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Alternative Representations for Scaled *R* Curves in a Titanium Alloy

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ABSTRACT Data from a study of the effect of geometric variation on J–R-curves for a titanium alloy are presented for large amounts of ductile tearing in fully plastic deep notch three point bending tests. Four series of tests were made covering 17 different combinations of thicknesses, widths, notch depth ratios, and absolute sizes. A first analysis, made in terms of J_m, showed a wide spread of J–R-curves for the various configurations. When plotted against a normalised abscissa of $\Delta a/b_0$ a tightly grouped family of curves was found.

Analysis is now made in terms of the dissipative work rate per unit crack extension, $dU_{dis}/B\,da$. This term represents an alternative resistance curve which, for the fully plastic test condition, falls with crack growth but approaches a steady state after about 20–30 percent of the ligament is torn. Within each series of tests this quantity scales with $(\Delta a/b_0)\,(S/b_0)$ where S is span and b_0 is original ligament, thereby giving a falling resistance curve independent of ligament size for a given thickness. A corresponding rising R-curve can be drawn, independent of ligament size, if a J-like term, J_{dis}, rather than a conventional term such as J_m, is used for the ordinate and if the abscissa is scaled to $\Delta a/b_0$.

Attention is drawn to the separate physical meanings of R-curves based on cumulative work done or on the dissipation rate as a function of crack growth. The former must always rise, whereas for fully plastic tearing the latter falls. The present dissipation rate data also scales with size up to large amounts of growth and it is this feature that merits attention as a term more useful in engineering analysis than the conventional rising J–R-curve.

Notation

Geometric terms

a_0 Initial crack length
Δa 'Crack' extension, or more properly, extension of ductile tearing
a Crack length, $a_0 + \Delta a$
b_0 Initial size of ligament, $(W - a_0)$
b Ligament, $(W - a)$
B Thickness
S Span
W Width

Material characteristics

E Young's modulus
J_i Initiation toughness; a concept to which values such as J_{Ic} are a working approximation
σ_Y Yield or proof stress

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Terms used in discussing tearing resistance

J	The so-called J-integral as derived experimentally from $\eta U/Bb$
J_m	The particular derivation of J defined by Ernst in **(9)**
J_{dis}	A particular derivation of a J-like term defined herein by equation (3)
R	A generic term for resistance to tearing with dimensions of work per unit area
R_G	The linear elastic component of the fracture resistance, R
U	Work done
w	Internal energy (not necessarily recoverable)

Suffixes applied to the tearing terms

dis	Dissipation, including both plastic and fracture terms
el	Elastic (linear)
pl	Plastic
total	The combined elastic, plastic, and fracture terms.

Introduction

The tearing toughness of ductile metals in the elastic–plastic fracture mechanics (epfm) regime is normally expressed in terms of J or COD resistance curves which are a rising function of the amount of tearing, Δa. Most published data are expressed through J derived from the work done and that style will be followed here. It is commonly seen that such a J–R-curve is not independent of geometry unless restricted to a small amount of growth and to certain sizes of piece such that so-called J-controlled growth is found. The general objective of the present analysis and other related work is to seek terms to describe ductile tearing resistance in a way that is less dependent on the dimensions of the test piece used.

The underlying argument is that unless data can be translated from one test condition to another then confidence in their use for actual structural cases is undermined although at the present stage only deep notch bend type configurations are considered.

Analysis of published data had already suggested that in a rather broad brush way some R-curve data could be unified by re-plotting to an abscissa of $\Delta a/b_0$. The results of a particular series of ductile tearing tests on a variety of sizes and proportions of a titanium alloy, **(1)**, when plotted in terms of J_m versus Δa, showed a wide spread. When re-plotted against the scaled abscissa, $\Delta a/b_0$, the R-curves showed little more than a small trend with thickness and random scatter of an amount not uncommon in fracture testing, **(2)**. These results are summarised below as a starting point. In an attempt to understand the role of a scaled abscissa in accounting for the effects of ligament size on such R-curves, re-analysis is made in terms of the dissipative work per unit area of tearing, $dU_{dis}/B\,da$. Such curves are a direct interpretation of the experimental data but in the fully plastic regime of interest here, fall with growth of tearing whereas the conventional J–R-curves rise. A rising R-curve, denoted J_{dis}, can however be constructed from the $dU_{dis}/B\,da$ term, if desired, but that term is independent of geometry only for certain definable cases.

It will be argued that after initiation, the d/da rate term, $dU_{dis}/B\,da$, is closely analogous to the Griffith concept. In the tearing regime (i.e., after initiation) it is seen as more fundamental than any of the J-based terms that are in practice derived from normalised total work since the cumulative work, however normalised, departs from the original conceptual meaning of J as either an energy release rate or a characterising stress field parameter.

The immediate object of this paper is to present one set of data analysed in terms of $dU_{dis}/B\,da$, partly to show the trends that emerge and partly to encourage similar analysis of other experimental R-curve data since the terms required, though readily available from raw test records using the unloading compliance method, cannot be recovered from J–R-curves in the usual published form.

Factors affecting *R*-curves

The effect of change of initial geometry on the form of an R-curve in bending is not generally agreed. A qualitative unification within sets of disparate R-curves from a number of published results was offered in **(3)–(6)**. The reasoning is outlined here as a preliminary to the present analysis.

From their tearing analysis Rice, Drugan, and Sham, (RDS) **(7)**, deduced that in contained yield a J-based R-curve was likely to be a function of $\Delta a/c$ where $c = 0.2EJ/\sigma_Y^2$, a measure of the plastic zone size, which is itself a measure of material toughness. It was argued in **(5)** that the bringing together of R-curves for a number of sizes or proportions of test pieces in **(3)** and **(4)** by use of normalised abscissa, $\Delta a/b_0$ or $\Delta a/B$, was simply an extension of the RDS case to uncontained yield. It is relevant to note that the original meaning of J as an energy release rate (albeit for non-linear elastic rather than actual elastic–plastic behaviour) is not preserved in the usual experimental derivations whereby a J–R-curve is found by measuring the work absorbed during stable growth. It follows that such an R-curve will be a function of whatever factors inhibit work dissipation and for the fully plastic tearing observed in most small test pieces those factors may well be dominated by geometry rather than material property.

Conceptually, for a very tough material the work would be entirely dissipated in general plastic deformation with no tearing at all. For stable tearing in contained yield the limit is the tearing toughness for the generation and propagation of a plastic zone. For the linear elastic fracture mechanics (lefm) case of plane stress, the plastic zone is dictated by the thickness of the material. Where tearing occurs in uncontained yield the dissipated work is an interplay between toughness and size. Either ligament or thickness or possibly both may control an R-curve, according to which governs the extent of plasticity. Although the intent in deriving an R-curve may well be to isolate the fracture term from the total work done term, there is little evidence that this can be done after initiation from the conventional type of epfm analysis. Although such a separation is not deduced here, certain features of the present analysis suggest how this might be done, and that argument has been followed in a paper to the ESIS Conference, Turin, 1990.

