\rho$ which is not accounted for in the present model. It must be emphasized that the Carpinteri's model represents a very simple and easy-to-use fracture mechanics approach. The calculation of the brittleness number is sufficient to evaluate the failure mode for a RC beam. The simplicity of the model, of course, disregards other secondary phenomena, as the cohesive zone and the steel slippage.

## MINIMUM PERCENTAGE OF REINFORCEMENT

Recently, it has been theoretically proved that the minimum percentage of reinforcement that enables the element to prevent brittle failure, depends on the scale [2]. With a classical approach these results are not predicable.

Minimum reinforcement Code predictions are not definitively settled and the corresponding formulas are based on limit analysis and do not display size effect for geometrically similar beams. The minimum reinforcement percentage versus the beam depth is reported in Fig. 1.b for four different concrete grades. In the same figure, the corresponding curves obtained from EC2 are reported, while the ACI 318 Code provides a single percentage of steel. The dependence of $N_P$ on the beam depth causes the decreasing of the minimum steel percentage with increasing the beam depth, while the values supplied by the codes are independent of the beam depth. The relation $M_P > M_F$ must be satisfied so to obtain a stable and ductile response, $M_P$ being the bending moment of steel yielding and $M_F$ the bending moment of first concrete cracking [2, 6]. The minimum percentages of reinforcement determined by applying the formula experimentally derived by Bosco and Carpinteri [6] for the same concrete grades are reported in Fig. 1.b. The brittleness number describes the ductile-brittle transition for the tests performed in this Round-Robin very satisfactorily. The same trends have been found in the numerical simulations [7].

Ruiz, Elices and Planas [3] proposed an expression for the minimum reinforcement, $\rho_{min}$, using the effective slip-length model previously introduced. A minimum reinforcement formula was obtained fitting the numerical results, as a function of the beam depth $h$, the reinforcement cover $c$, the steel yielding strength $f_y$ and the bond shear strength $\tau_c$:

$$\rho_{min}(h, c, f_y, \tau_c) = \frac{0.174}{(1-\gamma)} \frac{1 + (0.85 + 2.3\beta_1)^{-1}}{f_y^* - \eta_1 \phi}, \qquad (9)$$

where $\beta_1$, $\gamma$, $f_y^*$, $\eta_1$ and $\phi$, are five nondimensional parameters depending on the bond strength and cohesive fracture behaviour. The curves of minimum reinforcement to be used for beams of different depth, according to different formulas, are reported in Fig. 1.c. The curves are related to beams of normal strength concrete with a steel yield strength of 480 MPa and a cover $c$ =40 mm, which is assumed to be constant rather than proportional to the beam depth (this gives an apparent size effect in the curve of the Codes). The full-line curves shown in the figure correspond to the minimum reinforcement as given by eq. (9).

The curves based on fracture mechanics models show similar trends, which contrast with those of the recommendations in the Codes. In particular, in the slip-length model as in the Carpinteri's model, the minimum reinforcement based on fracture mechanics shows a sharp decrease with increasing depth for small beam depths. This trend becomes milder for larger sizes.

## CONCLUSIONS

From the theoretical models and from the experimental results performed in the Round Robin test programme, it is possible to confirm the existence of different behaviours at failure, from ductile to brittle, the latter occurring when the beam depth overtakes certain sizes and/or the reinforcement content is sufficiently low. Carpinteri's model, as confirmed by the experimental results [4, 7], is a consistent approach for low reinforced concrete beams, the crack mouth opening displacement compatibility condition, taken into account to determine the force transmitted by the reinforcement, being very close to the real failure mechanism.

The value of the critical brittleness number $N_{PC}$, that separates brittle from ductile collapses, is confirmed by the experimental as well as by the numerical results. Such a theoretical approach appears to be very useful for estimating the critical condition between stable and unstable crack propagation and thus the minimum reinforcement.

The Standard Codes still consider only the concrete compressive strength, whereas they do not the tensile post-peak behaviour. The results of the Round Robin evidence how the Standard Codes should be revised and take into account such an important scale transition.

## REFERENCES

(1) Bosco, C. and Carpinteri, A., "Scale effects and transitional failure phenomena of reinforced concrete beams in flexure", ESIS Technical Committee 9, Round Robin Proposal, Department of Structural Engineering, Politecnico di Torino, Italy, 1993.

(2) Carpinteri, A., J. Struct. Eng. (ASCE), Vol. 110, 1984, pp. 544-558.

(3) Ruiz, G., Elices, M. and Planas, J., "Size effect and bond-slip dependence of lightly reinforced concrete beams", Draft submitted for publication to Task Committe 9 of ESIS, Ed. A. Carpinteri, 1997.

(4) Lange-Kornbak, D., Karihaloo, B.L., "Fracture mechanical prediction of transitional failure and strength of singly-reinforced beams", Draft submitted for publication to Task Committe 9 of ESIS, Ed. A. Carpinteri, 1997.

(5) Karihaloo, B.L. In *Fracture Mechanics of Concrete Structures*, Ed. F.H. Wittman. AEDIFICATIO, Freiburg, 1995, pp. 1111-1123.

(6) Bosco, C. and Carpinteri, A., J. Eng. Mechanics (ASCE), Vol.118, 1992, pp. 1564-1577.

(7) Carpinteri, A., Ferro, G., Bosco, C. and Elkatieb, M., "Scale effects and transitional phenomena of RC beams in flexure", Draft submitted for publication to Task Committe 9 of ESIS, Ed. A. Carpinteri, 1997.

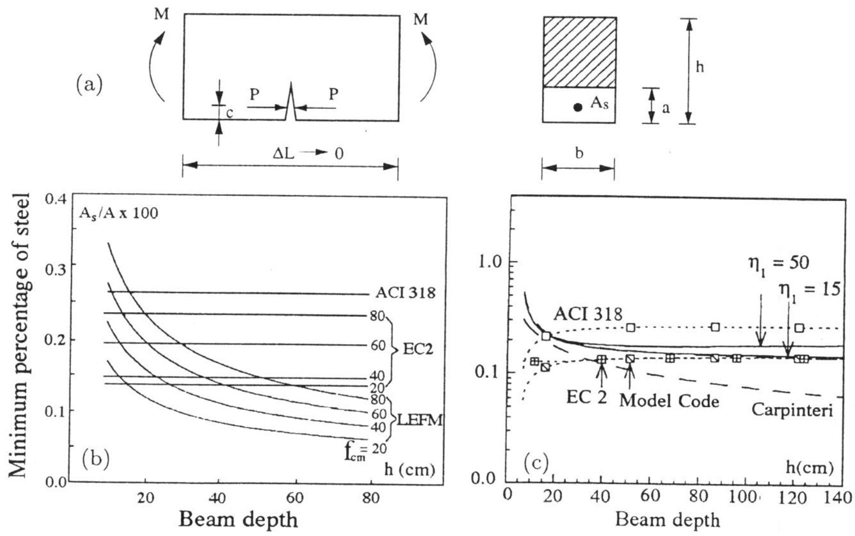


Figure 1: (a) Cracked beam element. (b) Minimum steel percentage versus beam depth: Carpinteri's brittleness number model for four different grades of concrete and (c) bond-slip model [3] versus Standard Codes.