Review of the Subsurface Strain Path Approach to Component's Fatigue Life Assessment

G. Shatil

Senior Lecturer, Faculty of Computing, Engineering and Mathematical Science (CEMS), University of the West of England, Bristol BS16 1QY, UK. Email: g.shatil@uwe.ac.uk

ABSTRACT. Fatigue lives obtained from complex testing and monitoring of different components often involve some degree of discrepancy in results due to geometrical variation, even when they are tested under controlled conditions, and have similar surface cyclic strain range at the critical location. Recently, several fatigue models were developed to improve correlation of specimen lives using a critical 'process zone' that surrounds the damaged material. A review of such an approach is presented. The approach is based on critical subsurface strains and consists of fatigue damage summation procedure in the affected area. The fatigue life prediction model is applied to two structural materials using three geometries subjected to biaxial cyclic stresses. These include notched bar, rhombic plate and car component. The subsurface strains are evaluated by using a detailed elastic-plastic finite element analyses and by considering critical subsurface fatigue paths. It is shown that in the several cases investigated the subsurface approach appears to have overcome surface life conservative predictions.

INTRODUCTION

In the past, multiaxial fatigue theories have been developed and were fairly successful in predicting the fatigue life of components subjected to complex loads. Low cycle fatigue (LCF) theories often used strain-based parameters that correspond to material deformation and microstructure behaviour. A similar microstructure approach was also adopted to predict the failure of components subjected to high cycle (HCF) multiaxial fatigue where elastic conditions prevailed during the majority of life.

An example of geometrical aspect in fatigue is shown in Fig. 1 [1]. Results from biaxial fatigue of thin wall specimens (1mm thickness) are compared to uniaxial fatigue of solid specimens (8mm diameter) using the Lohr-Ellison strain parameter. Simulation has shown that the biaxial specimen's thin wall geometry approaches plane-stress state, with almost no strain or stress gradient across the specimen wall. The fatigue lives of the uniaxial specimens, when tested under similar surface strain conditions were about three times greater than the lives of the hollow biaxial specimens, Fig. 1. The difference

in lives is associated with decreasing inward radial stress/strain gradient from the surface to the midsection of the solid specimens.



Figure 1. Fatigue lives of biaxial hollow specimens and uniaxial solid specimens [1].

Several subsurface fatigue models have been proposed to overcome the stress or strain gradient effects on fatigue life. In general, the models were either used for high cycle fatigue (HCF) [2] and sometimes to modify the endurance limits, or for low cycle fatigue (LCF) where the plasticity was considered by using strain based parameters. The fatigue models were based either on a critical plane multiaxial fatigue criterion or based on using an energy approach [3]. The subsurface models could be separated into those using a critical depth [4] and those that accumulate the fatigue damage up to a certain critical depth [5]. Other types of models have introduced Linear Elastic Fracture Mechanics (LEFM) principles to evaluate limit life of notched components by employing critical distance within the so called 'process zone' using the line and point calculation methods for short cracks fatigue critical distance [6].

Figure 2 illustrates the basic types of 'process zone' models for improving the geometrical differences in life prediction, using subsurface parameters. The models typically used one of the following (Fig. 2): 1. Reference point. 2. Reference path, 3. Reference plane, and 4. Reference volume.



Figure 2. Illustration of subsurface fatigue model types.

SUMMARY OF THE SUBSURFACE STRAIN PATH APPROACH

The subsurface strain path model, Shatil and Smith [5], belongs to case 2, Fig. 2 and uses the following assumptions:

- a) A critical high strain path up to a critical depth is numerically calculated.
- b) A subsurface multiaxial strain parameter along a critical path is divided into equal increments, and using the material strain-life relation, the life corresponding to the average strain from each increment is obtained.
- c) Contribution to the fatigue damage process from each increment of strain under the surface is weighted and assumed to decrease with the distance from the surface.
- d) A linear accumulation of the subsurface damage is carried out along a critical path. The average strain from each increment is calculated as, Fig. 3a:

$$\overline{\varepsilon}_n = \frac{\varepsilon_i - \varepsilon_{i-1}}{2} \tag{1}$$

where $\overline{\varepsilon}_n$ is the average incremental strain, *n* is the increment number with i = n-1.

The incremental damage parameter is calculated using the simulated strain gradient divided by the total strain gradient:

$$D_{n} = \frac{\varepsilon_{i} - \varepsilon_{i-1}}{\Delta \varepsilon_{Cr}}$$
(2)

where $\Delta \epsilon_{Cr}$ is the total strain gradient at a typical critical distance.

The relative distance from the surface of each strain increment is introduced through a function that modifies the damage values with regard to surface distance, for example:

$$D_n^* = D_n (1 - (\sum_{1}^{n-1} D_n^*))$$
(3)

where D_n^* is the modified damage parameter.

After calculating the modified incremental damage, the total life to failure is summed as, Fig. 3b:

$$N_{fD^*} = \sum_{1}^{n-1} (N_{fn} D_n^*)$$
(4)

where N_{fD^*} are the modified cycles to failure for a particular surface strain range and N_{fn} is the number of cycles to failure at a certain depth along the critical path, corresponding to the average incremental strain $\bar{\varepsilon}_n$ at that depth.