Dissipation rate as a measure of toughness

The R-curve data for titanium were also analysed in **(1)** in terms of $\mathrm{d}U_{\mathrm{dis}}/B\,\mathrm{d}a$, where U_{dis} is the combined dissipation for plasticity and fracture. This is found simply by subtracting the current elastic strain energy, $w_{\mathrm{el}} = Qq_{\mathrm{el}}/^2$ from the total work done up to the point of interest. Thus, at each step

$$\mathrm{d}U_{\mathrm{dis}} = \mathrm{d}U_{\mathrm{total}} - \mathrm{d}w_{\mathrm{el}} \tag{1a}$$

Since $U_{\mathrm{pl}} = U_{\mathrm{total}} - U_{\mathrm{el}}$ where U_{el} is the summation of the initial elastic work prior to tearing plus changes in both the internal elastic strain energy, w_{el}, and the elastic component of the fracture work, denoted R_{G} here, during tearing

$$\mathrm{d}U_{\mathrm{dis}} = \mathrm{d}U_{\mathrm{pl}} + \mathrm{d}U_{\mathrm{el}} - \mathrm{d}w_{\mathrm{el}} = \mathrm{d}U_{\mathrm{pl}} + R_{\mathrm{G}}B\,\mathrm{d}a \tag{1b}$$

Thus U_{dis} differs from U_{pl} by the elastic fracture work term $\Sigma R_{\mathrm{G}} B\,\Delta a$. In the lefm limit the term $\mathrm{d}U_{\mathrm{dis}}$ is compatible with the fixed load case which may be regarded as the limiting behaviour for stable tearing. In that limit, $\mathrm{d}U_{\mathrm{dis}}$ is $\mathrm{d}U_{\mathrm{el}}/2$ since half of $\mathrm{d}U_{\mathrm{el}}$ increases the internal strain energy, $\mathrm{d}w_{\mathrm{el}}$, whilst the other half provides the fracture work $R_{\mathrm{G}}B\,\mathrm{d}a$.

In the well known case of lefm plane stress there is a plastic work increment from the formation of a plastic zone comparable to the thickness although in algebraic terms there appears to be no dissipative term unless it is identified with the so called plastic zone correction. Indeed, in experimental work use of lefm is justified where the unloading line passes through the origin to within experimental limits, even after large amounts of tearing. In short, physically there is an identifiable plastic zone of extent similar to the thickness but a conventional lefm analysis, even when 'corrected' for plastic zone size, treats the whole fracture plus plastic dissipation as one term. The present argument simply extends that view point into epfm and fully plastic behaviour. Although G is often written in terms of strain energy as $G = \partial w/B\,\partial a|_{\mathrm{q}}$ a more general definition for either G (for lefm) or J (for non-linear elasticity) is $-\mathrm{d}P/B\,\mathrm{d}a$ where P is potential energy. For stable growth in an elastic system the corresponding dissipation must be the total derivative $\mathrm{d}U_{\mathrm{dis}}/B\,\mathrm{d}a$. In lefm R-curve testing it seems common to assess G, if need be with a plastic zone correction, rather than to measure the dissipation. In epfm J testing, total work is indeed directly measured and in the unloading compliance method the elastic energy is easily identified so that the dissipative work increment is both a virtually direct experimental observation and the direct continuation of the lefm term. It is extraordinary that it is does not seem to have been thoroughly explored as a fracture parameter.

The test data

The material, test methods, and conventional R-curve results for the present tests have already been given fully in **(2)** so only a brief summary is given here. The material used was a 6Al-2Cb-1Ta-1Mo titanium alloy of 728 $\mathrm{MN/m^2}$ proof stress and 828 $\mathrm{MN/m^2}$ tensile strength. Stable tearing tests were conducted using an unloading compliance method taken to large crack growths of some 50 percent or more of the original ligament. The geometries were all of the three point bend type, with span to width ratio, $S/W = 4$, and covered four series with the nominal sizes:

(i) $B = 35$ mm, $a_0/W = 0.55$, for various widths, $W = 17, 25, 30$, and 35 mm
(ii) $B = 17.5$ mm, $W = 35$ mm, for various notch depth ratios, $a_0/W = 0.2$, 0.35, 0.55, and 0.75
(iii) $W = 35$ mm, $a_0/W = 0.55$, for various thicknesses, $B = 4, 8, 12, 17.5$, and 35 mm
(iv) $a_0/W = 0.55$, $B = W$, for various absolute sizes, B or $W = 10, 20, 30, 35$, and 40 mm

The results of the tests, expressed in terms of J_{m}, were shown in **(2)** plotted against both Δa and $\Delta a/b_0$. As described in more detail there, indiviual R-curves were adjusted to a mean initiation toughness value of 0.17 MN/m to eliminate an apparent random scatter of initiation, itself not of direct interest here. In brief the results versus Δa showed no effect of geometry for series (ii) (various a_0/W), except for the shallow notch case, $a_0/W = 0.2$, or for series (iii) (various thicknesses). Results for series (i) (various widths) and (iv) (various sizes) showed a strong trend with size, the curves for the wider or larger pieces being lower than for the narrower or smaller. Excluding the shallow notch case, $a_0/W = 0.2$, all results for J_{m} as a function of $\Delta a/b_0$ are shown in Fig. 1 as two groups for $B \leqslant 20$ mm or $B \geqslant 30$ mm, all for $0.35 \leqslant a_0/W \leqslant 0.75$ with a small effect of thickness but no discernable effect of width. Thus the use of a scaled abscissa is encouraged even at this stage but in the following text both ordinate and abscissa are changed. The point plotted in Fig. 1 at about (0.05, 0.05) is also omitted as being an obvious experimental error, but all the other data are re-analysed.

Considering first the ordinate, the same test data, evaluated as $\mathrm{d}U_{\mathrm{dis}}/B\,\mathrm{d}a$ from the slopes of graphs of the experimental values of $U_{\mathrm{dis}} = U_{\mathrm{tot}} - w_{\mathrm{el}}$ versus Δa, are plotted against Δa for the four series in Figs 2(a)–(d). Here, U_{tot} is the total work done up to the point of interest and w_{el} is the elastic strain energy at that point. Such graphs differ in two ways from the conventional R-curve where

