

Strain Energy Density Based Modeling of Crack Growth for Sugar Mill Roll Shafts

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ABSTRACT. *The strain energy density factor criterion is used to model the growth of both circumferential and semi-elliptical fatigue cracks in sugar cane mill crusher shafts. Shaft dimensions and loads are based on the mill shafts of the CAI “Guillermo Moncada” sugar mill in Cuba. About 85% of the Cuban sugar mill shaft failures occur in the shoulder of the bearing nearest to the square box coupling. In terms of failure potential, the top shaft is the most critical. Calculations show that inspection intervals as short as once every 43 days are needed to avoid unexpected failure. Application of the strain energy density criterion is slightly more complicated and gives slightly shorter predicted fatigue lives than the equivalent stress intensity factor criterion. However, it offers more information about the expected growth direction of the crack which is potentially beneficial during NDE inspections. Calculated crack inclination angles are found to be in good agreement with crack angles reported for failed shafts.*

INTRODUCTION

Sugar cane mills use high compression loads in the extraction of juice from sugar cane. The cane is first sliced in a process that opens the plant cells and facilitates the juice extraction. The prepared sugar cane is then compressed by passing through a series of three to six mills each containing three or four crushing rollers. The lower crushing roller rotates in a fixed position while the upper crushing roller rises and falls freely according to the variation in sugar cane flow. Figure 1 shows a schematic diagram of a typical three-roller sugar mill.

The crushing rollers are made of coarse grain cast iron and are interference fitted on to steel shafts that transfer the needed torque. Shafts are mounted on lateral frames known as “virgins”. Mills usually operate with hydraulic forces of between 300 and 700 tons and the angular speed of the rollers is low, normally between 3 and 10 rpm.

Local stresses are very high in some locations of the mill roller shafts and fatigue is a frequently observed failure mode. Several researchers have carried out studies on the loads in the mill shafts [1-6]. Most failures occur in the bearing shoulder closest to the square box coupling of the shaft where the driving torque is applied. Several studies have analysed fatigue failures in sugar mill shafts based on simplified assumptions of

the stresses and modes of loading [1, 7]. The life span of shafts does not usually exceed the processing time for three or four sugar cane crops, i.e. about 120 days [8]. Unexpected shaft failure during the processing of a crop is expensive both in terms of lost wages, damage to other equipment and decrease in the sugar quality if the cane is not crushed promptly after harvest. For this reason, it is desirable to have in place a non-destructive evaluation plan that includes detection of cracks in the regions where the most critical faults occur.

ANALYSIS METHOD

Evaluation of typical loads on the mill shafts and a fracture mechanics analysis based on a strain energy release rate equivalent stress intensity factor criterion have been previously reported [9].

Sih [10] proposed a criterion for mixed mode loading known as the strain energy density factor, S . It is based on the strain energy density around the crack tip given by

$$S = a_{11}K_I^2 + 2a_{12}K_I K_{II} + a_{22}K_{II}^2 + a_{33}K_{III}^2 \quad (1)$$

where the coefficients under plane strain are

$$\begin{aligned} a_{11} &= \left(\frac{(1+\nu)}{\pi 8 E} \right) [(3-4\nu-\cos\theta)(1+\cos\theta)] \\ a_{12} &= \left(\frac{(1+\nu)}{\pi 8 E} \right) [2 \sin\theta \{ \cos\theta - (1-2\nu) \}] \\ a_{22} &= \left(\frac{(1+\nu)}{\pi 8 E} \right) [4(1-\nu)(1-\cos\theta) + (1+\cos\theta)(3\cos\theta-1)] \\ a_{33} &= \frac{(1+\nu)}{\pi 2 E} . \end{aligned}$$

Here, θ is the angle between the crack plane and the principal stress plane. The necessary and sufficient conditions for crack growth are

$$\begin{aligned} \frac{\partial S}{\partial \theta} &= 0 \quad \text{at } \theta = \theta_o \\ \frac{\partial^2 S}{\partial \theta^2} &> 0 \quad \text{at } \theta = \theta_o . \end{aligned} \quad (2)$$

