Fatigue Crack Growth Trajectories under Biaxial and Mixed Mode Loding

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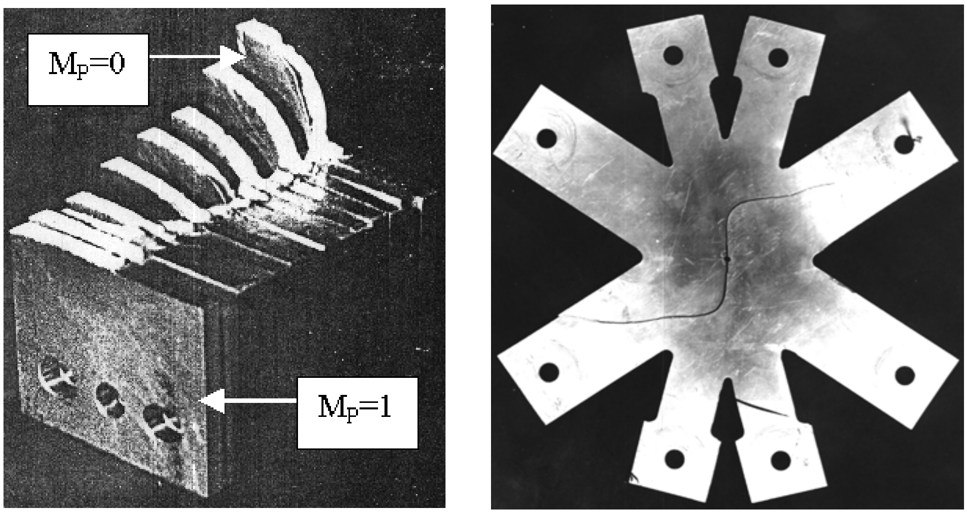
ABSTRACT. *Elastic-plastic crack growth under mixed mode I and II in aluminum alloys and steels were investigated. Two approaches are developed for geometrical modelling of crack growth trajectories for the central notched and compact tension shear specimens respectively. The damage process zone size concept is used for calculations of mixed-mode crack path. The influence of specimen geometry, biaxial loading and properties of the aluminum alloys and the steels on both crack growth direction and crack path at the macroscopic scale is discussed.*

INTRODUCTION

Main feature of mixed-mode fracture is that the crack growth would no longer take place in a self-similar manner and does not follow a universal trajectory that is it will grow on a curvilinear path. It is known that a "bent" crack does not propagate in its initial orientation direction. For mixed mode crack propagation, the crack front is continuously changing shape and direction with each loading cycle. As a result, the angle of crack propagation θ^* changes continuously. At each successive position of the crack front, the stress intensity factors in a plate, K_1 and K_2, must be calculated. However, for the actual "bent" crack geometry, the expressions for K_1 and K_2 cannot be easily determined. To overcome this difficulty an approximate procedure has been used by many authors (see Refs.[1]). Essentially, the procedure involves replacing the bent crack with a straightline crack approximation. A fatigue crack may be assumed to grow in a number of discrete steps. From the given initial values of crack angle β, biaxial ratio η and crack length a, the crack deviation angle θ^* is determined by the crack growth direction criterion. After each increment of crack growth, the crack angle changes from the original angle β and so does the effective length of the crack. For the next increment of crack growth, one has to consider the new crack length a_i and crack angle β_i. Values of a_i and β_i can be determined using the vectorial method. The objective of the present paper is to computationally and experimentally study crack growth under Mixed-Mode I and II loading in central notched and compact tension shear specimens.