$$J = \eta U/Bb \tag{2a}$$

prior to initiation and

$$J + \mathrm{d}J = \eta(U + \mathrm{d}U)/Bb + f(\mathrm{d}a, \eta) \tag{2b}$$

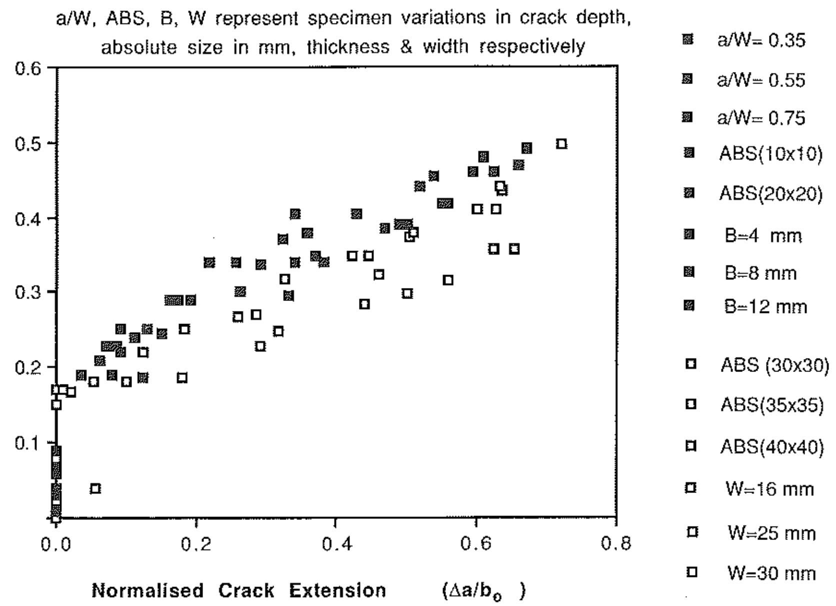

Fig 1 Normalised resistance curve J_m v $\Delta a/b$ for a range of sizes and proportions of three point bend tests. Solid symbols, $B \leqslant 20$ mm, open symbols, $B \geqslant 30$ mm. (After (2))

after initiation, subject to the niceties of various corrections for $f(\mathrm{d}a, \eta)$ such as J_{i+1} in **(8)** and J_m in **(9)**. The falling trends of Fig. 2 show what are essentially tearing reistance or R-curves expressed in terms of the work increment or rate, d/da, rather than through the normalised work quantity of equation (2). Much of the argument in this paper suggests that $\mathrm{d}U_{\mathrm{dis}}/B\,\mathrm{d}a$, the combined plastic plus fracture dissipation rate is more relevant and fundamental to plastic tearing than other quantities employing a J term derived from total work via equation (2), corrected as may be.

The new ordinate values are re-plotted Figs 3(a)(b), against a normalised abscissa, $(\Delta a/b_0)$ (S/b_0), where they appear to fall into two main groups, a rather flat curve for $B \geqslant 30$ mm, Fig. 3(a) and a more steeply falling curve for thickness $B \leqslant 20$ mm, Fig. 3(b). The former embraces the variable width series (for which $B = 35$ mm), supported by the thicker of the variable B series and the larger of the variable size series. The latter embraces the variable a/W series (for which $B = 17.5$ mm), supported by the thinner of the variable B series and the smaller of the variable size series. It is unlikely that there are just two separate curves for 'thick' and 'thin' but rather a trend from one to the other. Since there are only two cases, $B = 20$ mm from (iii) and $B = W = 30$ mm from (iv), that fall between thicknesses of 17.5 mm and $B = 35$ mm, and only three pieces, from series (iii) with $B < 17.5$, only two categories are considered here. At first only the groups of 35 mm (series i) and 17.5 mm thickness (series ii) were considered to define the single full line curve

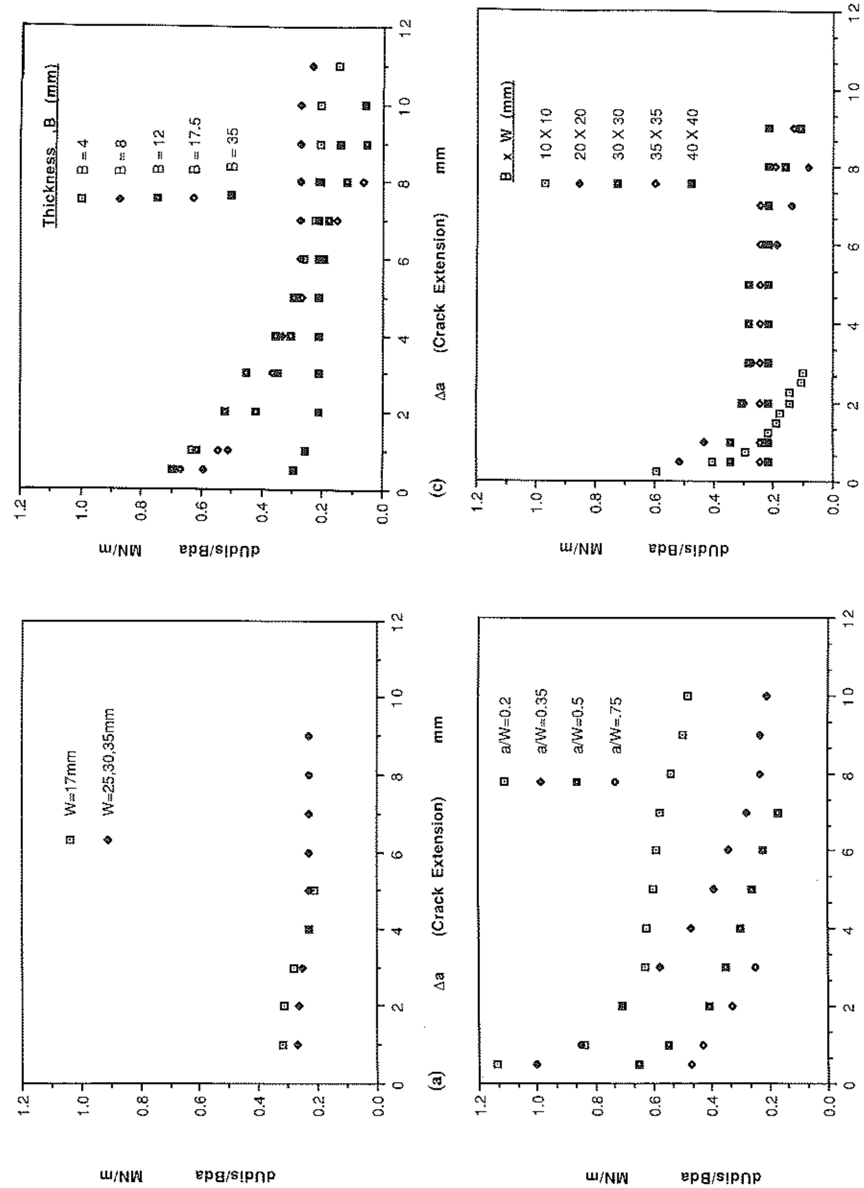

Fig 2 Curves of $\mathrm{d}U_{\mathrm{dis}}/B\,\mathrm{d}a$ versus crack growth, Δa. (a) for various W, (b) for various a/W, (c) for various B and (d) for various sizes

