TWO - PARAMETRIC ELASTIC - PLASTIC FRACTURE CRITERIA

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Fracture of many materials and structures is a complex process and it has both the plastic and the brittle features of fracture. In such cases, the limit fracture characteristics can be described most successfully using two parameter, reflecting the brittle and the plastic fracture, criteria.

INTRODUCTION

The limit state in case of a brittle fracture normally is characterized by the coefficient of intensity (K_{IC}, K_{IIC}, K_{IIIC}). In case of the elastic - plastic fracture, it is possible to characterize the limit sates by I- interval, δ - critical opening of a crack, and by other parameters. The evaluation of both cases is required when materials or structures fracture according to brittle and plastic features of the fracture. It is necessary to take into account the real deformation properties of the material but not to use the ideal material models only.

Two Criteria Method

According to the approximations of Irwin and Dagdale (1, 2), for a crack of size 2a in an infinite flat plate, subjected to a remote uniform tensile test in case of plane stress, one may write:

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for the Irwin's model

$$K_I = \sigma\sqrt{\pi(a + r_y)} = \sigma\sqrt{\pi a}\sqrt{1 + \frac{1}{2\pi a}\left(\frac{\sigma\sqrt{\pi a}}{\sigma_o}\right)^2} \qquad (1)$$

or after rearrangement

$$K_{ICP} = K_{IE}\sqrt{1 + \frac{1}{2}\left(\frac{\sigma}{\sigma_o}\right)^2} \qquad (2)$$

for the Dugdale - Barenblat model

$$\delta = \frac{K_{ICP}^2}{E\sigma_o} = \frac{8\sigma_o a}{\pi E}\ln\left[\frac{1}{\cos\left(\frac{\pi}{2}\frac{\sigma}{\sigma_o}\right)}\right] = \frac{8\sigma_o\left(\sqrt{\pi a}\right)^2}{\pi 2\sigma^2 E}\ln\left[\frac{1}{\cos\left(\frac{\pi}{2}\frac{\sigma}{\sigma_o}\right)}\right] \qquad (3)$$

for the Hence (3)

$$K_{ICP} = K_{IE}\sqrt{\frac{8}{\pi^2}\left(\frac{\sigma}{\sigma_o}\right)^2 \ln\frac{1}{\cos\left(\frac{\pi}{2}\left(\frac{\sigma}{\sigma_o}\right)\right)}} \qquad (4)$$

From where

$$\frac{K_{IC}}{K_{ICP}} = \frac{1}{\sqrt{\frac{8}{\pi^2}\left(\frac{\sigma_o}{\sigma}\right)^2 \ln\frac{1}{\cos\left(\frac{\pi}{2}\frac{\sigma}{\sigma_o}\right)}}} \qquad (5)$$

According to the author (4),

$$\left(\frac{K_I}{K_{IC}}\right)^{4p} + \left(\frac{\sigma}{\sigma_o}\right)^2 = 1 \qquad (6)$$

For brittle material (p=1), this criterion is written as

$$\left(\frac{K_I}{K_{IC}}\right)^4 + \left(\frac{\sigma}{\sigma_o}\right)^2 = 1 \qquad (7)$$

Equating $K_I=K_{IE}$ and $K_{Ic}=K_{ICP}$, the equation (6) can be rewritten

$$\frac{K_{IE}}{K_{ICP}} = \left[1-\left(\frac{\sigma}{\sigma_o}\right)^2\right]^{4p}, \qquad \text{.......(8)}$$

and equation (7),

$$\frac{K_{IE}}{K_{ICP}} = \left[1-\frac{\sigma}{\sigma_o}\right]^{4} \qquad \text{.......(9)}$$

For steel 08Ch18N10T, for which σ_o=1600 MPa, K_{IE}/K_{ICP} may be calculated using different criteria (2), (5), (6), and (7). Calculation results are submitted in Table 1.

TABLE 1 - Magnitudes of K_{IC}/K_{ICP} in Accordance to Various Criterions

	K_{IC}/K_{ICP}			
σ MPa	Criterion (2)	Criterion (5)	Criterion (9) p=1	Criterion (8) p=0,67
600	0,967	0,96	0,963	0,429
800	0,943	0,943	0,93	0,477
1200	0,885	0,85	0,813	0,654
1600	0,8165	0	0	0

The K_{IE}/K_{ICP} values, submitted in Table 1, are obtained using criterion (8) and deformation coefficient p calculated from formula (4)

$$p = 1-\exp\varepsilon_{x_o} \qquad \text{.......(10)}$$

The strain in the direction of the crack plane ε_{xo} for steel 08Ch10N10T is equal to 0,4 and the deformation coefficient p=0,67. As is seen from Table 1, the brittle fracture criteria including the correction of the plastic zone is not equal to zero at the yield limit and indicates the brittle fracture above this yield limit. Criteria (5), (8), (9), when the yield limit is reached, demonstrates the end of the brittle fracture.

While criteria (2), (5), (9) reflect the brittle fracture, the values of K_{IE}/K_{ICP}, obtained from criterion (9), are applied for evaluation of real deformations at the tip of a crack including the plastic strengthening of the materials. Therefore, the criterion (8) enables to analyze both the elastic and plastic types of fractures and it takes into account the real deformation laws at the tip of a crack.

It is well known, that describing the fracture process of the materials and structures by the R6 method, the relationship between K_{IE}/K_{ICP} and P/P_L may be written as follows:

$$\frac{K_{IE}}{K_{ICP}} = f\left(\frac{P}{P_f}\right) \qquad (11)$$

Then, it is possible to state, that the increments of function $f(P/P_f)$ are straight proportional to the increments of K_{IE}/K_{ICP}.
Comparing the brittle fracture, when p=1, and the fracture of real plastic materials, when p=0,67 (steel 08Ch10N10T), one can see that in the last case the ratio K_{IE}/K_{ICP} is smaller or, by other words, the influence of K_{IE} is weaker. When the stresses approach to the yield limit, the K_{IE} approaches to zero.

CONCLUSIONS

Two - parameter fracture criteria analysis demonstrates that:

1. The brittle fracture single - parameter criteria, accounting the plastic zone at the tip of a crack for plastic materials, can be applied for very low fracture stresses values only.
2. Approaching the yield limit, it is necessary to apply the two - parameter fracture criteria reflecting the process of plastic deformations.
3. The two - parameter fracture criterion, elaborated by the author, enables to analyze the materials as in the brittle as well in the purly plastic state.

SYMBOLS USED

K_I	= stress intensity factor for toughness in plane strain ($N\cdot mm^{-3/2}$)
σ	= nominal stress (N/mm^2)
σ_o	= yield stress (N/mm^2)
a	= size (mm)
r_y	= plastic zone (mm)
K_{IE}	= elastic stress intensity factor ($N\cdot mm^{-3/2}$)
K_{ICP}	= corrected value for plasticity stress intensity factor ($N\cdot mm^{-3/2}$)
ε_{xo}	= strain in the direction of the crack plane
P	= tensile or compressive load (N)
P_f	= plastic collapse tensile or compressive load (N)

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