

FE-SIMULATION OF CRACK GROWTH USING DAMAGE PARAMETER  
AND COHESIVE SURFACE CONCEPT

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The damage initiation and growth in ductile materials is simulated with the use of the Rice & Tracey's damage parameter and the element elimination techniques, and also with the use of the cohesive surface concept. The force-displacement curves for the deformed specimens are determined. The effect of the critical level of the damage parameter on the force-displacement curve and critical load is studied.

INTRODUCTION

In this paper, the damage initiation and evolution, and fracture in ductile materials is simulated with the use of element elimination technique (EET) and cohesive surface concept (CSC). The purpose of this work is to test the advanced numerical techniques in simulation of damage evolution and fracture in ductile materials, and to study the interrelations between the damage and cohesive surface parameters of crack growth.

ELEMENT ELIMINATION TECHNIQUE

The main idea of the element elimination technique (EET) is that the finite elements in which some value exceeds a critical level is excluded from the FE mesh. With this method, one can simulate the initiation of microcracks and pores, their coalescence and crack growth ((references (1)-(4)). When a finite element is eliminated, all components of the stress tensor in the elements (and, therefore, local forces) are set to be equal to zero. Wulf (4) has shown, however, that in eliminating elements, the

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Drucker condition of stability breaks down, what can lead to the losses of stability and bifurcations in calculations. The idea to eliminate the integration points instead of the finite elements is discussed by Wulf as well (4). In order to avoid the instability of solution caused by the used procedure of the element elimination, the reduction of local stresses to zero is distributed over several loading steps. These steps are called "relaxation steps" and at these "relaxation steps" no further loading is applied (4). So, the local failure, which is simulated as a stepwise disappearance of finite elements, is numerically presented as gradual process. The amount of the relaxation steps should ensure the gradual adaptation of the stress and strain field to the elimination of an element, on the one side, and not lead to drastical increase in the computation time, on the another side. Among the main advantages of EET, there are the possibility to apply this method for mutiphase materials, automatic modification of FE mesh, without necessity of the data transmission to a new FE mesh, as well as the possibility to simulate both micro- and macrocrack evolution.

#### CRITERIA OF ELEMENT ELIMINATION

The criterium which controls the elimination of finite elements should be determined from the concrete physical problem. Sveshnikov (5) has used the Pisarenko-Lebedev condition of local failure as a criterion for the element elimination. The criterion is applicable for both plastic and brittle materials, and is written as the equality of the following equivalent stress to some critical level:

$$\sigma_{eq} = \chi\sigma_i + (1 - \chi)\sigma_1 \quad (1)$$

where  $\sigma_i$  - root mean-square of the principal shear stress,  $\sigma_1, \sigma_2, \sigma_3$  - principal stresses, and  $\chi$  - ratio of critical stresses at uniaxial tension and compression. Lippmann et al. (2) have used two-criteria model of element elimination for the simulation of AlSi cast alloys: elements which were assigned to hard (Si) particles, were eliminated on the basis of a normal stress criterion, and the elements which are located in aluminium matrix were eliminated with the use of Rice & Tracey's void growth model. In our calculations, the criterion for element elimination based on the void growth model of Rice and Tracey was chosen:  $D = D_c$ , where  $D_c$  - some critical level of the parameter  $D$ , and the parameter  $D \propto \ln(const * r/r_0)$ , where  $r$  and  $r_0$  - current and critical radii of a void being initiated as a result of local plastic deformation. The value  $D$  presents the damage parameter which is defined as follows (4):

$$D = \alpha \int_0^{\epsilon_{pl}} \frac{d\epsilon_{pl}}{\epsilon_f} = \int_0^{\epsilon_{pl}} \exp\left(\frac{3}{2}\eta\right) d\epsilon_{pl} \quad (2)$$

where  $\eta = \sigma_H/\sigma_V$ ,  $\sigma_H$  - hydrostatic stress,  $\sigma_V$  - equivalent von Mises stress,  $\eta$  - stress triaxiality,  $\epsilon_{pl}$  - equivalent plastic strain,  $\epsilon_f$  - failure strain,  $\alpha$  - proportionality coefficient.