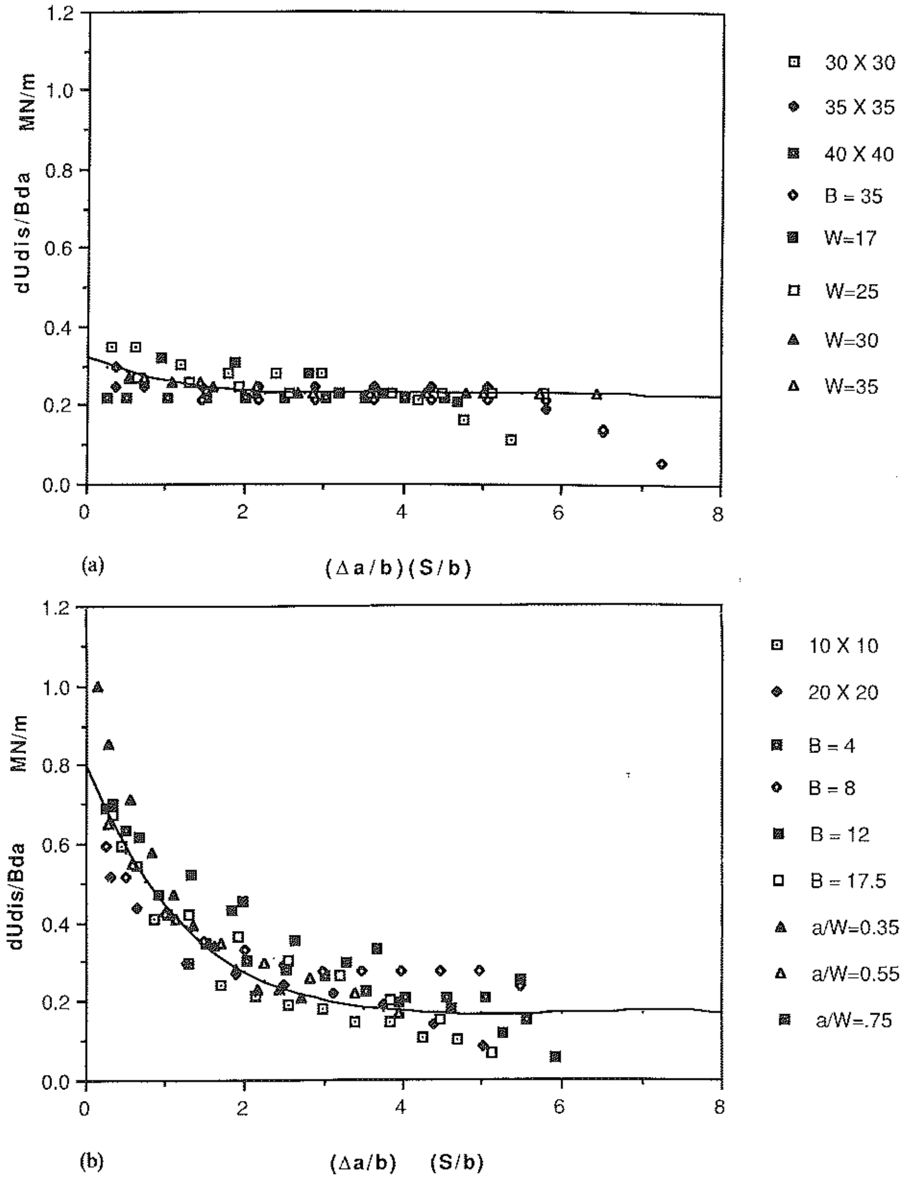

Fig 3 **$dU_{dis}/B\,da$ versus normalised crack growth $(\Delta a/b_0)$ (S/b_0). (a) $B \geqslant 35$ mm ('thick'). (b) $B \leqslant 20$ mm ('thin')**

on each figure but inclusion of the data for the other cases seems to make little difference. It may be a peculiarity of the particular material that the effect of thickness appears unusually small in relation to data on steel or aluminium alloys and as already noted no effect at all is seen in terms of J_m versus Δa, (2).

It is then supposed that for a given thickness (or group of thicknesses) there is a unique curve of $dU_{dis}/B\,da$ versus $(\Delta a/b_0)$ (S/b_0) for variation in initial size of ligament, as shown by the full curves in Figs 3(a) and (b). In all cases in Fig. 3(a) the variation in initial size of ligament, b_0, is due to variation in width, W. Series (ii), where the ligament changes due to change of a/W at constant width, provides most but not all of the data in Fig. 3(b).

A corresponding *J–R*-curve

A rising *R*-curve, corresponding to a conventional *J–R*-curve, can be constructed from $dU_{dis}/B\,da$. Thus

$$J_{dis} = J_i + \Sigma\, dJ_{dis} \tag{3a}$$

where

$$dJ_{dis} = \eta\, dU_{dis}/Bb \tag{3b}$$

It should be noted that J_{dis} after initiation is defined through the initiation value, J_i and the post initiation increments, dJ_{dis}. The distinction between this and other formulations such as the deformation term J_d, the term used in interim test standards, J_{i+1}, other J_m formulation is that the term added for dJ in these other formulations is itself derived by differentiating J to give both the 2nd and 3rd terms in equation (2b). Equation (3b) does not involve the Δa dependent 3rd term. This is seen to be in accord with the Rice *et al.* (RDS) argument, (7), that the *R*-curve should not be an explicit function of Δa. The physical reason for defining dJ without differentiating J is that, in the authors' opinion, the processes of blunting prior to initiation and of tearing subsequent to initiation are discontinuous in the macro sense so that equation (2a) is not differentiable through the point of initiation. No doubt that implies strictly that the J terminology should not be applied to the post-initiation regime unless indeed contour integral values are used for its evaluation.

Since the experimental values of $dU_{dis}/B\,da$ for a given small range of thicknesses have here been shown to be a function of $(\Delta a/b_0)$ (S/b_0), the resulting J_{dis}–*R*-curve may be a function of several variables. For cases where η and S/b_0 are constant then the resulting *R*-curve will be unique in in terms of J_{dis} versus $\Delta a/b_0$ for variations of width. The *R*-curves for a series with various a_0/W values at fixed S/W would not therefore be expected to be ligament width independent. Though none are discussed here, shallow notch results would also differ from the deep notch cases because of variation of η at small a_0/W, even if the curves of $dU_{dis}/B\,da$ were to be the same.

If it is accepted that the single curve shown on Fig. 3(a) is unique for the group of thicker pieces of about 35 mm then the single *R*-curve found, the lower curve on Fig. 4, is relevant to any such 'thick' pieces subjected to deep notch three point bending with $S/b_0 = 8.9$ (which corresponds to $S/W = 4$, $a_0/W = 0.55$ used for the present tests), irrespective of the absolute width or a_0/W value, i.e., with S/b_0 fixed rather than S/W fixed. Using the steeper curve

