Elastic-Plastic Fatigue Crack Propagation under Mixed-Mode (I+III) Cyclic Torsion and Axial Loading

Hirohisa Kimachi[1], Keisuke Tanaka[2], Yoshiaki Akiniwa[2] and H. Yu[3]

[1]Department of Mechanical Engineering, Meijo University, Japan
[2]Department of Mechanical Engineering, Nagoya University, Japan
[3]Mechanical Properties Lab., Beijing Institute of Aeronautical Materials, Beijing China

ABSTRACT. *The propagation behavior of a circumferential crack in a cylindrical bar of a stainless steel was studied under various combinations of cyclic torsion and axial loading. The J-integral was found to be the most appropriate fracture mechanics parameter for characterizing flat-type fatigue crack propagation under mode I and III loading. In order to estimate the J-integral value under mixed mode I and III loading, elastic-plastic stress analysis of a circumferential crack under mode I, III and mixed-mode loading, was conducted by the finite element method. The J-integral value under mixed-mode loading obtained by the simple method was in good agreement with that estimated from the energy method. The relation between the J-integral value under mixed-mode loading and various fracture mechanics parameters, for example crack tip opening displacement and crack tip sliding displacement, was examined. The transition condition from a factory-roof type to a flat type of the mode III fatigue fracture surface was proposed on the basis of the crack sliding displacement at the crack tip.*

INTRODUCTION

Fatigue cracks often show a mixed-mode propagation of mode I and III in power train shafts subjected to cyclic torsional and axial loading. Those cracks are often accompanied with excessive plasticity. The mode III fatigue fracture surface was changed from a factory-roof type to a flat type with increasing the load level [1, 2]. Various fracture mechanics parameters for flat-type fatigue crack propagation with excessive plasticity were proposed, for example, the plastic strain intensity factor [1, 3, 4] and the crack tip sliding displacement [1, 2]. Tanaka et al. proposed the *J*-integral range as an appropriate parameter controlling mode III fatigue crack propagation [5], and further applied it to elastic-plastic fatigue crack propagation under mixed-mode loading [6].

In the present work, the propagation behavior of a circumferential crack in cylindrical bars of a stainless steel was studied under cyclic torsion and axial loadings. First, the finite element analysis was conducted to investigate the applicability of the simple method of *J*-integral estimation by comparing with the *J*-integral value determined by the energy method. Next, the mixed-mode fatigue crack propagation behavior was analyzed by using the *J*-integral. The effect of mode mixity on the crack propagation rate was examined from the view points of the *J*-integral and the striation spacing.

J-INTEGRAL ESTIMATION FOR MIXED-MODE CRACK

Energy Method

Consider a circumferentially cracked cylindrical bar under an axial load, *P*, and torque, *T*, as shown in Fig. 1(a). The *J*-integral value can be determined by the change of the strain energy with crack extension under constant-displacement, *u*, or constant-twist angle, θ (Fig. 1(b)). The *J*-integral values of mode I under axial loading, J_{I}, and of mode III under torsion, J_{III}, were given by

$$J_{\mathrm{I}} = -\frac{1}{2\pi b}\int_0^u \left(\frac{\partial P}{\partial a}\right)_u du = -\frac{1}{2\pi b}\left(\frac{\partial U_{\mathrm{I}}}{\partial a}\right)_u \tag{1}$$

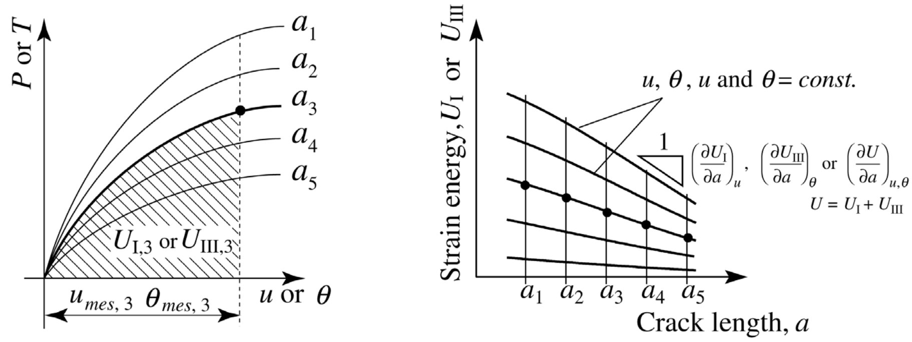
$$J_{\mathrm{III}} = -\frac{1}{2\pi b}\int_0^\theta \left(\frac{\partial T}{\partial a}\right)_\theta d\theta = -\frac{1}{2\pi b}\left(\frac{\partial U_{\mathrm{III}}}{\partial a}\right)_\theta \tag{2}$$

where U_{I} and U_{III} are energy for mode I and III loadings, respectively.

