A STUDY OF MULTIAXIAL FATIGUE DAMAGE ACCUMULATION IN CARBON STEEL

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Engineering components experience multiaxial variable amplitude loading, which may be non-proportional. Fatigue design codes do not cover explicitly non-proportional paths, as cycle counting methods are rarely defined for multiaxial strains. Low cycle fatigue test results are presented here for Case A and B loads on a medium carbon steel, together with two level cumulative damage data. It is shown that life may be predicted from Stage I and II crack propagation rules for short cracks, driven by maximum shear strain range and a Rankine (Mode I) equivalent strain respectively. For physically realistic modelling, crack coalescence should be included to obtain optimum correlation. Crack propagation models provide a viable basis for life prediction under multiaxial low cycle fatigue, and offer an alternative to the widely used Miner linear damage summation rule.

INTRODUCTION

Generally engineering components are subjected to multiaxial variable amplitude loading, which may be non-proportional. One of the best known low cycle fatigue design codes that addresses non-proportional loading problems is the ASME Boiler and Pressure Vessel Code, but this does not include a cycle counting method (1, 2). Fatigue loading is defined by the greatest equivalent strain range that can be derived from a block of load history. But cycle counting is a requirement for analysis of variable amplitude strains found in service, where the three in-plane strains at the root of a notch may be multiaxial, out-of-phase and individually stochastic processes with their own frequencies and waveforms. Traditional rainflow methods however are usable only if there is a single load varying with respect to time, or where multiaxial loads are proportional.

Multiaxial cycle counting methods have been introduced recently by Bannantine and Socie (3), and Wang and Brown (4), where the strain history on critical planes for fatigue damage is assessed on a cycle by cycle basis throughout the lifetime. The two methods differ as planes experiencing either the maximum level of damage (3) or the maximum plastic shear deformation (4) are critical planes, that are associated with crack initiation and growth. Damage accrued in each reversal is determined after cycle counting, and then summed using Miner's linear damage law. Although it is recognised that the linear rule is flawed, no better damage summation techniques have been identified in practice.

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Life prediction procedures are usually validated with tension/torsion tests (the Case A regime of stress/strain states), if laboratory work is conducted. But for many components, e.g. pressure vessels, loading falls in the first quadrant of stress space (the Case B regime, where the minimum principal strain is normal to the free surface). This paper gives low cycle fatigue (LCF) results spanning Cases A and B, using a tubular specimen under strain control, with axial tension/compression, cyclic torsion, internal pressure and external pressure. This combination generates the full range of biaxial stress states on the specimen surfaces, with control of the principal axis direction. The axial, torsional and diametral strains were monitored on the external surface, and controlled to follow a triangular waveform. Failure was defined by a 10% drop in the stable load range, which also approximates to breakthrough of the 2 mm thick specimen. The material is a medium carbon steel (En 8), for which the short crack behaviour has been extensively studied elsewhere. Constant amplitude base-line data are presented which characterise multiaxial LCF behaviour. Secondly a range of two level cumulative damage (CD) tests have been completed for three different strain states, under proportional straining for each test with high-to-low and low-to-high sequences. The CD stress states studied fall within the Case A and B regimes, being plane strain, equibiaxial and a combination of tension and torsion.

FATIGUE TEST RESULTS AND THE LOCAL STRAIN APPROACH

Forty two multiaxial tests have been conducted over nine strain states, and strain life curves plotted for each multiaxial state. Fitting of curves for constant amplitude tests in the conventional Coffin-Manson and Basquin form to each biaxial stress state permits interpolation of data to find the fatigue strengths at a given endurance, which are shown in Fig. 1 in the form of a Gamma plane (plotting the shear and normal strains on the maximum shear plane). The endurance contours exhibit separate behaviour typical of Cases A (inclined lines) and B (vertical lines). However in Case B lower endurances were observed when fatigue cracks initiated in the bore of the tube from honing marks, produced during polishing of the test pieces. This distorts the Gamma plane contours, moving the Case A results and Case B with axial crack systems to higher strengths compared to uniaxial. This shift is seen by comparison with the predicted lines in Fig. 1.