Figure 3. (a) The subsurface average strain and strain increments; (b) The predicted life to failure using the calculated average strain.

APPLICATIONS

Fatigue of Axisymmetric Notch Bar (Isotropic and Anisotropic Structural Steel)

Two batches, isotropic and anisotropic, of the structural steel EN15R (BS150M36) were used in an extensive experimental programme reported elsewhere [1,5,7]. The cyclic strains at the notch root of the axisymmetric notched bar specimens were estimated using elastic-plastic finite element analysis. The finite element simulation also provided subsurface strains that were used to evaluate life based on the subsurface strain model. A separate analysis was carried out for each batch of the material.

Fatigue strain-life master curves were evaluated from uniaxial smooth solid and biaxial hollow specimen test results (Fig. 1), using the Manson-Coffin relation and the Lohr-Ellison equivalent strain parameter. The surface and subsurface notched specimen lives were estimated by using elastic-plastic finite element strains and employing Eqs 1 to 4.



Figure 4. Predicted and experimental lives of axisymmetric notch specimens using biaxial data: a. Surface analysis and, b. Subsurface analysis [5].

In Fig. 4 [5] the notched specimens experimental lives are compared to predictions obtained by using the biaxial fatigue of the thin walled specimens and the finite element results. Comparing the surface analysis predictions (Fig. 4a) and the subsurface analysis predictions (Fig. 4b), the conservative trend due to the difference in the specimen geometry is reduced, particularly for the isotropic material.

Fatigue of Rhombic Plate Anticlastic Bending (Aluminium Alloy 2024)

Experimental results of a rhombic plate subjected to anticlastic bending to obtain cyclic biaxiality of stress and strain are reported elsewhere [8], as well as details of the elastic-plastic finite element analysis to estimate the surface and subsurface cyclic strains. The subsurface strain method was applied to the anticlastic bending test results and the experimental lives of the rhombic plates were compared to surface life predictions, Fig. 5. The life prediction procedure was carried out at several subsurface paths by using the maximum shear strain obtained from finite element simulations.

The strain lives prediction calculated up to 1mm thickness at two different subsurface paths and are compared to surface predictions, Fig. 5, using uniaxial strainlife master-curve. The predicted subsurface model lives are somewhat non-conservative but the trend is consistent with the notched specimens subsurface analysis to reduce the conservative surface life predictions.



Figure 5. Experimental and predicted lives of the rhombic plate specimens using a subsurface shear strain damage parameter and two different strain paths [8].

Fatigue of a Metro Car Suspension Arm (Isotropic Structural Steel)

The subsurface strain path analysis was also used to estimate service component life the Metro car suspension arm [1]. The car component was made of the isotropic batch of the EN15R material mentioned previously. Critical surface and subsurface elasticplastic strains were estimated from separate plane-stress and plane-strain finite element analyses and these strains were used to calculate several biaxial fatigue cyclic parameters. The component life was predicted by using a biaxial fatigue master curve otained from hollow specimen tests, similar to the notched specimens life prediction.



Figure 6. Experimental and predicted lives of the Metro car suspension arm [1].

In Fig. 6 the predicted lives are compared to the experimental data using strains from the plane-stress and plane-strain finite element analyses at two cyclic loads. Although the experimental results have shown much scatter in lives between components, the life prediction from the analysis using a critical subsurface path were less conservative in comparison to the surface analysis. This was independent of the type of finite analysis used and was in agreement with the analyses of the laboratory specimens shown previously.

DISCUSSION

The reviewed subsurface strain path model appears to improve the conservative surface life prediction of components based on critical strain state. The model is independent of the choice of biaxial strain parameter or critical path, and is particularly useful in situations when fatigue master-curve, obtained using plane-stress specimens, is used to predict specimens that are in a state approaching plane-strain. It is argued that in this later case the strain gradient under the surface 'delay' the fatigue failure process [1, 5, 8].

To calculate the model parametres a critical fatigue strain path under the surface is required and this is geometry and loading dependent. Sometimes the choice is obvious, as for example the paths used with the notched specimens and the suspension arm life predictions. However, finding the critical path in the case of the rhombic plate tests was not straight forward and several paths aligned at increments of 15^0 from the surface were investigated [8]. This required a very detailed finite element simulation and careful consideration of element meshing prior to the analysis. It maybe argued that, in general,

shortest life is calculated for the critical path and this could be obtained by using a numerical procedure.

Current limitations of the subsurface model include the following; it does not contain a direct calibration with material fatigue micromechanics damage and/or constitutive behaviour and the subsurface distance in which the model applied is not well defined. Implementation of a subsurface critical distance parameter related to the material microstructure [6] and the geometrical constraint at the critical areas required further investigation. However, the model includes the use of the stress-strain response in the simulation stage, the choice of a suitable multiaxial fatigue parameter and the material strain-life relationship (master-curve).

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