In mixed-mode (I+III) loading, the *J*-integral value is determined by the change of the total strain energy with crack extension. Since the total strain energy is the sum of the mode I and III strain energies under constant-displacement and twist angle, the *J*-integral value of mixed mode, $J_{\mathrm{I+III}}$, was given by

$$J_{\mathrm{I+III}} = -\frac{1}{2\pi b}\left(\frac{\partial U_{\mathrm{I+III}}}{\partial a}\right)_{u,\theta} \tag{3}$$

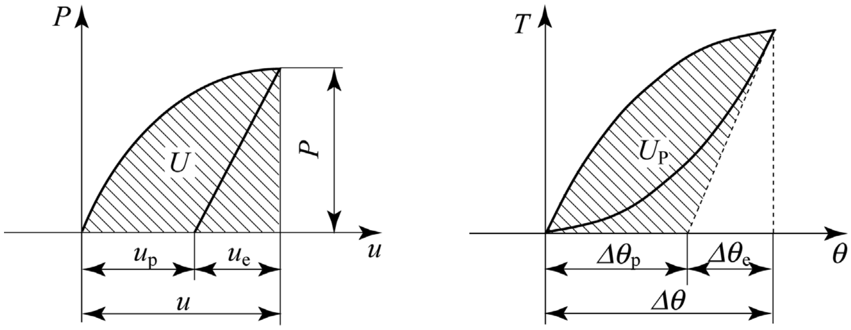
Although *J*-integral values estimated from the energy method are exact, it is very difficult to apply the energy method to fatigue tests.



(a) Load vs. displacement or Torque vs. twist angle

(b) Strain energy vs. crack length

Figure 1. *J*-integral estimation from the energy method.



(a) Load vs. displacement (b) Torque vs. twist angle

Figure 2. *J*-integral estimation from the simple method.

Simple Method

A simple estimate of the *J*-integral can be obtained from the loading part of load vs. displacement or torque vs. twist angle curve. Figure 2 illustrates typical curve of load vs. displacement and of torque vs. twist angle under mode I and III loadings.

If the plastic deformation zone extends only in the ligament part of a specimen, the mode I *J*-integral value, J_{I}, is estimated by [7]

$$J_{\mathrm{I}} = \frac{1-\nu^2}{E} K_{\mathrm{I}}^2 + \frac{1}{2\pi b^2}\left(3\int_0^u P du - Pu - \frac{1}{2} P u_e \right) \tag{4}$$

where K_{I} is the mode I stress intensity factor, P, u and u_{e} are also shown in Fig. 2 (a). The mode III *J*-integral value, J_{III}, is estimated by [7]

$$J_{\mathrm{III}} = \frac{1+\nu}{E} K_{\mathrm{III}}^2 + \frac{3}{2\pi b^2} U_{\mathrm{p}} \tag{5}$$

where b is the radius of the ligament of the specimen, K_{III} is the mode III stress intensity factor and U_{p} is the energy corresponding to the shaded area shown in Fig. 2 (b).

For the case of mixed-mode (I+III) loading, we assume the *J*-integral value, $J_{\mathrm{I+III}}$, equals the sum of J_{I} and J_{III}:

$$J_{\mathrm{I+III}} = J_{\mathrm{I}} + J_{\mathrm{III}} \tag{6}$$

In the next paragraph, the applicability of the simple method was discussed in comparison with the *J*-integral value estimated from the energy method on the basis of the elastic-plastic finite element analysis.

