CONTRIBUTION TO THE DEVELOPMENT OF A J ESTIMATION SCHEME FOR CIRCUMFERENTIAL THROUGH-WALL CRACKS IN PIPES

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This paper presents J-estimation formulae for through-wall cracked cylinders under tension, out-of plane bending, pressure and combined loadings. The present work is based on the KJ95 rule developed for surface cracked cylinders and elbows under combined pressure and bending (1). The objective was to select specific problems showing the weaknesses of the existing schemes (GE-EPRI method (2), R6 rule (3) as well as KJ95) and to present new formulations. The next step will be to propose to include these improvements in the RSE-M code developed by EDF for in service inspection and maintenance of Pressurised Water Reactors.

INTRODUCTION

In Leak-Before-Break analyses, a key issue is the critical crack size determination. This size may be obtained from the crack stability condition expressed in terms of the crack driving force J. With regard to the risk of Double Ended Guillotine Break, the most severe cracks are located in circumferential pipe cross sections. Several J estimation schemes are available for circumferential through-wall cracks in pipes, such as the GE-EPRI scheme or the R6 rule. However, the GE-EPRI method is not well suited for complex cases such as high level of pressure, presence of torsion or combined pressure and axial loading. The option 2 of the R6 rule allows to take into account such type of loadings, but is less accurate. Furthermore, the determination of the R6 reference stress from the limit load is no more valid for cracks non centred at the most severe location. These problems are not of concern for current Leak-Before-Break analyses. However, in sensitivity analyses, safety margins may be applied to pressure and the effect of high level of axial force or torsional moment on J estimation may be addressed. This paper proposes a scheme based on the R6 approach, but applicable to short cracks, off-centred cracks under tension, bending or pressure. The treatment of combined loadings is presented but its validation is under way.

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THEORETICAL FOUNDATIONS

The KJ rule expresses J as a product of J^e by a yield function developed by Ainsworth and on which is based R6 Option 2 is based.

$$\gamma = \frac{J}{J^e} = \frac{E\varepsilon_{ref}}{\sigma_{ref}} + 0.5\frac{L_r^2}{1 + L_r^2} \qquad (1)$$

where $\sigma_{ref} = \sigma_y L_r$ and ε_{ref} is associated to σ_{ref} by the stress-strain law.

In R6 Option 2, for through-wall cracks, L_r is defined as the ratio of the « total applied load giving rise to primary stresses to the limit load of the structure ». In the KJ95 scheme, instead of the limit load, we consider a reference load which corresponds to the loss of yield containment around the crack tip. This load, at which crack tip and global yielding interact, depends on the crack location and the strain hardening parameters. Reference loads and interaction conditions for surface cracks in pipes and elbows have been given in (1). The determination of the reference load requires first the derivation of an accurate limit load expression for the uncracked component Q_{ync}, which characterises the global plastic behavior. In a second step, this limit load is corrected to represent factors accelerating (crack size and location μ_c, and global strain-hardening effects μ_h) or delaying the crack tip yielding (crack location effect μ_g, triaxiality and local strain hardening effects μ_t) :

$$Q_{ref} = \mu_g \, \mu_h \, \mu_c \, \mu_t \, Q_{ync} \qquad (2)$$

In the cases examined in this paper, $\mu_g = 1$. We compare R6 (based on the limit load $Q_{yc} = \mu_c \, Q_{ync}$), a formulation KJLim, where the limit load is corrected by the triaxiality factor $\mu_t = 1 - 0.5*\beta/\pi$ (3), a formulation KJEcm accounting for the global hardening effect, and the KJ95 rule described in (1) which also introduces a correction for short cracks.