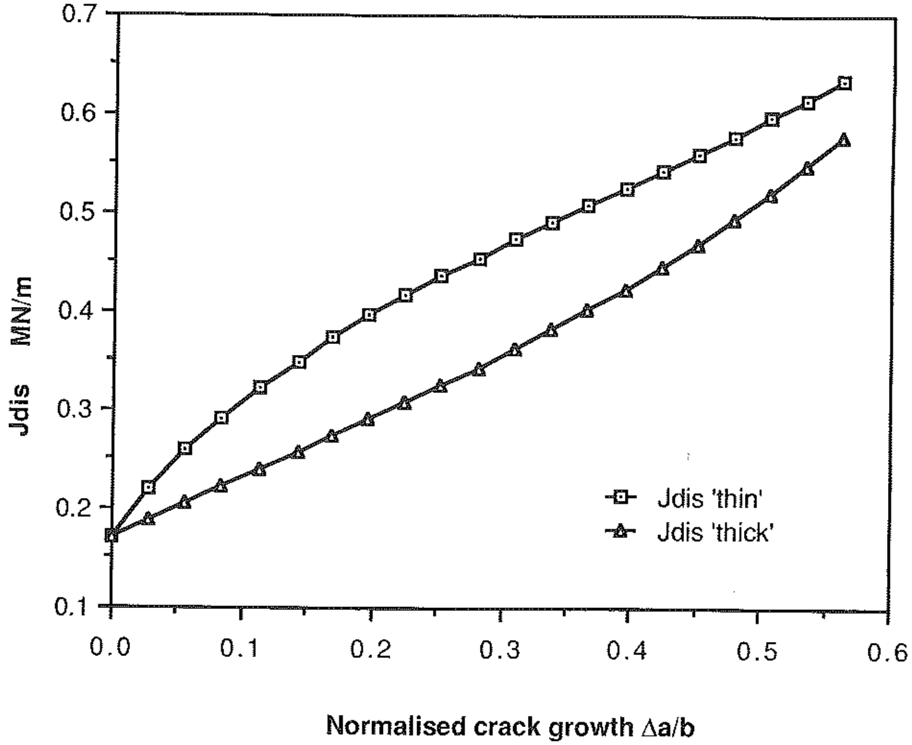

Fig 4 ***R*-curves derived from Fig. 3 for 'thick' or 'thin' three point bend pieces of any size for a given *S/b* ratio, in terms J_{dis} versus normalised growth $\Delta a/b_0$**

of Fig. 3(b) for the 'thin' pieces, the higher *R*-curve of Fig. 4 is found. This curve would be relevant to any such 'thin' piece, subject to the same restrictions on S/W and η. It may well be that if more tests were done at other values of thickness a trend of the curves of Figs 3 and 4 with thickness would emerge, rather than data falling into just the two groups shown although the small effect of thickness found over a 9-fold range of thickness is presumably a characteristic of the particular material tested.

Attention is drawn to the experimental data being treated in several ways which might affect the degree to which a particular presentation seems effective. For Fig. 1 and in the several comparisons that follow, the value of J_m has been calculated for each experimental point giving typically five useful points per curve. For $dU_{dis}/B\,da$ and J_{dis} each set of data have been smoothed by reading from the best fit-by-eye curves for individual test pieces. For Fig. 4, the raw data have been expressed via equation (1a) and then smoothed *en masse* by plotting a 'best curve' on Fig. 3(a) or (b) to emphasise the effect of the normalised abscissa. This eliminates both experimental scatter and any small geometric differences that may be hidden by the variations in Figs 3(a)(b). In Figs 5–8 individual values of $dU_{dis}/B\,da$ are used from Figs 2(a) and 2(d)

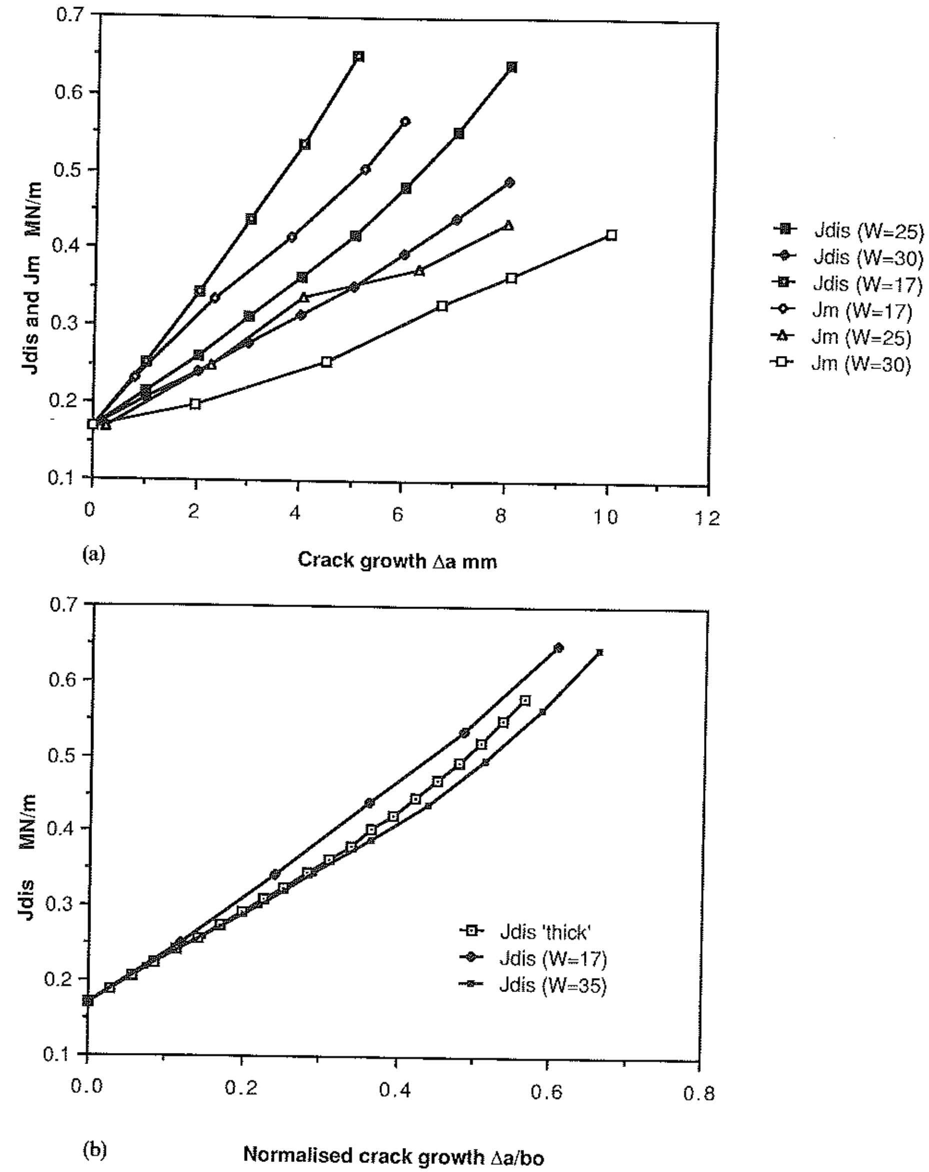

Fig 5 **Individual 'thick' *R*-curves for $B = 35$ mm, a/W nominal = 0.55. (a) J_{dis} and J_m–*R*-curves for $W = 17$, 25 and 35 mm, versus Δa. (b) A unique J_{dis}–*R*-curve versus $\Delta a/b_0$ for $W = 17$ and 35 mm; also the 'thick' curve of Fig. 4**

together with equations (3a) and (4) but the 'individual points' now refer to the steps of Δa (typically 0.5 or 1 mm) at which the slopes from the graphs of U_{dis} were read off to form Figs 2 and 3. This reduces scatter but retains any geometric trends otherwise swamped by the single 'best curves' for two regimes of