Elastic-Plastic Finite Element Analysis

The elastic-plastic analysis of a cylindrical bar with a circumferential crack under tension and torsion was conducted by the finite element method. The analysis was carried out by the software MARC. The analyzed rectangular region and dimensions are shown in Fig. 3. In this analysis, the crack length was varied from 2.0 to 5.0 mm. Only the rectangular region shown in Fig. 3 was analyzed because of the axisymmetry with respect to $y = 0$. The analyzed region is composed of 460 axisymmetric eight-node isoparametric elements and 1493 nodes. Displacement, u_{app}, or/and twist angle, θ_{app}, was applied to the end of the model. In mixed-mode (I+III) case, the ratio of u_{app}/θ_{app} is set to be 0.4.

Material Properties
A material modeled in FEM analysis was a stainless steel (JIS SUS316NG). The material has a Young's modulus of 196 GPa and a Poisson's ratio of 0.265, and is modeled as elastic–plastic materials following the von Mises yield condition. The constitutive equation between the equivalent stress, $\bar{\sigma}$, and strain, $\bar{\varepsilon}_p$, is given by

$$\bar{\sigma} = c(d + \bar{\varepsilon}_p)^n \tag{7}$$

where $c = 1250$ MPa, $d = 0.0245$ and $n = 0.439$.

Comparison between Simple Method and Energy Method by FEM analysis
Figure 4 shows the relation between the J-integral values obtained by above two methods and crack lengths for the case of the mixed-mode (I+III) loading. J-values by the energy method show solid lines and the values by the simple method are symbols in Fig. 4. The J-integral value estimated from the simple method was in good agreement with that estimated from the energy method in the range of displacement $u < 0.0625$ mm and twist angle $\theta < 0.025$ rad for cracks longer than 2 mm. Therefore, the simple estimate method is applicable to long circumferential cracks in cylindrical bars under a combination of tension and torsion.

MIXED-MODE FATIGUE CRACK PROPAGATION

Material and Specimen
A cylindrical bar of a stainless steel (JIS SUS316NG) with a diameter of 16 mm was solution treated. The 0.2 % offset stress was 254 MPa, and the tensile strength was 564 MPa. A circumferential notch of 3.0 mm depth was made, and a pre-crack of a depth of 0.40 mm was introduced below a circumferential notch by cyclic compression. All the specimens were annealed at 1173 K for 30 min before fatigue testing.

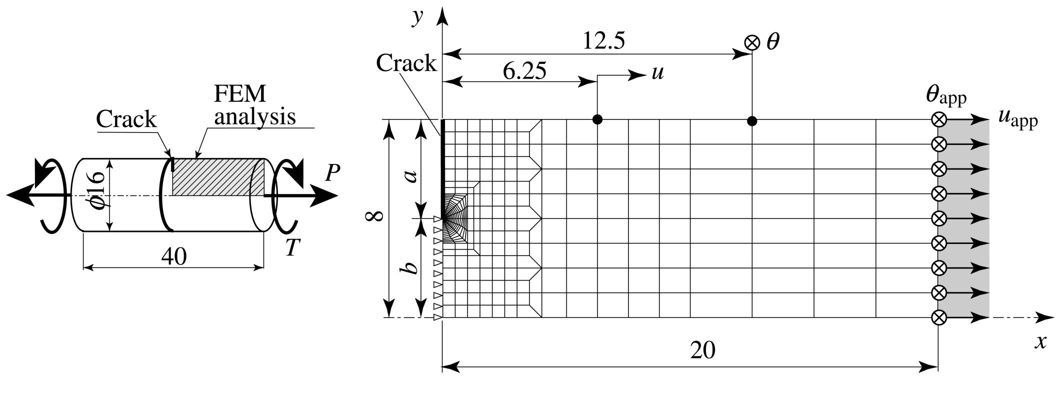


Figure 3. Finite element model.