The results may be analysed by the local strain approach, using the critical plane cycle counting methods (3, 4). Results are presented in Figure 2, which shows reasonable correlation for the constant amplitude tests, but large errors can arise in CD high-to-low predictions caused here by the intrinsic scatter in base life data. Two local analyses are presented using uniaxial material properties. The first employed an empirical multiaxial equivalent strain to determine base lives that are entered in Miner's rule, where

$$(\gamma_{\max} + S.\delta\varepsilon_n) / (1 + \nu' + S(1 - \nu')) = (\sigma'_f - \sigma_n)(2N_f)^b / E + \varepsilon'_f (2N_f)^c \quad (1)$$

Here $\delta\varepsilon_n$ is the normal strain excursion between two turning points of maximum shear strain $\gamma_{\max}$, σ_n is the mean stress normal to the maximum shear plane, and ν' is the effective Poisson's ratio. The empirical constant S is 0.2 for Case B, and 1.3 for Case A, reflecting the slopes on the Gamma plane in Fig. 1. The second correlation is derived from a Mode I crack propagation concept that stress intensity factor governs multiaxial fatigue life. Andrews (5) derives values of S = 0 for Case B and S = $(1 + \nu')/(1 - \nu')$ for Case A.

It can be seen there is only a small increase in accuracy gained by moving to an empirically fitted equivalent strain parameter here, for the majority of tests.

Both methods depend on Miner's linear rule for damage accumulation. This is the source of error in two level CD tests, where the problem of the linear law is magnified. Improved prediction is found in more random service loading histories (2). This arises because the physical mechanism of fatigue involves crack propagation by at least two stages, tensile and shear crack extension respectively, so that Miner's linear law should be replaced by a two stage damage summation to reflect the response of fatigue cracks.

CRACK PROPAGATION MODELLING OF MULTIAXIAL FATIGUE

The short crack growth laws for the carbon steel are developed here to predict CD response, treating short microstructural cracks as Stage I with extension on the maximum shear plane. Tensile fatigue cracking corresponds to Stage II growth and coalescence, which is observed after completion of the shear crack nucleation phase (6). Treatment of fatigue as a two stage process extends the damage accumulation law beyond Miner's linear rule, which should therefore show improvement over the local strain approach (7).

Cracks are driven by applied deformation, so Stage I cracks respond to the maximum shear strain range in persistent slip bands (8), and Stage II cracks to a Rankine strain range. For a Stage I Mode II crack in polycrystalline material, the microstructural influence on growth is principally a barrier to propagation of surface length d, such that for a crack depth c in En 8 medium carbon steel (9)

$$[dc/dN]_{II} = 36700(\Delta\gamma_{\max})^{3.51}(\beta d/2 - c) \qquad (2)$$

This equation embodies the remaining plastic slip band ($\beta d/2$ - c) between the crack tip and the barrier, as predicted in dislocation models of the mechanism of Stage I cracking. The aspect ratio β for a 3-D crack is $2c/a$ (surface length $2a$), and βd/2 is taken as 116 μm for the carbon steel. Fatigue life can be determined by integrating equation (2) from $c = 0$ up to the transitional crack length c_i, for a Stage I Mode II crack.