THEORETICAL AND METHODICAL ASPECTS

The experimental part of our work is performed on plane compact tension shear (Fig.1,a) and eight-petal specimens (Fig.1,b). The first two specimen configurations with single-edge and central initial cracks were tested under uniaxial loading with variation of the initial crack orientation angle relative to the loading axis. Compact tension shear specimens are made from 30Cr steel types A, B and C (see Table 1) and used for mixed-mode fracture test with the loading direction to an angle β_0 to the initial crack plane. Values of β_0 are varied from 0° up to 90°. Non self-similar crack growth is realized in the compact tension shear specimen by using a set of S-shaped grips developed by Richard [2] such that a different mixity parameter, M_E or M_P, can be obtained corresponding to the different proportions of tensile and shear loads. The mixed-mode parameter, M_E, expressed through the Mode I and Mode II stress intensity factors, was varied by changing the load direction, β_0. Specimens were precracked under cyclic loading until they reached a crack length of a_0 = 44 mm. After the precrack had been created, each specimen was subjected to a different mixed mode fatigue load. As a result of cyclic mixed mode loading a branched fatigue crack is formed as it is shown in Fig.1,a.



a) b)

Figure 1. Compact tension shear (a) and eight-petal (b) specimens tested at different mixed mode loading

Eight-petal specimens (Fig.1,b) are made from aluminum alloys (see Table 1) and used for biaxial test. Mixed-mode fracture tests under biaxial loading with stress ratio η $=0.5$ are performed such that the mixed mode parameter, M_E, was varied by changing the inclined angle of initial crack, β_0, from 0° up to 90°. Using the specimen with a slant crack any desired K_1 and K_2 combinations can be induced by various both crack angle and biaxial loadings. In our work the experimental results on the 30Cr steel types A,B,C and eight aluminum alloys are used to compare with the computational data. Their main mechanic characteristics are presented in Table 1. The tests were carried out at the room temperature under cycle loading.

Table 1. Mechanical properties of aluminium alloys and 30Cr steel type A, B and C

Material	E (GPa)	σ_0 (MPa)	σ_t (MPa)	σ_f (MPa)	ε_f	n
AMG6	71	160	320	384	0.182	4.293
01420T	75	225	390	446	0.135	4.813
1163AT	72	285	439	525	0.178	5.569
D16AT	72	310	445	528	0.171	6.197
1201AT	71	320	420	475	0.122	7.441
1163ATM	72	369	478	536	0.115	7.441
01419	70	300	345	376	0.086	11.588
B95AT1	72	506	563	625	0.104	11.594
Steel A	200	1514	1750	2333	0.288	7.791
Steel B	200	1039	1136	2064	0.599	6.425
Steel C	200	444.8	761.2	1438	0.635	4.300

Here σ_0 is the yield stress, σ_t is the tensile strength, σ_f is the true fracture stress, ε_f is the true fracture strain, *n* is the strain hardening exponent.

Many of the fracture mechanics theories are based on a critical distance local to the crack tip. It has been considered as fundamental characteristic parameter that distinguishes damage at the microscopic and macroscopic scale level. Within the fracture damage zone some microstructural damage accumulates until crack growth takes place at the macroscopic scale level. In the present paper the critical distance r_c ahead of the crack tip is assumed to be located where the stress strain state in the element reaches a certain critical value that can be measured from a uniaxial test. A relative fracture damage zone size $\overline{\delta}_c = r_c/a$ was introduced by Shlyannikov [3]

$$\overline{\delta}_c = \left\{ \frac{\overline{S}_2 \pm \left[\overline{S}_2^2 - 4\left(\overline{W}_c^* - \overline{S}_3\right)\left(\overline{S}_1 + \overline{S}_p\right)\right]}{2\left(\overline{W}_c^* - \overline{S}_3\right)} \right\}^2 \quad (1)$$

where $\overline{W}_c^* = \left(\dfrac{\sigma_0}{\sigma_{yn}}\right)^2 \left[\dfrac{1}{2}\overline{\sigma}_f^2 + \dfrac{\alpha n}{n+1}\overline{\sigma}_f^{n+1}\right]$ and $\overline{\sigma}_f \approx \overline{\sigma}_u^{true} = \dfrac{\sigma_u}{\sigma_0}(1-\psi)$ (2)

All stresses in these equations are normalized by the yield stress σ_0, and σ_u is the true ultimate tensile stress, ψ is the reduction of area. In equation (1) $\overline{S}_i\left(i = 1,2,3\right)$ and