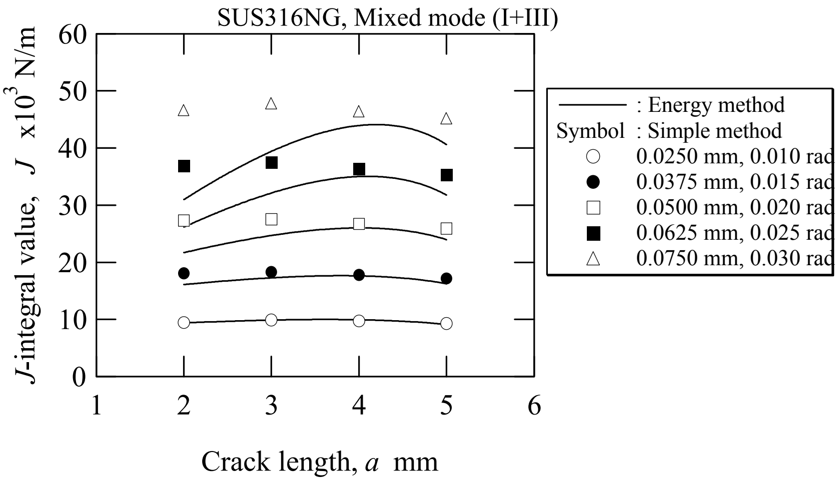


Figure 4. Relation between *J*-integral value and crack length.

Fatigue Testing

Fatigue tests were conducted in a computer-controlled electro-servo hydraulic tension-torsion fatigue testing machine (Shimadzu EHF-ED10/TQ-40L). Mixed-mode fatigue tests were conducted under the torsional-angle and axial-displacement controlled conditions. The wave shape was triangular and the displacement was completely reversed under the in-phase condition. The range of the *J*-integral was kept constant for pure mode III crack propagation tests. The value of *J*-integral range during mixed-mode fatigue tests was calculated by the simple method described in the following paragraph. The amount of crack extension was determined by the d.c. electrical potential method [6]. After the crack was propagated about 1 mm, the specimen was fractured under low stress cycling with a high stress ratio of 0.5. The fracture surface was examined with scanning electron microscopy (SEM).

J-Integral Estimate

The *J*-integral range was estimated from the simple method during fatigue tests. Figure

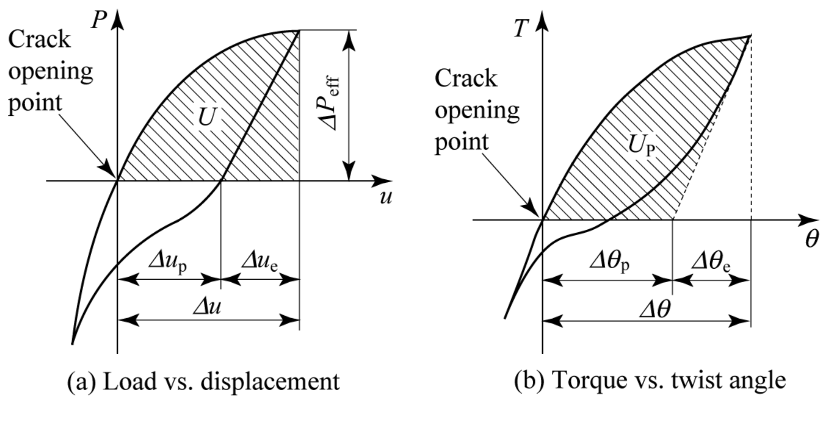


Figure 5. Estimation of *J*-integral range of mixed-mode (I+III).

5 illustrates the hysteresis loops of load vs. displacement and of torque vs. angle of twist obtained under mixed-mode (I+III) loading. The crack-tip opening point was determined from the unloading compliance method.

The mode I and mode III *J*-integral ranges, ΔJ_{I} and ΔJ_{III}, are estimated from the loading part above the opening point by the following equations.

$$\Delta J_{\mathrm{I}} = \frac{\Delta K_{\mathrm{Ieff}}^2}{E}(1-\nu^2) + \frac{3}{2\pi b^2}\left(3\int_0^{\Delta u} P du - \Delta P_{\mathrm{eff}} \Delta u - \frac{1}{2}\Delta P_{\mathrm{eff}} \Delta u_{\mathrm{e}}\right) \tag{8}$$

$$\Delta J_{\mathrm{III}} = \frac{K_{\mathrm{IIIeff}}^2}{E}(1+\nu) + \frac{3}{2\pi b^2} U_{\mathrm{p}} \tag{9}$$

Where ΔK_{Ieff} and $\Delta K_{\mathrm{IIIeff}}$ are the mode I and III effective stress intensity ranges. E is Young's modulus, ν is Poisson's ratio and b is the radius of the ligament of the specimen. ΔP_{eff}, Δu and Δu_{e} are also shown in Fig. 5(a), and U_{p} is the energy corresponding to the shaded area shown in Fig. 5(b). *J*-integral values for pure mode I and III were estimated from Eqs. (8) and (9).