SIMULATION OF CRACK GROWTH USING EET

In order to study the efficiency of the element elimination technique for modelling of crack propagation in ductile materials, the following simulation of 3-point bending test of specimen from Al/SiC (20 %) was carried out. The calculations were carried out for a material with averaged elastic properties of the specimen ( $E = 99400$  MPa,  $\nu = 0.323$ ). The purpose of the simulation is to study the effect of the choice of the element elimination criterion on the correctness of the simulation. To do it, the simulation was to be compared with the experiments and also with simulation presented in reference (4). In the experiments from (4), 3-point bending specimens with sizes  $20 \times 100 \times 1$  mm were loaded till failure. This process was simulated by Wulf (4), with the use of the EET. The critical plastic strain was chosen as a criterium of the element elimination. The criterion of element elimination different from that used in (4) was applied in our calculations: instead of the critical plastic strain, the Rice & Tracey damage parameter was used. The difference between these two approaches (i.e. the critical plastic strain and the Rice & Tracey based damage criterion for element elimination) is determined among others by the fact that the Rice & Tracey based damage criterion is very sensitive to the degree of triaxiality in the deformed material, what is not the case for the critical plastic strain criterion. The finite elements which are located on the loading surface were taken to be elastic and not damageable. Fig. 1 shows the numerical force-displacement curves for the critical damage parameters 0.2, 0.15 and 0.1 as well as the experimental curve from (4). One can see that both curves with  $D_{cr} = 0.2$  and 0.15 are very close to the experimental data, although only the force - displacement curve with  $D_{cr} = 0.2$  gives the peak load and displacement which are equal to the experimental data. The last part of the curve at which the behaviour of failed specimen is described can not be obtained in these calculations.

SIMULATION OF CRACK GROWTH USING CSC

The crack propagation through the specimen was simulated using a cohesive surface model. It allows the evolution of crack initiation from the free surface or from a pre-existing crack, crack growth and crack arrest to be described. It is worth noting that this model differs essentially from the so-called "cohesive zone models" (e.g. (10)) where the prescribed crack path has been characterized as a thin material layer with its own elastic or elastic-plastic constitutive relation (traction - separation law). However, these relations are such that, with increasing crack opening, the traction reaches a maximum, then decreases and eventually vanishes so that complete decohesion occurs. It is questionable whether the softening part in the constitutive relation (where the traction reduces from the assumed maximal value to zero) does not affect the correctness (i.e. existence and uniqueness of the solution) of the mathematical model problem. This results very often in highly mesh dependent FE solutions. Recently, some attempts were made to regularise the model problem by introducing the so-called length scale parameters into the formulation. Up to now it is not clear whether some physical meaning of these parameters can be found or

they are introduced just for facilitating a little the calculations. It is true that when chosen appropriately the length scale parameters can eliminate the dependence of the solution on the element size but the question about their physical sense is still open. From the same drawbacks suffers, in general, the element elimination technique as well. In order to avoid the above mentioned difficulties, the following approach is suggested: it is postulated that a criterion for decohesion is controlled by the normal traction transmitted through the cohesive surface. Decohesion under Mode I occurs, if the normal traction reaches a critical value (cohesive strength of the material). In our formulation the decohesion criterion is embedded in the boundary value problem as an additional boundary condition along the prescribed crack path. The addition of the non-overlapping condition for the bonded part of the prescribed crack path, leads to a variational formulation with inequalities. The analyses were conducted within the framework of a plane strain formulation. The loading is applied by a prescribed vertical displacement of the loading beams. The  $J_2$  (Von Mises) flow theory of plasticity is employed to characterise the material. Due to the symmetry only one half of the model needs to be considered. A rigid surface was defined along the symmetry line in order to simulate a symmetric Mode I crack. In such a way the crack propagation is simulated as decohesion between the rigid surface and the half of the model under consideration. In order to get closer agreement with the experimental results, the cohesive strength was determined iteratively by changing incrementally the cohesive model parameter. Best agreement with the experiment was obtained for critical normal traction of 550 MPa (see Fig. 2). For comparison purposes in the same figure are shown the force-deflection curves when values of 540 MPa and 560 MPa are assumed for the critical traction. It can be concluded that the developed finite element model provides a detailed quantitative description of Mode I crack propagation in the presence of large strains.

#### COHESIVE ZONE AND DAMAGE CONCEPTS: INTERRELATIONS

In many works, the mechanism of crack propagation was described as a result of formation of voids or secondary crack in front of crack tip and their joining with the crack by the mechanisms of tearing or secondary cleavage (6,7). Such a physical process corresponds fully to the described above approach to crack growth modelling, in which the unit step of crack propagation is presented as the elimination of small volume of material (FE) caused by the growth of a void in this small volume. Consider shortly interrelations of this approach with the cohesive zone model. The local failure of a small cell of material caused by void growth and coalescence is influenced by the stress triaxiality to a large extent, whereas the concept of cohesive surface is based on the approximation of a number of growing voids in front of the crack by an area subject to uniaxial (in simplest case) tension. The size of this area  $L$  as well as the critical stress  $T_n$  at which the area becomes a new surface depend on the conditions of problem and material properties. This event (i.e. the transformation of the cohesive surface into a free surface) corresponds to the elimination of  $N$  cells in front of the crack in the cell model of material (8). Following the reasonings of Broberg (8) and

Johnson (9), unit cell is assumed to correspond to a finite element in FE discretization. If the size of a cell is  $l_0$ , the transformation of the cohesive zone into a free surface in the cohesive surface concept corresponds to the failure (elimination) of  $N = L/l_0$  cells.