$\overline{S}_p$ are elastic and plastic coefficients respectively. The work [3] contains more details about the determination of these coefficients $\overline{S}_i = \overline{S}_i(\theta, \kappa, \beta, \eta, Y_I, Y_{II})$ and $\overline{S}_p = \overline{S}_p(n, \nu, I_n, M_p, \widetilde{\sigma}_e, Y_I, Y_{II})$ for the general case of mixed-mode elastic-plastic fracture. These coefficients are different for various geometric configurations. Thus, the radial distance r_c normalized by the crack length a may be found from relation (1) to be a function of the angular direction θ, the material properties, the stress strain state and the mixed-mode parameter M_p.

When calculating the crack growth trajectory it is necessary to distinguish the following principal moments. Firstly, proceeding from theoretical precondition one can estimate the crack front shape as a set of successive positions of the assumed crack tip on its propagation trajectory, as was made by Shlyannikov and Dolgorukov [4]

$$\begin{cases} a_i = \left[a_i^2 + \Delta a_i^2 - 2a_{i-1}\Delta a_i \cos\left(\pi - \theta_{i-1}^*\right)\right]^{1/2} \\ \beta_i = \beta_{i-1} \arcsin \dfrac{\Delta a_i \sin\left(\pi - \theta_{i-1}^*\right)}{a_i} \end{cases} \tag{3}$$

where θ_{i-1}^* is crack growth direction or crack deviation angle. Secondly, in fatigue life calculations it is necessary to connect the crack length increment Δa_i along its growth trajectory with the corresponding number of loading cycles ΔN_i.

Let Δa_i in Eq. 3 have the physical sense of the fracture damage zone size δ_i. Then Eq.1 can be applied to the crack path prediction for the two typical geometric configurations containing the single-edge and the central initial cracks of length a_0 and obliqueness β_0 as shown in Fig.2,a and b. Crack path prediction for the mixed modes I and II initial crack can be carried out making use of the following scheme. This scheme involves replacing a bent crack with a staightline crack approximation, as shown in Fig.2. The principal feature such modeling is determination of the crack growth direction and definition of crack length increment in this direction. Crack may be assumed to grow in a number of discrete steps. After each increment of crack growth, the crack angle changes from the original angle β_0 and so does the effective length of the crack. For the next increment of crack growth, one has to consider the new crack length a_1 and crack angle β_1. As shown in Fig.2, OA is the initial crack length a_0 oriented at an angle β_0. Let r_0=AB be the crack growth increment for the first growth step. It would correspond to the fracture damage zone size. Making use of Eq.1, $\bar{\delta}$ and hence $r_0 = \bar{\delta} a_0$ can be computed. The value r_0 is then extended along AB with the angle θ_0^* whose value is determined by the crack growth direction criterion. For the single-edge crack geometry (Fig.2,a) the first step of crack growth obtained as $\phi_0 = \theta_0^*$ and