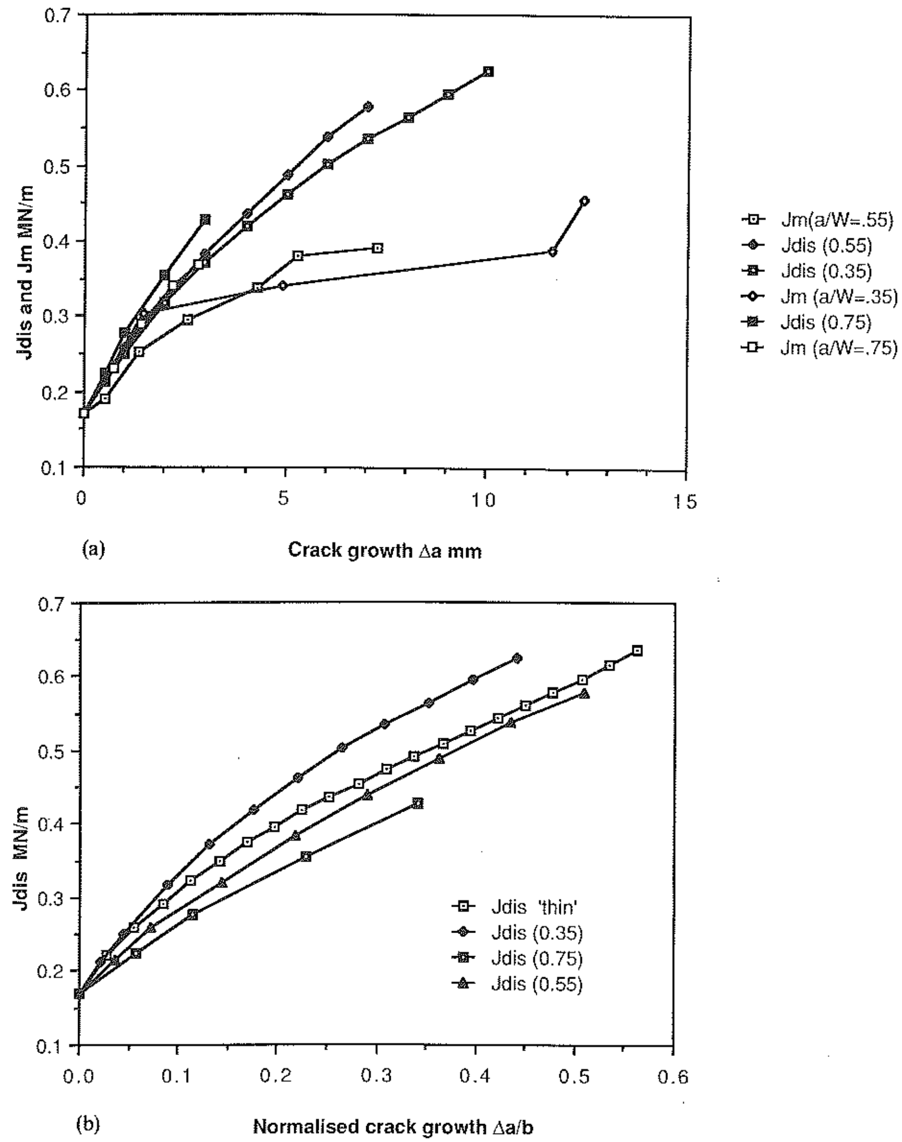

Fig 6 **Individual *R*-curves for cases with various a/W; $B = 17.5$ mm, $W = 35$ mm. (a) J_{dis} and J_m–*R*-curves for $a/W = 0.35$, 0.55 and 0.75 versus Δa. (b) J_{dis}–*R*-curves for the same cases showing a small effect of S/b**

thickness. For the first series with various W or b, the presentation against Δa for three widths, Fig. 5(a), shows a distinct trend of 'wider gives a lower curve' although use of J_{dis} gives curves that are steeper than for J_m. Plotting against the normalised abscissa, $\Delta a/b_0$, Fig. 5(b), brings the J_{dis} data together, just as it did for J_m in Fig. 1; the other widths all fall between the cases shown.