Mode I crack growth is frequently described for linear elastic conditions by a Paris law, which can be modified to incorporate a strain intensity factor $(\Delta K/E)$ that governs both LEFM and EPFM behaviour, giving a more general expression than Paris (10). Replacing $\Delta\sigma/E$ for push-pull with an equivalent strain range $\Delta\gamma_R$ derived to represent the strain intensity factor, the Paris law with a threshold term becomes for En 8 steel (9)

$$[dc/dN]_I = 0.427(\Delta\gamma_R)^{2.06}c - 0.00212 \qquad (3)$$

$$\Delta\gamma_R = \Delta\sigma_1 . \Delta\varepsilon_{eq} / \Delta\sigma_{eq} \qquad (4)$$

where dc/dN is in μm/cycle and c is in μm. The subscript eq represents von Mises equivalent values. Integrating this equation from c_i (crack depth for the Stage I to Stage II transition) to c_d (final crack depth for Mode I) gives the Mode I propagation life. Cracks grow on critical planes defined by the applied strain field, i.e. Stage I on the shear plane under $\Delta\gamma_{max}$ control, and Stage II on the first principal plane under $\Delta\gamma_R$ control. The crack speed (11) will be the fastest of those derived from equations (2) and (3). A Stage I and II model provides a simple but effective model for fatigue. However for non-proportional loading, the shear modes of growth may dominate over Stage II, and further shear growth laws may be needed in analysis, as well as Mode I (10). Once again, the mechanism of growth is selected by the fastest growth rate criterion (11).

The crack growth laws are integrated between fixed initial crack length of zero for Stage I and a final crack length (160 μm) that is fitted with torsional data. Predicted lives are shown in Fig. 3, as a "single crack integration". Good life prediction is found, but life is overestimated when cracks form in the specimen bore at honing marks, or in 2 level CD tests. The final crack depth is physically unrealistic, compared to the 2 mm wall thickness.

However validation of the predicted lives against the experimental data in Fig. 3 shows that surface finish in the specimen bore and the coalescence of LCF cracks should be taken into account. Integration of a single crack's extension tends to overestimate life, but the inclusion of a coalescence model for 15 microcracks provides better correlation. The final crack depth is taken at 10% of the specimen cross section (the failure definition), or 2.7 mm for a semicircular crack. More cracks (i.e. 23) are able to initiate in the bore from a honing mark, giving the reduced endurances observed. We use here the crack coalescence model of Gao (7, 8), where no coalescence is seen for short Stage I cracks, but after transition to Stage II, coalescence progresses at a constant rate until failure. For n subcracks of combined depth C (the sum of all the depths), in Mode I

$$[dC/dN]_I = 0.427(\Delta\gamma_R)^{2.06} C - 0.00212.n \qquad (5)$$

The initial number of cracks n (15 or 23) is the critical number that coalesce to form the single final failure crack, and therefore the rate of growth of these cracks is a specific measure of fatigue damage. Gao (7) observed that the rate of coalescence reduced n linearly with crack depth C. Integration of equation (5) from $n.c_i$ gives the Stage II life.

The random process in fatigue have many contributory factors, but the correlation of multiaxial response suggests this deterministic model provides the essential description of multiaxial behaviour. Particularly important statistical factors arise from a) coalescence, b) the microstructural barrier d, c) uncertainty over the applied loads, d) non-proportional loading, e) mixed-mode branching and different crack planes, f) material properties, and g) geometrical inaccuracy in manufacture. Figure 4 shows the probability of failure for these tests, compared to previous work for non-proportional loading in tension/torsion for En15R steel (2, 4). The similar probability distributions, even after normalising to take out multiaxial path dependency, show that the equivalent strain in the local method gives almost the same accuracy as the propagation approach for random loads, except for the special case of two level CD tests.

CONCLUSIONS

More accurate damage analyses are feasible replacing Miner's linear law with a double-linear, multiaxial rule for Stage I and II cracks with coalescence in Stage II. Multiaxial behaviour can be predicted from uniaxial crack growth laws, without the need for multiaxial correlation factors, for the full range of stress states.