For the case of mixed-mode (I+III) loading, the total range $\Delta J_{\mathrm{I+III}}$ can be given as the sum of ΔJ_{I} and ΔJ_{III}:

$$\Delta J_{\mathrm{I+III}} = \Delta J_{\mathrm{I}} + \Delta J_{\mathrm{III}} \tag{10}$$

In the following, we denote $\Delta J_{\mathrm{I+III}}$ by ΔJ for simplicity.

Fatigue Crack Propagation

Mixed-mode fatigue tests were conducted under $\Delta J_{\mathrm{III}}/\Delta J_{\mathrm{I}}$= 0.5, 1, and 2 at about ΔJ

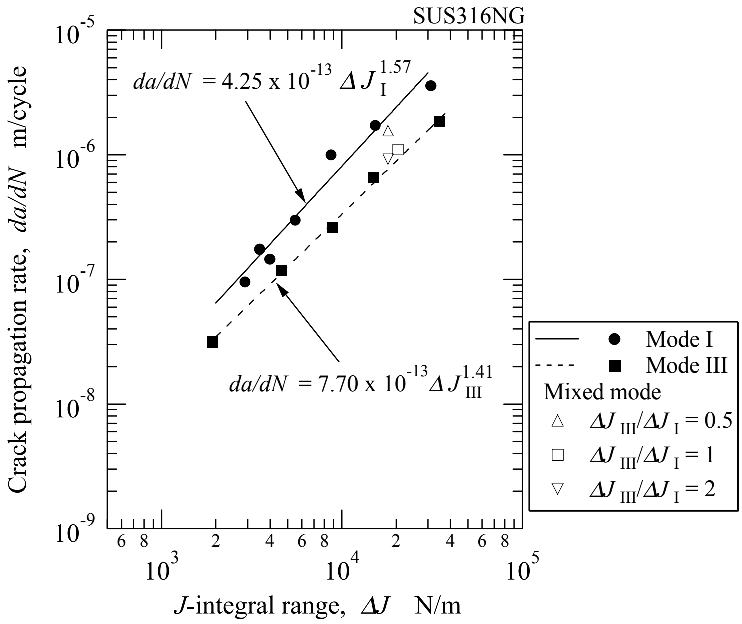
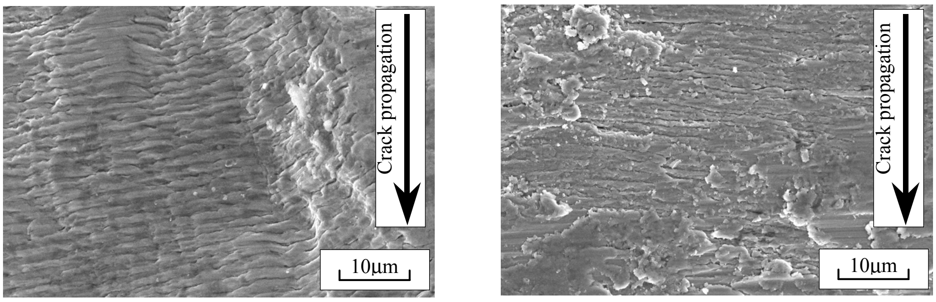


Figure 6. Relation between crack propagation rate and ΔJ.



(a) $\Delta J_{III}/\Delta J_I = 0.5$ (b) $\Delta J_{III}/\Delta J_I = 2$

Figure 7. SEM micrographs of mixed-mode fatigue fracture surface.