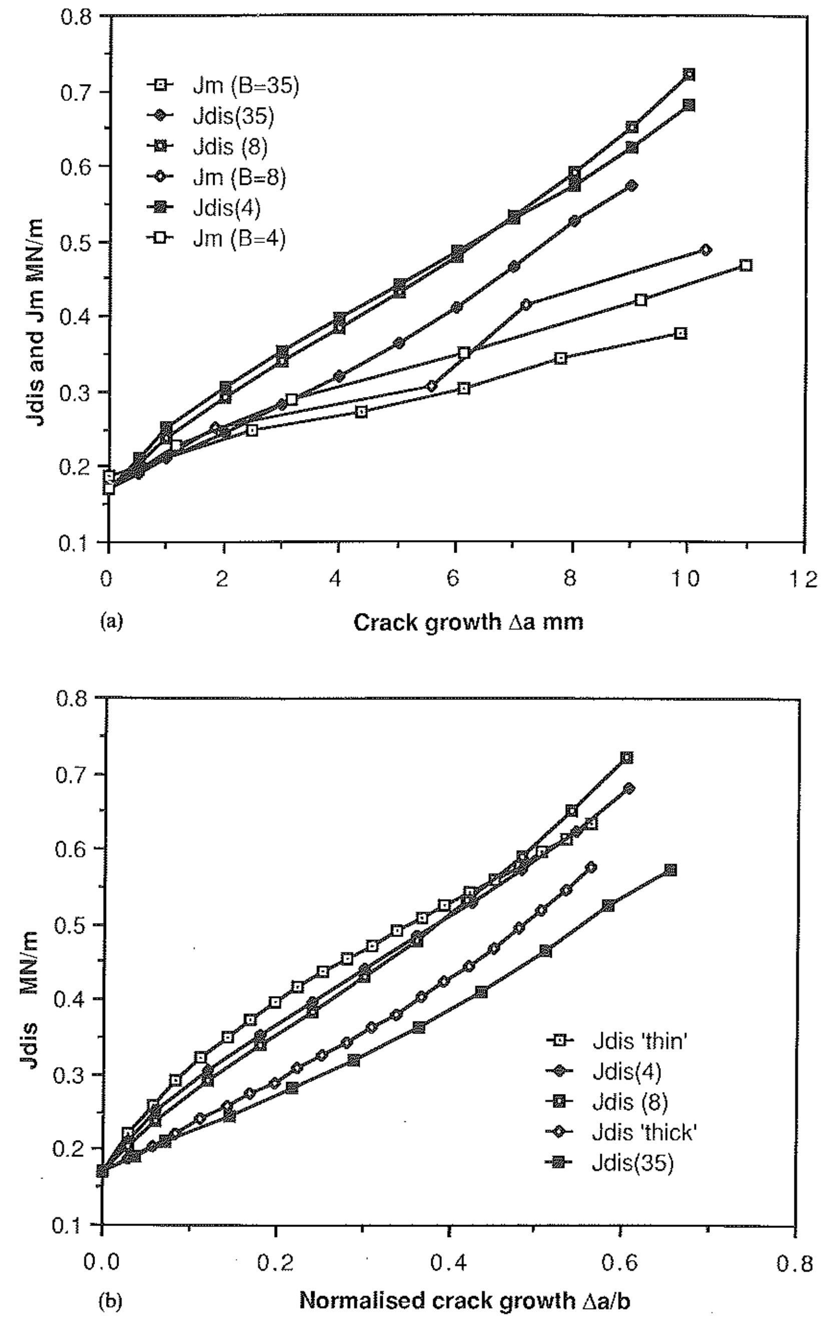

Fig 7 **Individual *R*-curves for various thicknesses, $W = 35$ mm, $a/W = 0.55$ nominal. (a) J_{dis} and J_m–*R*-curves for $B = 4$, 8 and 35 mm versus Δa. (b) J_{dis} versus $\Delta a/b_0$ for the same cases; also both curves from Fig. 4, showing a small effect of thickness**

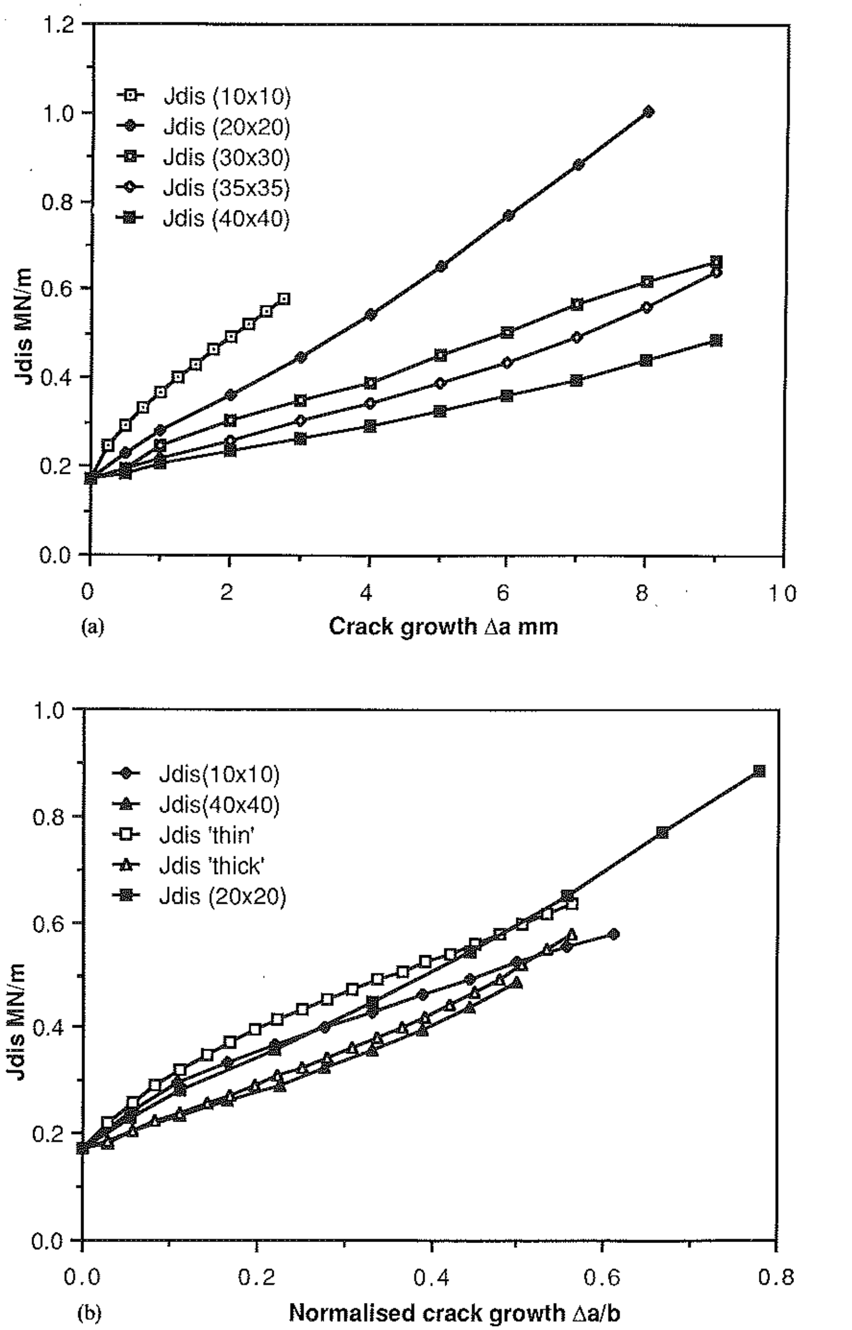

Fig 8 **Individual *R*-curves for various sizes, $a/W = 0.55$ nominal. (a) J_{dis} versus Δa showing a big effect of size. (b) J_{dis} versus $\Delta a/b$; also both curves from Fig. 4, showing only the small effect of thickness**

For the second series with various a_0/W, Fig. 6(a) shows a group of results, with the J_{dis} treatment again above the J_m treatment, with possibly a slight trend, of little more than the scatter, to 'larger a_0/W (smaller ligament) gives a higher curve' in both treatments. In terms of J_{dis} versus $\Delta a/b_0$ an effect of a_0/W is seen in Fig. 6(b) either side of the 'thin' curve from Fig. 4. This is to be expected since varying a_0/W at constant S/W, as in these tests, alters S/b_0 and thus violates the condition for which the curve of J_{dis} versus $\Delta a/b_0$ is expected to be unique.

For the third series with various thicknesses two 'thin' cases, $B = 4$ and 8 mm, and one 'thick' case, $B = 35$ mm, are shown versus Δa, Fig. 7(a). The J_{dis} data are again above the J_m version and a small effect of thickness shows, again little more than the scatter when the raw experimental points are used as for J_m. In terms of $\Delta a/b_0$, the J_{dis} treatment, Fig. 7(b), seemingly shows a distinct difference for the 4 and 8 mm cases and the 35 mm case which of course leads to the two regimes of Fig. 4. The other thicknesses are here omitted for clarity; the 12 mm case falls slightly above the 4 mm case, perhaps due to its rather larger a_0/W ratio; the 17.5 mm case falls in between.

Finally, in Fig. 8(a) the J_{dis} results for the fourth series with various sizes are shown versus Δa. J_m results are in **(2)** so are omitted here for clarity. They again fall below the J_{dis} analysis with both treatments showing a very distinct trend to 'larger sizes give lower curves'. In terms of $\Delta a/b_0$, and in so far as a single 'thick' or 'thin' curve is justified for a group of thicknesses, Fig. 3, the 10×10 and 20×20 mm cases in Fig. 8(b) relate to the upper curve from Fig. 4 whilst the 40×40 mm cases relate to the lower. The 35×35 case falls very near the 40 mm size with the 30×30 case in between those shown.