$$x_0 = r_0 \cos\theta_0^*, \quad y_0 = r_o \sin\theta_0^* \tag{4}$$

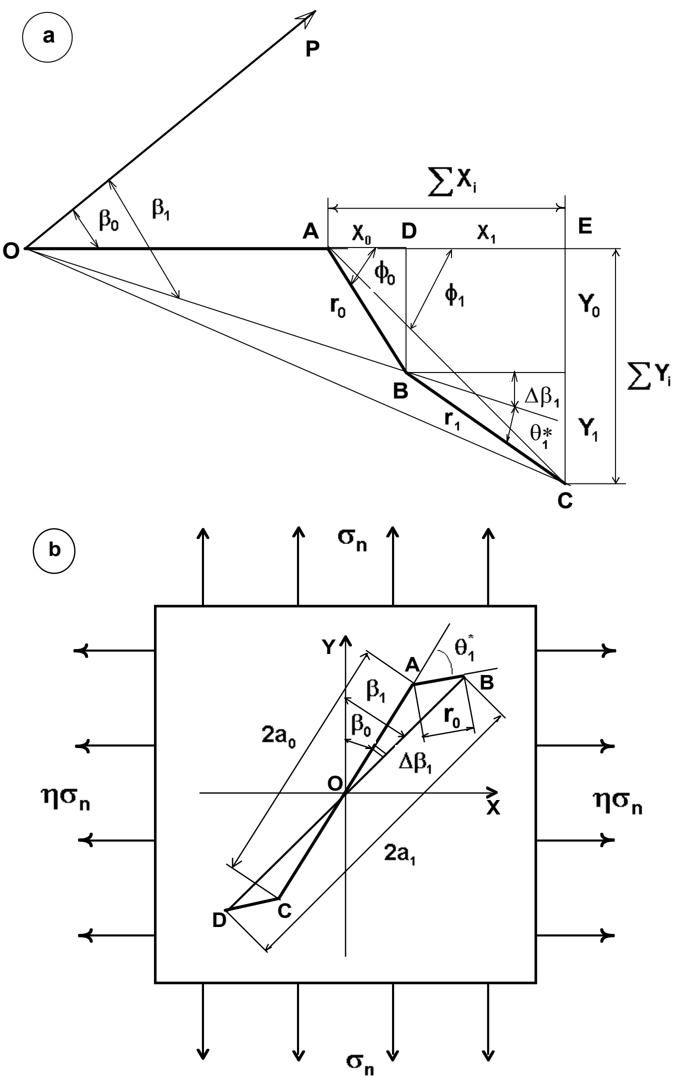


Figure2. Crack growth trajectory approximation, (a) single-edge crack geometry, (b) central notched biaxially loaded crack geometry

The next step plotting r_1 along BC oriented at the angle θ_1^*. In this case AC=$\sqrt{\sum x^2 + \sum y^2}$ and

$$\phi_1 = \tan^{-1}\left(\sum y / \sum x\right),\ x_1 = r_1 \cos\gamma_1,\ \ y_1 = r_1 \sin\gamma_1 \tag{5}$$

where

$$\gamma_1 = \Delta\beta_1 + \theta_1^*, \sum x = x_0 + x_1,\ \ \sum y = y_0 + y_1 \tag{6}$$

and so on. For the central crack geometry subjected to biaxial loads (Fig.2,b) the crack path can be determined using the formulae Eq.1.

RESULTS AND DISCUSSION

In this section the experimental results on the 30Cr steel types A,B,C and eight aluminum alloys are used to compare with the computational data. Criterion [1] is applied for analyzing the fatigue crack growth trajectories in specimens the above geometries. On the compact tension shear specimen was realized the full range of mixed mode fracture from tensile (pure Mode I) to shear (pure Mode II) loading. The eight petal specimens are subjected to biaxial tension at η =0.5 as the initial inclination angle, β_0, is varied from 0° up to 90°.

Figures 3,a-d present a comparison of both computational and experimental crack growth trajectories for aluminum materials and 30Cr steel with different properties subjected to biaxial and uniaxial tension at $\eta = 0.5$ and $\eta = 0.0$, respectively. Their conformity suggests the validity of the straightline crack concept and hence Eqs.3 may be used in fatigue life calculations. A characteristic feature of Eq.3 as against other equations in Refs [1] is the fact that they take into account an influence of both the materials properties (strain hardening exponent) and the nominal stress σ_{yn} on the crack growth trajectory via the angle of crack propagation θ^*. As it is shown in Fig.3,a the fatigue trajectories under uniaxial and biaxial tension for brittle aluminum alloy 01419 almost coincide. However, for ductile aluminum alloy AMG6 the crack paths under the same types of loading are very different. Under uniaxial tension when these cracks propagate, they gradually rotate to align normal to the applied principal stress directions.

Brittle and ductile materials have different curvature of the crack trajectories. This is confirmed by the experimental results that are presented in Fig.3,b. The occurrence of mixed mode growth is dictated by the crack angle, but the occurrence of any mode is believed to be dependent on both stress state and microstructure. Usually when viewed on the macroscopic scale with respect to the material structure the fatigue crack path may generally be regarded as smooth. However, on smaller, microscopic scale, the crack path is generally very irregular. It can be noted that our approach based on the of fracture damage zone concept allows to describe the crack behavior on the microscopic scale.