The energy needed for elimination of one cell is equal to  $A_1 = \int_0^{\epsilon_f} \sigma d\epsilon$ . Substituting the relation between  $\epsilon_f$  and  $D$  from (4), and assuming that the damage evolution law (damage growth rate vs. damage parameter relation) is given in the form of some function  $\dot{D} = \psi(D)$ , one can obtain the relation between the energy of cell failure  $A_1$  and the critical damage parameter:  $A_1 = \int_0^{\alpha\psi(D_c)/\dot{\epsilon}_{pl}} \sigma d\epsilon$ , where  $\dot{\epsilon}_{pl}$  - plastic strain rate,  $\dot{D} = dD/dt$ . Taking into account the relation between the work of separation per unit area  $A_2$  and the traction stress  $T_n$ :  $A_2 = \alpha_1 T_n$  (10), where  $\alpha_1$  - proportionality coefficient depending on the accepted model of crack opening, and setting  $A_1 = A_2$ , we obtain:

$$T_n = (N/\alpha_1) \int_0^{\alpha\psi(D_c)/\dot{\epsilon}_{pl}} \sigma d\epsilon \quad (3)$$

This equation relates the traction stress  $T_n$  (parameter of crack growth in the cohesive model) and the critical Rice & Tracey damage parameter  $D_c$ .

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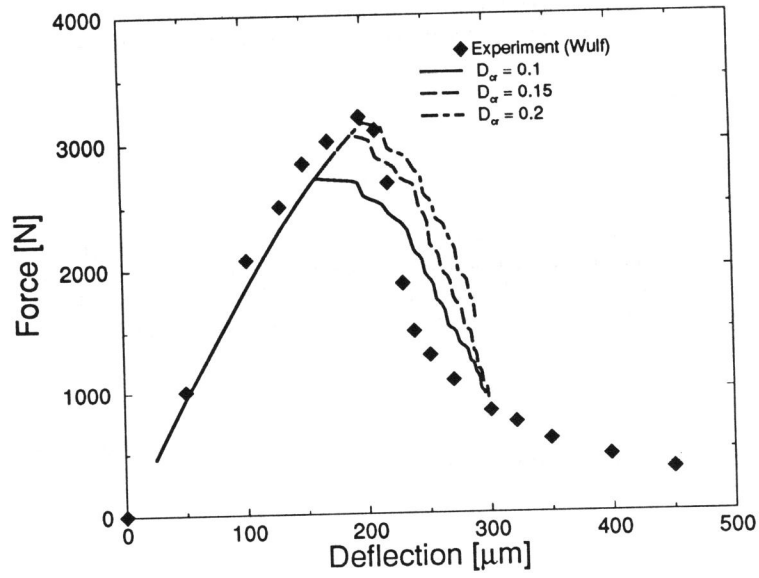


Figure 1 Force-deflection curves obtained by the EET

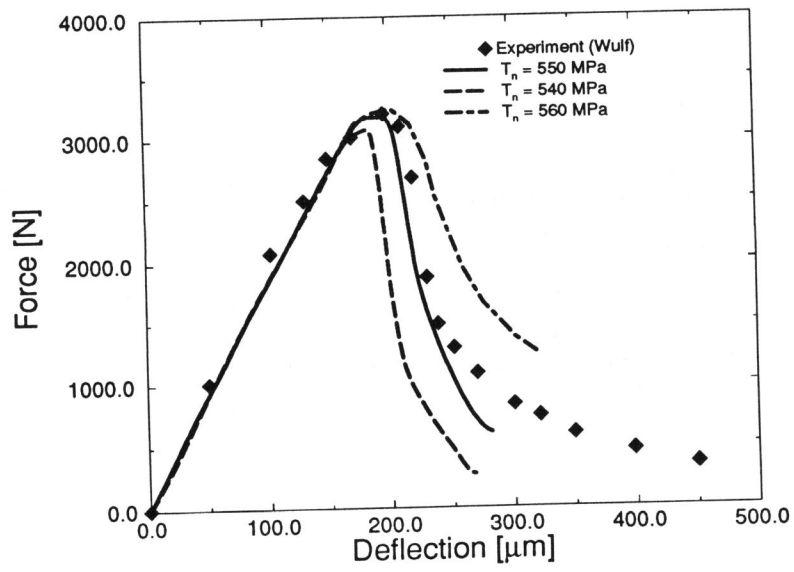


Figure 2 Force-deflection curves obtained by the CSC