DAMAGE TOLERANCE PREDICTION USING PROBABILISTIC MODELLING OF A RANDOM STRESS-STRAIN RESPONSE

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In this paper an estimation of cumulative damage for dynamic loading conditions is described. Basis of the method consists in specification of an equivalent amplitude of the harmonic cycle with zero stress and strain mean values. At the same time, a relative damage must be the same as the damage of the actual cycle which has been identified by the rainflow method. The submitted approach enables to utilize the knowledge of a cumulative damage process and a fatigue life prediction under harmonic loading and to use it for any complex loading spectrum with possible impact events.

INTRODUCTION

Prediction of a damage tolerance is not only a material problem, but also loading conditions, technology design and environmental properties are very significant. Especially, the time history of an operational loading considerably effects a stress-strain material response and a fatigue life of structures. Traditional investigation methods, i. e. experimental research in laboratories or in a real service, are very expensive and time-consuming. Therefore, a computer-simulation methods are very perspective in this case.

The basis of the methods consists in a creation of both a probabilistic model of service conditions and corresponding cumulative hypotheses or failure criteria. Under random loading conditions, an equivalent amplitude of a closed cycle must be calculated, so that an actual cycle increases a cumulative damage identically as a cycle with the equivalent amplitude without residual stresses consideration. The equivalent amplitude is calculated according to cyclic stress-strain response properties because it is dependent on a total plastic deformation that corresponds to the stress parameters.

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For specification of all required stress and strain parameters of an actual closed cycle, a suitable simplified interpretation of a time history is very desirable. It can be made using a rainflow decomposition of a loading process. Moreover, regarding to a long-time loading history, a simulation must be carried out in a real-time, and therefore, the procedure must be also optimally proposed, so that the on-line signal generation could be possible. The algorithm of simultaneous signal generation and rainflow counting can be effectively used in this case [1].

CUMULATIVE DAMAGE EVALUATION

In order to evaluate the fatigue damage under dynamic loading, the cumulative representation of a relative damage of a closed cycle is accepted. The problem could arise, if the mean values of cycles are changing, because the relative damage of individual cycles cannot be simply added in this case, neither if the cycles have been identified according to the rainflow method [2]. The relative damage of a loading cycle with some definite amplitude and mean value under block loading need not correspond to the relative damage under pure harmonic loading. Moreover, the relative damage is significantly different in the case of a random loading.

The problem of the loading mean stress/strain effect is usually solved in such a way that we specify some equivalent amplitude σ_a^* in dependence of a stress mean value σ_m and strain mean value difference $\Delta\varepsilon_m$ [3]. We can derive the amplitude σ_a^* from a triaxial Haigh diagram (Figure 1) $\sigma_A = f(\sigma_M)$ for different $\Delta\varepsilon_m$, where $\Delta\varepsilon_m$ denotes the shift of strain mean value of the actual cycle from the mean value of the corresponding cycle on the cyclic curve. Supposing that the actual cycle is equivalent to the cycle with $\sigma_m = 0$ and $\Delta\varepsilon_m = 0$, we can express the equivalent amplitude as follows

$$\sigma_a^* = \sigma_a \frac{\sigma_P}{\sigma_H}. \qquad (1)$$

The relationship (1) can be linearized using a description of the triaxial diagram. Thus, we obtain the approximate relationship

$$\sigma_a^* = \sigma_a + \psi_\sigma \sigma_m + \psi_\varepsilon \Delta\varepsilon_m , \qquad (2)$$

where $\psi_\sigma = cotg\ \varphi_\sigma$ and $\psi_\varepsilon = cotg\ \varphi_\varepsilon$.

RAINFLOW DECOMPOSITION OF A RANDOM LOADING SIGNAL

In order to identify the strain mean value of the actual hysteresis loop, we can dismiss already closed hysteresis loops during cyclic process in the $\sigma - \varepsilon$ diagram, and we can only consider hitherto unclosed loops and open branches. At the same time, the first unclosed loop is called as a branch of the *zero* inclusion, the second unclosed loop that is included in the first one is called as a branch of the *first* inclusion, etc. The actual *N-th* unclosed loop is then called as a branch of the *(N-1)-th* inclusion.