Crack extension occurs when the strain energy density factor reaches a critical value in a direction defined by θ_o . This will be the direction of minimum strain energy density. For cyclic loading a cyclic strain energy density factor is defined as

$$\Delta S = 2 \left[a_{11}(\theta_o) K_I^{\text{mean}} \Delta K_I + a_{12}(\theta_o) (K_{II}^{\text{mean}} \Delta K_I + K_I^{\text{mean}} \Delta K_{II}) + a_{22}(\theta_o) K_{II}^{\text{mean}} \Delta K_{II} + a_{33}(\theta_o) K_{III}^{\text{mean}} \Delta K_{III} \right] \quad (3)$$

This equation includes both stress range and mean stress. Before computing the strain energy density factor the direction of crack growth θ_o must be determined from Eq. 2. The crack growth rate is directly related to ΔS

$$\frac{da}{dN} = C_s (\Delta S)^{\frac{m}{2}} \quad (4)$$

The value of the constants can be determined from the standard Paris crack growth constants as

$$C_s = C \left(\frac{2 \pi E}{(1-2\nu)(1+\nu)} \right)^{\frac{m}{2}} \quad (5)$$

For purposes of this analysis, crack propagation is not considered for negative values of normal stress the crack. In this case a crack is considered to be closed. A compressive normal stress also has the effect of increasing friction between the crack faces thus inhibiting shear crack.

To establish an adequate plan of inspection, it is necessary to define a minimum size of crack that can be reliably detected by the inspection personnel. This value depends on the equipment that is used, the accessibility and facility of detection, and finally and no less important, the qualification and experience of the personnel. For the current investigation a value of 0.5 mm is selected. The maximum permissible crack size is based on the expected minimum fracture toughness of the material, 80 MPa m^{1/2}, and the maximum expected overload stress in the critical location. The overload stress values chosen for the calculations are 40% greater than the normal operating stresses. Critical sizes of cracks were 44 mm, 148 mm and 97 mm for the top, front, and back shafts, respectively.

RESULTS AND DISCUSSION

Crack Growth

Based on the crack growth rate between the assumed detection limit size, $a_d = 0.5$ mm and the final fracture size, inspection intervals can be computed. Table 1 shows the

computed inspection interval both in cycles and days for the three shafts. For comparison, both semi-circular surface cracks and circumferential cracks are considered. As an example, predicted crack propagation in the shoulder of the top shaft is shown in Figure 2 for both semi-circular surface cracks and circumferential cracks.

Table 1. Predicted inspection interval for crack growth from a_d up to a_{final} .

| <i>Shaft</i> | <i>Semi-circular cracks</i> | | <i>Circumferential cracks</i> | |
|--------------|-----------------------------|-------------------------|-------------------------------|-------------------------|
| | ΔN_{max} (cycles) | Δt_{max} (days) | ΔN_{max} (cycles) | Δt_{max} (days) |
| Top | $3,3 \cdot 10^5$ | 56 | $2,5 \cdot 10^5$ | 43 |
| Front | $6,3 \cdot 10^7$ | 11 000 | $4,0 \cdot 10^7$ | 6 900 |
| Back | $3,0 \cdot 10^6$ | 520 | $2,0 \cdot 10^6$ | 340 |

If the average period of time required to process one crop is 120 days, it can be seen that based on these predictions the back shaft only needs to be inspected once for every two crop seasons. It is not necessary to inspect the front shaft because the computed inspection interval exceeds the useful life of the shaft. It only needs to be guaranteed that the shaft has no defects larger than 0.5 mm at the initiation of service. According to these calculations the highly critical top shaft should be inspected every 43 days of operation to assure that an unexpected fault will not happen as a result of a circumferential crack. The assumption of a circumferential crack in the current analysis is conservative because the formation of such a crack assumes the distribution of small flaws around the circumference of the shaft.

Crack Orientation

It has been observed in laboratory experiments that cracks tend to always turn and grow in a direction normal to the maximum principal normal stress [10]. In other words, a crack will try to propagate by mode I of loading whenever possible. Equation 2 illustrates that the predicted direction of crack propagation is the angle corresponding to minimum strain energy density. Figure 3 shows the predicted crack orientation for semi-elliptical cracks propagating in the shoulder of a sugar cane mill back shaft. This figure represents on the instantaneous propagation angle vs. depth. For example, a semi-elliptical crack in the back shaft has a crack inclination from 10° for a crack depth of 20 mm up to an inclination of 25° for a crack depth of 60 mm. In other words, crack will tend to gradually curve so as to reduce the strain energy density.