LOWER BOUND INTERACTION CONDITION

For a circumferentially trough-wall cracked cylinder under pressure, axial force, bending and torsional moments, the interaction surface is given by the following set of equations, valid if the crack is totally opened and $p^2 + m_t^2 < 1$ (4). Under simple load, these equations give directly the expression of μ_c.

$$\alpha_1 = -\frac{\pi}{2}\left[\frac{n_a}{\sqrt{1 - p^2 - m_t^2}} + \frac{\beta}{\pi}\left(1 + \frac{C\,p}{\sqrt{1 - p^2 - m_t^2}}\right)\right] \qquad (5a)$$

$$m_{fy} = \cos\alpha_1 \sin\alpha_2 \sqrt{1 - p^2 - m_t^2} - \frac{1}{2}\sin\theta_f \sin\beta\left[C\,p + \sqrt{1 - p^2 - m_t^2}\right]$$

$$m_{fz} = \cos\alpha_1 \cos\alpha_2 \sqrt{1 - p^2 - m_t^2} - \frac{1}{2}\cos\theta_f \sin\beta\left[C\,p + \sqrt{1 - p^2 - m_t^2}\right]$$

(respectively 5b and 5c) and where θ_f gives the location of the crack centre,

$C = \left(1 - 2\xi + 4\xi\lambda_p\right)/\sqrt{3}$, λ_p = crack face pressure/internal pressure, and α_1, α_2 are related to the θ_N angles defining the orientation of the neutral axis.

$$\alpha_1 = \frac{\pi}{2} + \frac{\theta_{N1} - \theta_{N2}}{2} \qquad \alpha_2 = \frac{\theta_{N1} + \theta_{N2}}{2} \qquad (6)$$

For a crack centred in the bending plane XZ, we have :

$$\theta_f = \pi/2 \ , \theta_{N1} = \alpha_1 \ , \theta_{N2} = \pi - \theta_{N1} \quad , m_{fz} = 0.$$

If $p^2 + m_t^2 = 1$, the position of the neutral axis is undetermined and

$$m_{fy} = -\frac{1}{2}\sin\theta_f \sin\beta\, C\, p \text{ and } m_{fz} = -\frac{1}{2}\cos\theta_f \sin\beta\, Cp \quad (7)$$

FINITE ELEMENT ANALYSES

Full three dimensional elasto-plastic finite element computations were conducted on circumferentially cracked cylinders having the same cross section (r_m = 300 mm, t = 60 mm). All the materials have a same Poisson's ratio (0.3) and follow Ramberg-Osgood stress-strain laws : $\varepsilon = \frac{\sigma}{E}\left[1 + \alpha\left(\sigma/\sigma_o\right)^n\right]$ (8)

The through-wall cracks are short (11.25 and 15 degrees). The meshes contain at least three layers of elements through the thickness. Computations were conducted using SYSTUS+ finite element code (5). The meshes are constituted with quadratic isoparametic solid elements. The Von-Mises flow rule is selected and calculations are made under the small displacement assumption. The crack driving force J has been computed through the use of the G-THETA domain integral method.

PURE TENSION

The Young's modulus value is 174700 MPa and the Ramberg-Osgood constants are σ_0 = 163 MPa, α =1 and n=7. The half crack angle value is 15 degrees. Figure 1 compares finite element results, GE-EPRI (7), R6 and KJ95 predictions. Our finite elements computations give the same results as the GE-EPRI scheme. The R6 and KJLim formulae underestimate J, on the contrary KJ95 give good predictions. In strain-hardening structures, the containment of plasticity around a small surface defect increases the defect influence on the reference stress. In the R6 rule, this effect is taken into account for surface cracks by a recharacterisation of the defect and the definition of a local yield load. The present finite element results show that for short through-wall cracks, the reference load is also a local yield load.

CRACK LOCATION EFFECT IN PURE BENDING

Material characteristics and crack size are the same as in the tension case. For the centred crack under bending, the same trend has been observed as in the tension case but the « short crack effect » is smaller under bending. Figure 2 makes clear that, for a crack which is 60 degrees off-centred from the bending plane, the J/J^e ratio is higher than in the centred case(GE-EPRI curve). The reverse would be expected from the limit load comparisons. When a crack becomes closer to the neutral axis, the limit load increases as predicted by our formula (5). The R6 J estimations have been made using the centred crack limit load, therefore the R6 underprediction is minimised by this approximation. The KJLim scheme gives the worst predictions, although the limit load used in this scheme accounts for crack location. To our opinion, this result comes from the effect of hardening on the stress distribution in the cross section under bending. The lower the hardening exponent of the Ramberg-Osgood law, the more the stress distribution departs from the plastic hinge model on which are based limit load estimations. The KJ95 scheme includes an hardening factor which reduces the reference load for surface cracks, assuming the material obeys a pure power law and the crack is small :