The ordinates $\sigma_1, \sigma_2, \ldots, \sigma_N$ (Figure 2) represent the origins of hitherto unclosed hysteresis loops. If we know the form of a cyclic curve (OSSRC): $\sigma = \Phi_{\pm}(\varepsilon)$, for $\varepsilon \gtrless 0$, we can specify the branches $\sigma_1 \rightarrow \sigma_2, \sigma_2 \rightarrow \sigma_3, \ldots, \sigma_{N-1} \rightarrow \sigma_N$ as follows [3]

$$\sigma = (1+\kappa)\ \Phi_{+}\left(\frac{\varepsilon}{1+\kappa}\right) . \tag{3}$$

for loading ($\frac{d\sigma}{dt} > 0$), and

$$\sigma = \left(1+\frac{1}{\kappa}\right)\ \Phi_{-}\left(\frac{\varepsilon}{1+\frac{1}{\kappa}}\right) , \tag{4}$$

for unloading ($\frac{d\sigma}{dt} < 0$), where $\kappa = -\frac{R_e^-}{R_e^+}$, whereby $R_e^+ > 0$ is a yield stress in tension and $R_e^- < 0$ is a yield stress in compression.

If $R_e^+ = - R_e^-$ (i.e. $\kappa = 1$), it holds (for $\varepsilon > 0$): $\Phi_+(\varepsilon) = -\Phi_-(-\varepsilon) \equiv \Phi(\varepsilon)$ and then the relationships, can be unified using the form

$$\sigma = \pm 2\ \Phi\left(\frac{|\varepsilon|}{2}\right) \ ; \ \text{for } \varepsilon \gtrless 0 , \text{ respectively.} \tag{5}$$

The experimental results suggest that cyclic curve can be expressed in the form: $\Phi(\varepsilon) = \pm K|\varepsilon - \varepsilon_e|^n$, for $\varepsilon \gtrless 0$, where ε_e is the elastic strain, and K and n are definite constants. Then, the relationship (5) can be adapted into the form

$$\sigma = \pm 2\ \mathrm{K}\left(\frac{|\varepsilon - \varepsilon_e|}{2}\right)^n \ ; \ \text{for } \varepsilon \gtrless 0 , \text{ respectively.} \tag{6}$$

Then we could step by step identify the corresponding values: $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N$
Let us introduce

$$\Delta\varepsilon_{m_N} = \varepsilon_{m_N} - \varepsilon_{m_0} , \tag{7}$$

where ε_{m_N} is the strain mean value of actual closed loop of the *N-th* inclusion, and ε_{m_0} is the strain mean value of hysteresis loop with the same σ_a and σ_m like the actual loop, but with the origin on the cyclic curve.

Using the denotation: ε_u - the maximum strain; ε_d - the minimum strain and ε_a - the strain amplitude of the hysteresis loop, it follows:

$$\varepsilon_{m_N} = \varepsilon_{u_N} - \varepsilon_{a_N} , \ \text{respectively}\ \varepsilon_{m_N} = \varepsilon_{d_N} + \varepsilon_{a_N} \tag{8}$$

and

$$\varepsilon_{m_0} = \varepsilon_{u_0} - \varepsilon_{a_0}\ ,\ \text{respectively}\ \varepsilon_{m_0} = \varepsilon_{d_0} + \varepsilon_{a_0}\ . \tag{9}$$

Because it holds $\varepsilon_{a_N} = \varepsilon_{a_0}$ in accordance with the requested condition, from eqn. (7) we obtain:

$$\Delta\varepsilon_{m_N} = \varepsilon_{u_N} - \varepsilon_{u_0}\ ,\ \text{respectively}\ \Delta\varepsilon_{m_N} = \varepsilon_{d_N} - \varepsilon_{d_0}\ . \tag{10}$$