Table 2 shows some actual crack orientation data for fractured sugar mill shafts [1,2]. Unfortunately the orientation measurement method is not reported and the crack depth is not reported. This is an area that should be further studied. Most data falls in the 6-15 degree range, which, according to Fig 3, corresponds to a depth of 12-35 mm.

It can be shown that a circumferential crack always propagates in the plane of the shaft cross section for mixed-mode I–III loading. On the other hand, a semi-elliptical crack for I-II-III mixed-mode of loading will initially propagate in the plane of the shaft cross section but will gradually turn. The exact direction shift will depend on the ratio K_{II}/K_I .

Table 2. Observed crack inclination for fractured shafts [1,2].

| Number of fractures | Crack inclination (°) |
|---------------------|-----------------------|
| 13 | 0 – 5 |
| 28 | 6 – 10 |
| 23 | 11 – 15 |
| 8 | 16 - 20 |

CONCLUSIONS

Non-destructive inspection intervals for crushing roller shafts for a sugar mill have been determined based fracture mechanics analysis. Due to the large shear stresses and the out-of-phase nature of the normal and shear stresses, a multiaxial crack growth criteria based on equivalent strain energy release rate has been used. The mill design evaluated is expected to be very severe in terms of fatigue loading.

Of the three shafts considered, it was determined that the front shaft was least critical in terms of fatigue failure and it should not be necessary to inspect this shaft during its normal design life. The back shaft should be inspected every 240 days, which is about once every two crop seasons, to ensure that no unexpected failures occur. The most heavily loaded top shaft should be inspected every 43 days. This interval is far less than one crop season even for the demanding crack detection size of 0.5mm and re-design of the shaft shoulder should be considered.

The strain energy density criterion is slightly more complicated than the equivalent stress intensity factor approach to multiaxial crack propagation analysis, however, it offers more information about the expected inclination of the crack that can be useful during NDT.

The predicted crack angle for semi-elliptical cracks is qualitatively in agreement with crack angles for failed sugar mill shafts. However, details of the crack angle measurements are not given.

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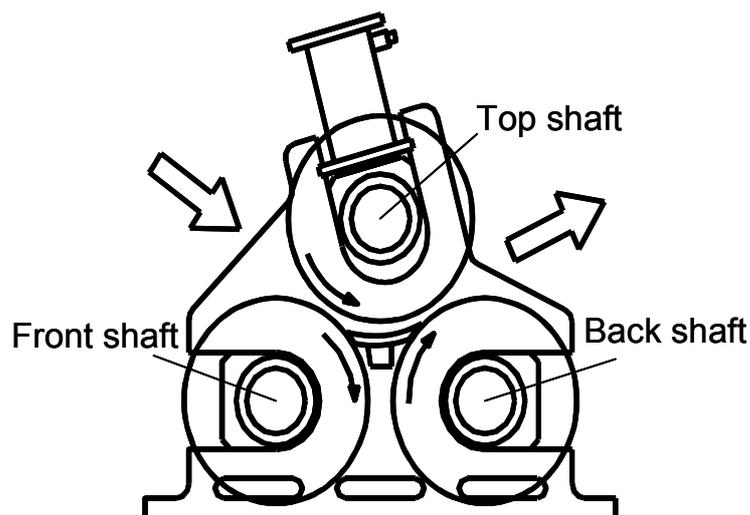


Figure 1. Schematic illustration of a typical three-roll sugar mill crusher.