$=2\text{x}10^4$ N/m. In displacement-controlled tests of mixed-mode loading, the total *J*-integral ranges, ΔJ, and the mode ratio of $\Delta J_{III}/\Delta J_I$ were nearly constant during crack extension of about 1 mm from the pre-crack, and the crack propagation rate was also nearly constant. In Fig. 6, the mean propagation rate was plotted against the total *J*-integral range. The relations between *da/dN* and ΔJ for mode I and III were also shown in Fig. 6. The crack propagation rate was expressed as a power function of the *J*-integral range for pure mode I, and also for pure mode III. The propagation rate is lower under mode III loading than under mode I at the same ΔJ value. All the data of mixed mode lie between the relations for mode I and III. Therefore, the relation between *da/dN* and ΔJ for mode I gives a conservative estimate for the case of mixed-mode loading. At the same ΔJ, the crack propagation rate decreases with increasing ratio of $\Delta J_{III}/\Delta J_I$.

Fractography

The fatigue fracture surface of all the specimens whose mixed mode data were indicated in Fig. 7 was macroscopically flat and did not show factory-roof type topography. Figure 7 presents SEM micrographs of fatigue fracture surface made under $\Delta J_{III}/\Delta J_I=$ 0.5 and 2. Striations can be seen in the micrograph for mixed-mode cases, although more amount of rubbing marks coexist as the mode III component becomes bigger. Those striations suggest that the mechanism of mode I crack propagation is operating even in mixed-mode propagation. On the other hand, no striations were observed on the fracture surface made under pure mode III. Only rubbing marks were seen extending perpendicular to the growth direction.

The striation spacing was measured on the fatigue fracture surface made under mode I and mixed-mode conditions. Figure 8 shows the relation between the striation spacing, *s*, and the crack propagation rate. In the range of crack propagation rate between 1×10^{-7} and 4×10^{-6} mm/cycle, the striation spacing is equal to the crack propagation rate.

CONCLUSIONS

(1) The *J*-integral value under mixed-mode (I+III) loading estimated from the simple method was in good agreement with that obtained by the energy method for long cracks.

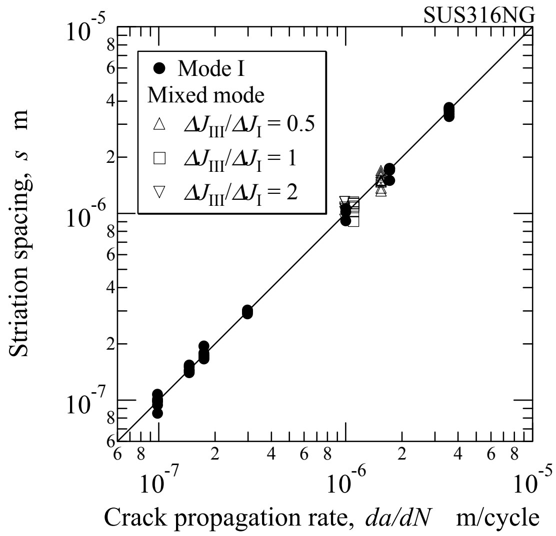


Figure 8. Relation between striation spacing and crack propagation rate.

(2) The relation between crack propagation rate and ΔJ under mixed-mode loading lie between those for mode I and mode III loading. At the same ΔJ, the crack propagation rate decreases with increasing ratio of $\Delta J_{III}/\Delta J_I$.

(3) Striations were observed on the fatigue fracture surface created under mixed mode loading. The striation spacing was equal to the crack propagation rate as in the case of mode I loading.

REFERENCES

1. Ritchie, R. O., McClintock, F. A., Nayeb-Hashemi, H. and Ritter, M. A. (1982) *Metallurgical Trans.* **13**A-1, 101-110.
2. Tschegg, E. K. (1983) *J. Mater. Sci.* **18**, 1604-1614.
3. McClintock, F. A. and Ritchie, R. O. (1981) In: *Mechanics of Fatigue*, *AMD*, **47**, pp. 1-9., Mura, T. (ed.), ASME, New York.
4. Tschegg, E. K. (1983) *Acta Metallurgica* **31**-9, 1323-1330.
5. Tanaka, K., Akiniwa, Y. and Nakamura, H. (1996) *Fatigue Fract. Engng Mater. Struct.* **19**, 571-579.
6. Tanaka, K., Akiniwa, Y. and Yu, H. (1999) *ASTM STP* **1359**, 295-311.
7. Itoh, Y. Z., Murakami, T. and Kashiwaya, H. (1988) *Engng Fract. Mech.* **31**, 967-975.