In the experimental and computational data presented in Fig.3,b the following may be mentioned. Under the same biaxial loading conditions the crack trajectory for some materials tends to be normal to the nominal tensile stress σ_{yn} direction, while for others this does not occur. This distinction on crack growth trajectories is connected with different properties of materials. Numerical and experimental results concerning the effect of the (a/w)-variation showed for compact tension shear specimens (Fig.3,c,d) that the fatigue crack path is sensitive to both the initial crack orientation and length change.

In Fig.4 initial parts of the fatique crack path corresponding to macrotrajectories displayed in Fig.3,b for η =0.5, β_0 =0° and β_0 =65° are presented, respectively. As it is seen in Fig.4 the degree of irregularity in crack path depends on the initial inclination

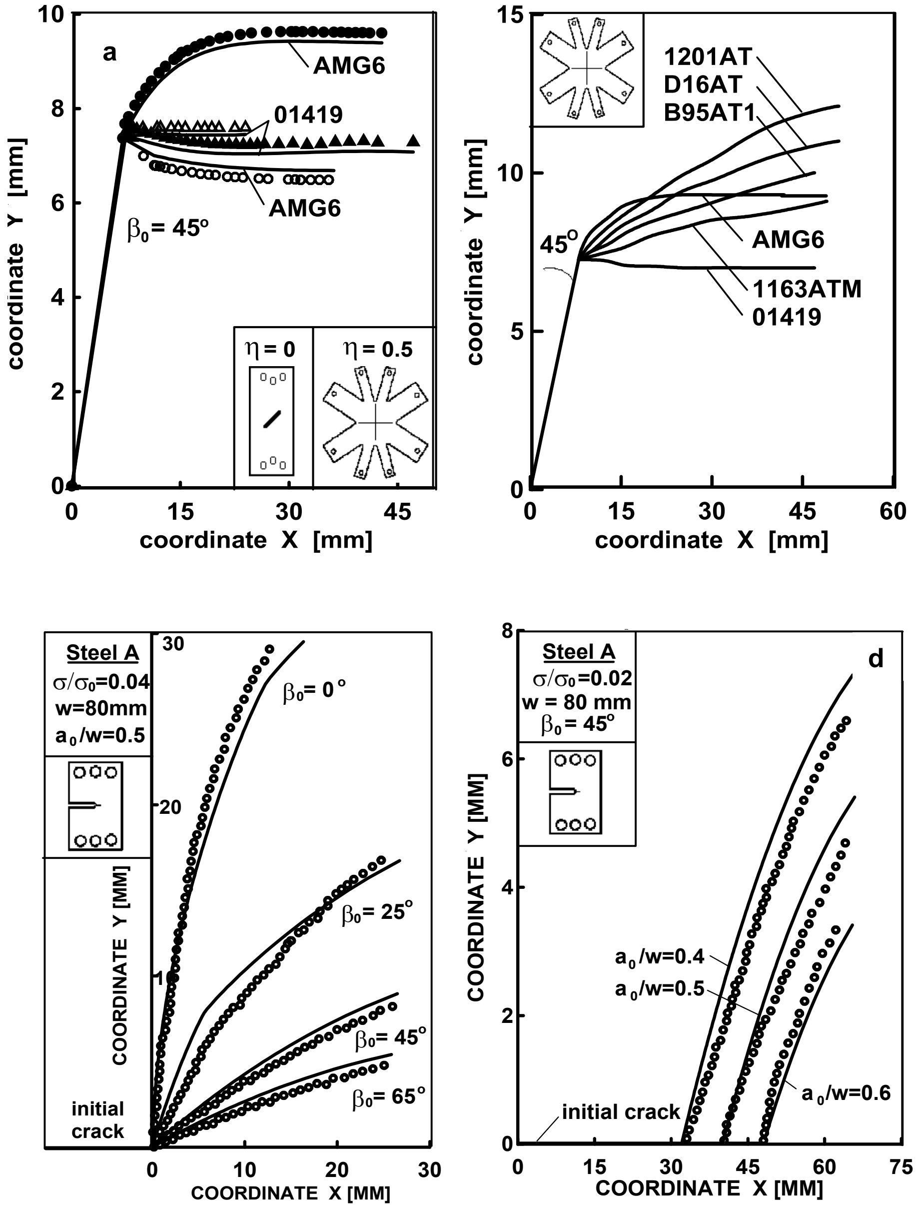


Figure 3. Theoretical (curves) and experimental (points) fatigue crack growth trajectories for (a,b) eight-petal and (c,d) compact tension shear specimens

angle, β_0. So, the situation in biaxially loaded eight-petal specimen for η=0.5 and β_0=0° (or η=2 and β_0=90°) concerns the state of unstable equilibrium, and the irregularity in crack behavior is