With respect to: $\varepsilon_{u_N} = \varepsilon_N$ (respectively $\varepsilon_{d_N} = \varepsilon_N$) and $\varepsilon_{u_0} = \Phi_+^{-1}(\sigma_N)$ (respectively $\varepsilon_{d_0} = \Phi_-^{-1}(\sigma_N)$), we further obtain the final relationship as follows

$$\Delta\varepsilon_{m_N} = \varepsilon_N - \Phi_\pm^{-1}(\sigma_N)\ , \tag{11}$$

where the sign *(+)* holds for $\sigma_{m_N} > 0$ and *(-)* for $\sigma_{m_N} < 0$.

For the start cycle, using an inversion of the cyclic curve, we get: $\varepsilon_1 = \Phi_\pm^{-1}(\sigma_1)$, where *(+)* holds for $\frac{d\sigma}{dt} > 0$ and *(-)* for $\frac{d\sigma}{dt} < 0$.

Regarding (3) and (4), for the branch of the first loop we then obtain:

$$\varepsilon_2 = \varepsilon_1 + \left(1+\kappa^{\mp 1}\right) \Phi_\mp^{-1}\left(\frac{\sigma_2 - \sigma_1}{1+\kappa^{\mp 1}}\right) ,$$

for the second branch:

$$\varepsilon_3 = \varepsilon_2 + \left(1+\kappa^{\pm 1}\right) \Phi_\pm^{-1}\left(\frac{\sigma_3 - \sigma_2}{1+\kappa^{\pm 1}}\right) ,$$

and generally for the integer $k \geq 1$:

$$\varepsilon_{2k} = \varepsilon_{2k-1} + \left(1+\kappa^{\mp 1}\right) \Phi_\mp^{-1}\left(\frac{\sigma_{2k} - \sigma_{2k-1}}{1+\kappa^{\mp 1}}\right) ,$$

$$\varepsilon_{2k+1} = \varepsilon_{2k} + \left(1+\kappa^{\pm 1}\right) \Phi_\pm^{-1}\left(\frac{\sigma_{2k+1} - \sigma_{2k}}{1+\kappa^{\pm 1}}\right) , \tag{12}$$

where the upper signs hold for $\sigma_1 > 0$, and the lower ones for $\sigma_1 < 0$.

For the most of structural materials, we suppose that $\kappa = 1$ and $\Phi_-(\varepsilon) = -\Phi_+(-\varepsilon)$, i.e. the cyclic curve for the entire range can be expressed as $\sigma = \Phi(\varepsilon)$. Then, it follows from (12), using $\varepsilon_1 = \Phi^{-1}(\sigma_1)$:

$$\varepsilon_N = \Phi^{-1}(\sigma_1) + 2\sum_{i=2}^{N} \Phi^{-1}\left(\frac{\sigma_i - \sigma_{i-1}}{2}\right) . \tag{13}$$

After substitution (13) into (11), for the loop of the *N-th* inclusion it holds

$$\Delta\varepsilon_m = \Phi^{-1}(\sigma_1) - \Phi^{-1}(\sigma_N) + 2\sum_{i=2}^{N} \Phi^{-1}\left(\frac{\sigma_i - \sigma_{i-1}}{2}\right) . \tag{14}$$

If the cyclic curve is analytically expressed by (6), we can put down an inverse relationship in the form

$$\varepsilon = \varepsilon_e \pm 2\left(\frac{|\sigma|}{2K}\right)^{\frac{1}{n}} = \frac{\sigma}{E} \pm 2\left(\frac{|\sigma|}{2K}\right)^{\frac{1}{n}} \;;\; \text{for } \sigma \gtrless 0 \,, \tag{15}$$

where E is the elasticity modulus.