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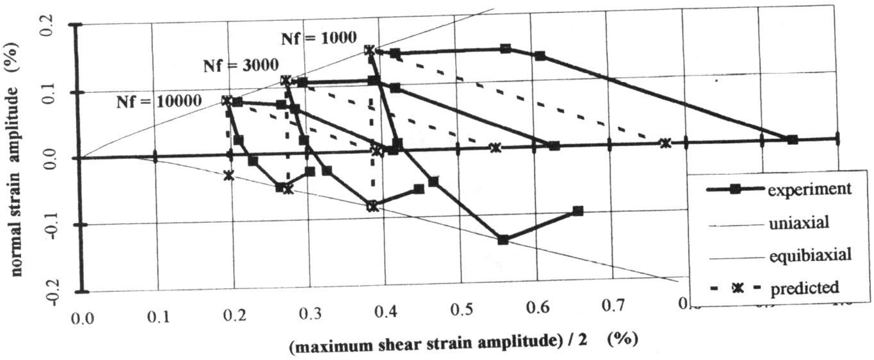

Fig. 1. Gamma plane for En 8 medium carbon steel, with predictions for Mode I cracking.

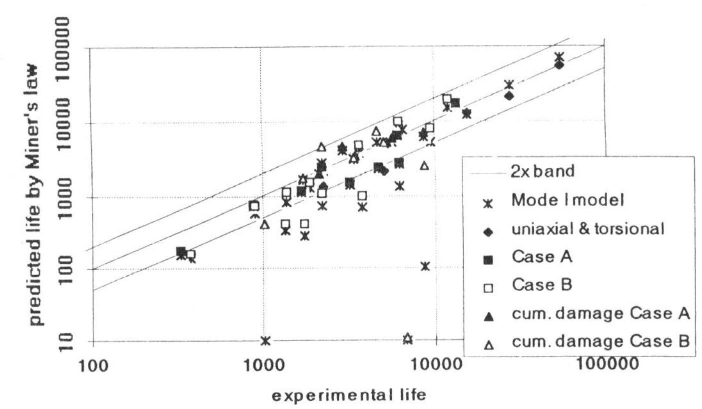

Fig. 2. Multiaxial fatigue life prediction for carbon steel by the local strain approach.

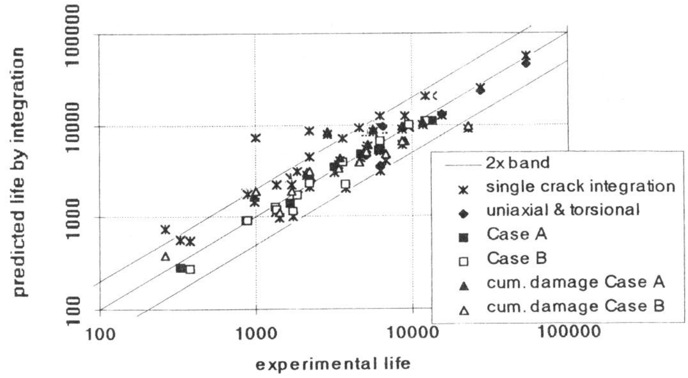

Fig. 3. Multiaxial life prediction from crack growth, with and without coalescence.

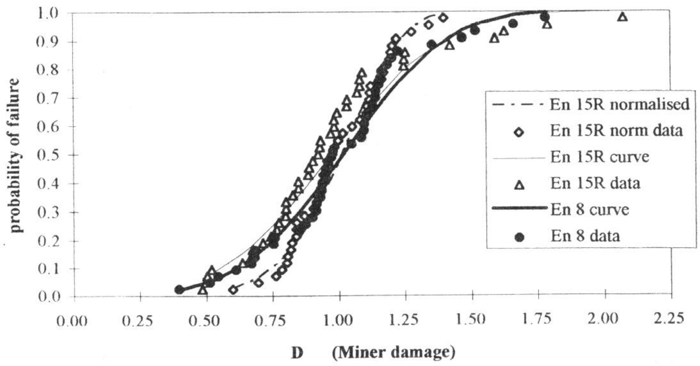

Fig. 4. Multiaxial fatigue probability of failure, for two steels with random loading CD.