$$\mu_{Ph} \cong (\sin\theta_c)^{\frac{1}{n}} \qquad (9)$$

In the trough-wall crack case, the KJEcm estimation takes into account crack location and triaxiality effects on the limit load (increasing factors) as well as hardening and short cracks effects (reducing factors). However, figure 2 shows that theses improvements are not sufficient to predict accurately J/J^e. This underestimation may be due to the fact that expression (9) has been derived neglecting the elastic part in the Ramberg-Osgood law.

Figure 3 compares J predictions for a large crack (β = 45°) and a slightly different Ramberg-Osgood law (σ_0 = 200 MPa, α = 1 and n=5). KJLim gives the same results as GE-EPRI but R6 and KJ95 overestimate J. The R6 overestimation comes from an underestimation of the limit load. For deep circumferential cracks in cylinders under tension or global bending, the stress state is no more uniaxial in the ligament. This stress triaxiality increases the limit load up to 20% and is magnified by local strain-hardening effects. We have derived the triaxiality factor formula (3) from GE-EPRI results relative to axisymmetrically cracked cylinders under tension. Figure 3 proves the applicability of this factor for trough-wall cracks.

PRESSURE AND AXIAL TENSION DUE TO PRESSURE

Young's modulus value is 150000 MPa and Ramberg-Osgood constants are σ_0 = 175 MPa, α =1.634 and n = 10.49. The half crack angle value is 15 degrees. Figure 4 indicates that the GE-EPRI method, which do not consider the hoop stress, is valid only for a pressure 25% lower than the limit pressure of the sound pipe. The KJ95 predictions are very slightly unconservative. The effect of crack face pressure is negligible in the large scale yielding regime.

CONCLUSION

For cylinders with short circumferential through-wall cracks in tension and for cylinders with off-centred cracks under bending, R6 Option 2 appears to be unconservative. On the contrary, the R6 rule is too conservative for large centred cracks. The GE-EPRI scheme is well suited to analyse cylinders under uniaxial loadings but for pressure its validity is limited to the small scale yielding regime.

0ur paper explains these inaccuracies and gives the bases of a formulation for circumferentially trough-wall cracked pipes under any kind of loading. The short crack effect could be accounted for by an empirical correction as in KJ95. For off-centred cracks, the global strain hardening coefficient, developed for surface cracks, has to be revisited in the through-wall crack configuration.

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SYMBOLS USED

E = Young's modulus (MPa) and σ_y = 0.2% proof stress (MPa).

M_{fy}, M_{fz}, M_t = In plane, out of plane bending and twisting moments (kN.m).

$p_{yPnc} = 4\beta\sigma_y/\sqrt{3}$ uncracked cylinder limit pressure (MPa), p = Pressure/p_{yPnc}.

$$M_{yPnc} = 4\,\sigma_y\, r_m^2\, h \qquad m_{fy} = \frac{M_{fy}}{M_{yPnc}} \qquad m_t = \frac{M_t}{2\sqrt{3}\,\pi\,\sigma_y\, r_m^2\, h}$$

$$n_x = \frac{N}{2\pi\sigma_y R t}$$ axial force due to pressure $n_p = \frac{1-2\xi}{\sqrt{3}} p$ $\quad n_a = n_x - n_p$

r_m = pipe mean radius (mm); h = pipe thickness (mm) $\xi = h/(2r_m)$

The superscript e refers to elastically computed quantities.

β = half crack length (degree) and $U = \beta/180$.