After substitution into (14), we obtain

$$\Delta\varepsilon_m = \frac{\sigma_1}{E} \pm \left(\frac{|\sigma_1|}{K}\right)^{\frac{1}{n}} - \frac{\sigma_N}{E} \mp \left(\frac{|\sigma_N|}{K}\right)^{\frac{1}{n}} + \\ + \frac{1}{E}\sum_{i=2}^{N}(\sigma_i - \sigma_{i-1}) \pm 2\sum_{i=2}^{N}\left(\frac{|\sigma_i - \sigma_{i-1}|}{2K}\right)^{\frac{1}{n}} \tag{16}$$

and after adaptation, finally

$$\Delta\varepsilon_m = K^{-\frac{1}{n}}\left[\pm|\sigma_1|^{\frac{1}{n}} \mp |\sigma_N|^{\frac{1}{n}} \pm 2^{\frac{n-1}{n}}\sum_{i=2}^{N}|\sigma_i - \sigma_{i-1}|^{\frac{1}{n}}\right], \tag{17}$$

where for $\sigma_1 > 0$, $\sigma_N > 0$ and $\sigma_i > \sigma_{i-1}$, it holds the upper corresponding sign, otherwise the lower one.

CONCLUSIONS

Hitherto, only few experimental verification results have been obtained, owing to big theoretic demands of the method and to a necessity to specify different characteristics for each case. Nevertheless, the already performed results fully confirm the presented method. It can be expected that the method will be used in progressive CAD/CAE technologies, and it will create a very effective tool for a specification of databases of various expert information systems, too.

ACKNOWLEDGEMENT

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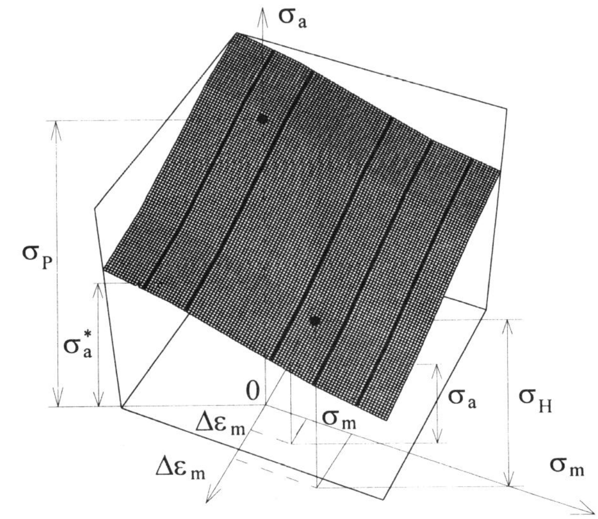

Figure 1 The triaxial Haigh diagram

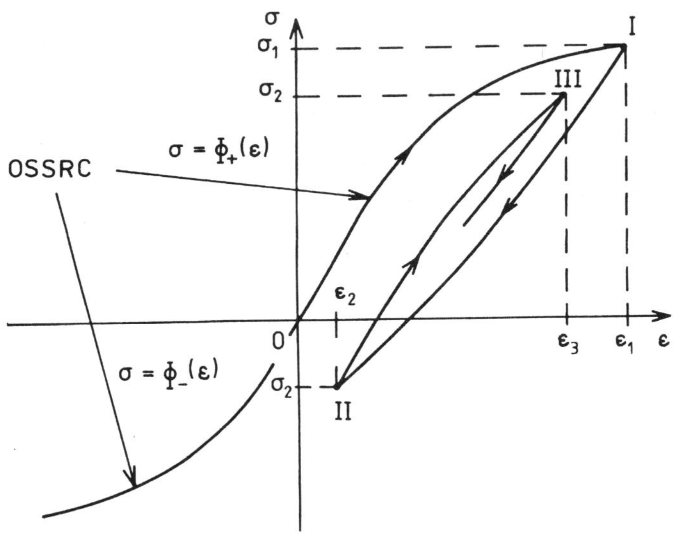

Figure 2 Branches of hysteresis loops