greater than for β_0=65°. The zig-zag path of a propagating crack may be explained by considering the advancement of a crack as consisting of distinctive steps, where voids and other discontinuities of the material, surrounding the crack tip, coalesce and create each kink for the crack. If the biaxial stresses are tensile (η>0) then a crack is directionally unstable and, following a small deviation, does not return to its initial plane. A crack, for example, in a eight-petal specimen under biaxial tension with η=0.5 and β_0=0° is directionally unstable in this sense, and a typical crack path is shown in Fig.4,a. Moreover, for these specimens under biaxial loading the amount of crack path curvature (Fig.4,b) is a function of the tensile properties of the aluminum alloys concerned Table 1.

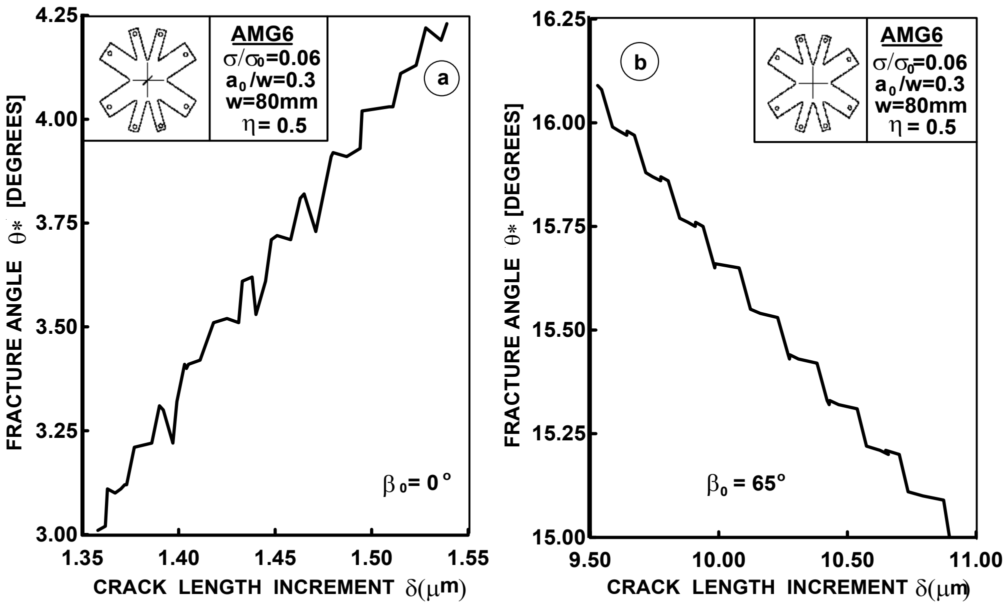


Figure 4. Computational predictions for the fatigue crack path at the microscale level

Thus, the influence of the geometry specimens, the initial crack length and the material properties were studied. Attention is focused on the mixed-mode crack trajectories. The behavior of the crack path under mixed-mode fracture is discussed with regard to microscopic and macroscopic scales.

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