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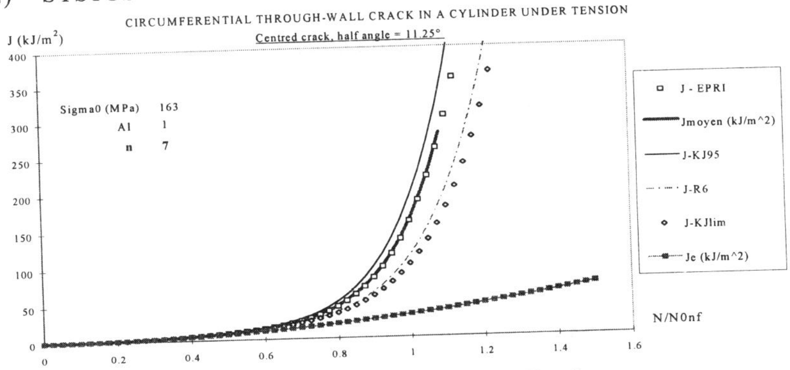

Fig. 1: F.E. computed and estimated J. Short crack - Tension.

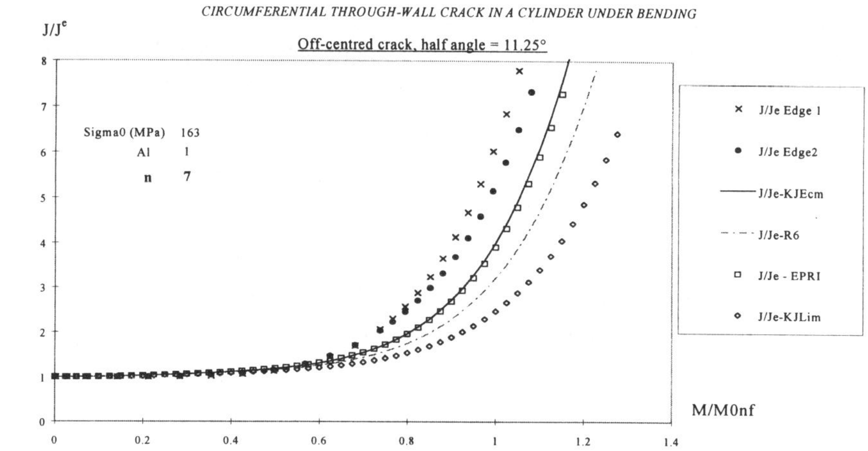

Fig. 2: F.E. computed and estimated J/J^e. Off-centred crack - Bending.

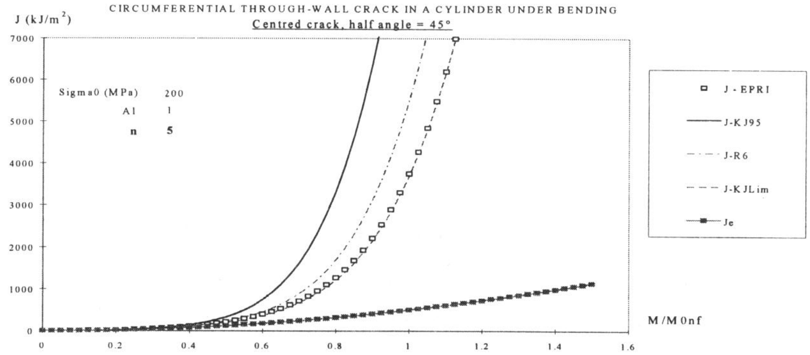

Fig. 3: F.E. computed and estimated J. Large crack - Bending.

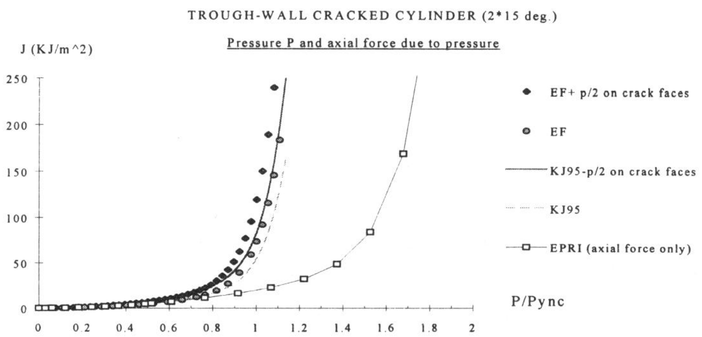

Fig. 4: F.E. computed and estimated J. Short crack - Pressure and end effect.