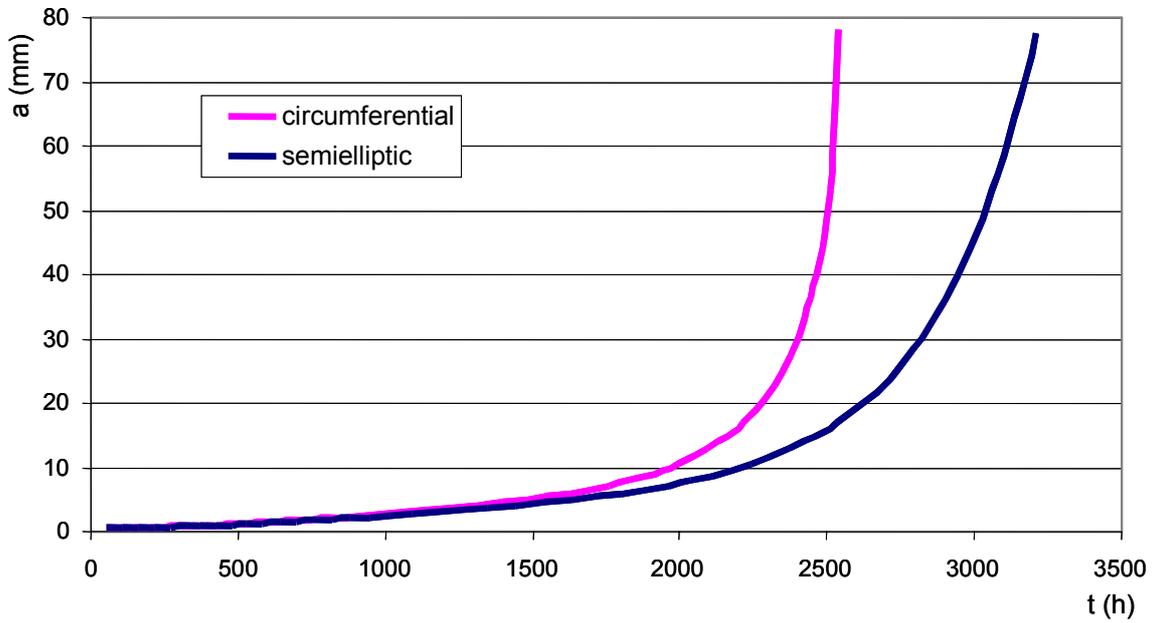


Figure 2. Crack growth predictions for the top shaft.

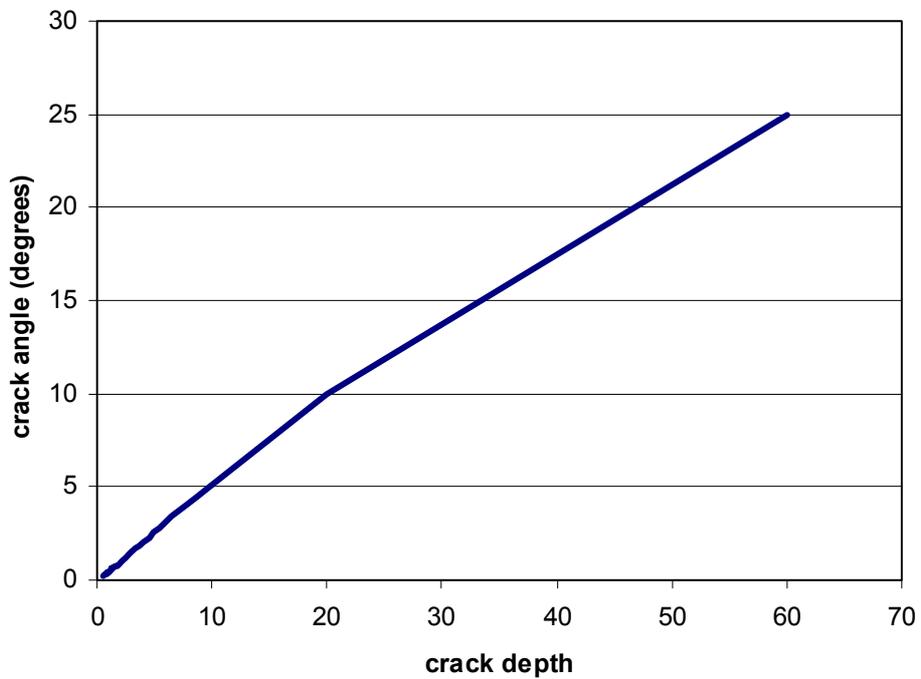


Figure 3. Predicted crack angle vs. depth for semi-circular crack in back shaft.