Discussion

There are two separate points of interest. The first is whether the terms $\mathrm{d}U_{dis}/B\,\mathrm{d}a$ and the associated J_{dis} are more meaningful than J_{i+1}, J_m or other. The second is whether a scaled abscissa is correct since even in terms of J_m the present data were unified in **(2)** much more when plotted against $\Delta a/b_0$ rather than just Δa. As remarked before, it is the normalisation of the abscissa that is the dominant effect in bringing the several geometries together, although use of $\mathrm{d}U_{dis}/B\,\mathrm{d}a$ puts that process on a more rational basis. As argued here, that rationale is the experimental evidence of Figs 3(a)(b) and the similar curves of **(6)** using data for A533B steel from **(10)**. Yet further examination of the present data suggests that all terms within equation (1b) may scale with the same geometric group, $(\Delta a/b_0)$ (S/b_0) whilst data in the literature with very different ratios of plastic zone to test piece size scale in a different but related manner. For those reasons, the points now made in Discussion and Conclusions, although best pin-pointed by the present data, are not seen as peculiar to this material or one set of results but as a way of re-studying virtually all fully plastic *R*-curve data.

An R-curve common to either the 'thick' or 'thin' data sets, subject to the restrictions on S/b_0 and η already discussed, has been derived provided the term J_{dis} is used for the ordinate and $\Delta a/b_0$ for the abscissa. Neither the present nor other macro-style analyses of which the authors are aware, offer a relation between 'thick' and 'thin' cases.

It should be clear that if the experimental data were perfect with no experimental scatter or material variability, and if such perfect data confirmed that the $dU_{dis}/B\,da$ curves of Fig. 3 were indeed unique for a given thickness, then the curves of J_{dis} versus $\Delta a/b_0$ would be exactly the same whether analysed directly from individual experimental data or via the separate curves of Figs 2(a)–(d) or the 'best fit' dU_{dis} curves of Fig. 3(a), for 'thick' or 3(b), for 'thin'. The differences between the results of the three analyses arise firstly because there is no smoothing when the individual points are analysed directly; secondly because the $dU_{dis}/B\,da$ data although smoothed for an individual set of test data, still reflects the actual a/W values (and hence S/b_0 values) which may differ from the nominal 0.55, being for example only 0.47 for the 'thick' case $W = 30$ mm but 0.51 for the $W = 17$ mm case as used Fig. 5(b); thirdly the 'best fit' curves of Figs 3(a) and (b) smooth the raw data and also eliminate any variations of a/W or indeed other small geometric effects that may exist. Whilst these different procedures for handling the data affect individual values (and recalling the use of a mean value for initation in **(2)**), it is thought that all the major trends are correctly shown.

As already remarked it was suggested in **(7)** that for full plasticity the dJ type parameter adopted should become a function of work increment but that for large growth it should not be an explicit function of Δa. It must also reduce to G for lefm. When J_m was introduced, it was the only J-like term to retain the former attribute so arranged as not to lose the essential latter attribute. As seen clearly in Figs 5(a), 6(a) and 7(a), the J_{dis} term derived from dU_{dis} via equation (3) is not identical to J_m. Nevertheless it possesses the attributes required in **(7)** and reduces to lefm as seen following equation (1b).

As seen in Fig. 1, the data for J_m versus $\Delta a/b_0$ fall into two bands, similar to, though less well defined than, the J_{dis} curves just discussed. Since the effect of thickness is accounted for in the curves of Fig. 3 for $dU_{dis}/B\,da$, the similarity seems to imply a relation between J_m and J_{dis}. This relation is that in the event that the elastic terms were negligible then

$$dJ_{dis} = dJ_m = dJ_{tot} = \eta\, dU_{tot}/Bb \tag{4}$$

Such neglect is quite often found for R-curve results in the fully plastic regime but is not made here where the elastic term accounts for some 10–20 percent of the increments of work. It is the commonality of the major dissipation term within dU_{tot} that accounts for the broad brush similarity of thickness and scaling effects of either term but it is the different way in which the elastic term affects J_{dis} and J_m that accounts for the closer scaling of the former. The J_{dis} analysis also reveals in a way that J_m does not, that the various a_0/W data should not be quite the same as the other cases because of the variation of S/b_0 here.

In so far as the $dU_{dis}/B\,da$ treatment appears to unify R-curve data up to large growth much more so than does the conventional treatment of a rising R-curve then since it is a function of $\Delta a/b_0$ rather than just Δa, it implies that there is no regime of so called J-controlled growth for the fully plastic case. Nevertheless, as seen in Fig. 1 and Figs 5–8, it is the use of a scaled abscissa rather than the choice of J that produces the major unification of the data, although J_{dis} has a simple physical basis (of combined dissipation) not perceived for J_m.

The evidence for a scaled abscissa as outlined earlier in this paper and illustrated in **(2)–(6)**, has not been refuted to the writers' knowledge whilst the supporting arguments, seen as an extension to uncontained yield of the contained yield picture of **(7)**, have matured to the present position. Whether or not the scaling aspects are correct, it may be asked why the conventional rising R-curve has been adopted for elastic–plastic behaviour over the past few years. Although curves for J_{dis} versus $\Delta a/b_0$ could be predicted from $dU_{dis}/B\,da$ data for some other values of S/b_0 and η (subject still to shallow notch effects on toughness and to differences between η_{el} and η_{pl} in some cases) it might be asked why turn to a rising R-curve if the falling one is in fact the more general?

Up to about the 1960s a falling R-curve was pictured, notably by Boyd **(11)**, e.g., **(12)** Fig. 2.66, the rising curve coming to prominence in 1961 due mainly to Krafft *et al.* **(13)**. In the opinion of the present writers, an R-curve representing a dissipative rate can either rise or fall with crack growth according to circumstance. Whilst the plastic zone increases with size, the R-curve will rise, Fig. 9(a), until it reaches a plateau dictated by either the material toughness or the material thickness, as in lefm plane stress. In epfm tests where full plasticity is reached initiation often occurs rather close to maximum load so that most of the rising limb corresponds to blunting rather than to actual tearing. The reducing regime follows almost at once, Fig. 9(b), since the volume of the active plastic regime is decreasing. Data for such a case are shown in **(10)** although the rising limb with blunting was not pointed out. The similar concept of a reducing zone of active plasticity and reducing apparent toughness has been explored **(14)** for metal forming amongst other things. Thus the complete dissipation rate R-curve for a body that starts to tear in the elastic regime but then passes the fully plastic regime as the ligament reduces in size would first rise and the fall as shown schematically in Fig. 9(c).

The introduction of the rising R-curve into epfm appears to have ensued from the convenience of carrying over evaluating J from equation (2a), prior to initiation, into equation (2b), after initiation. Regarding J here as a measure of the resistance curve, the term must by that definition relate to a dissipative action rather than to its original energy release rate meaning. Prior to initiation both work and work rate increase so the definition of η ensures both a

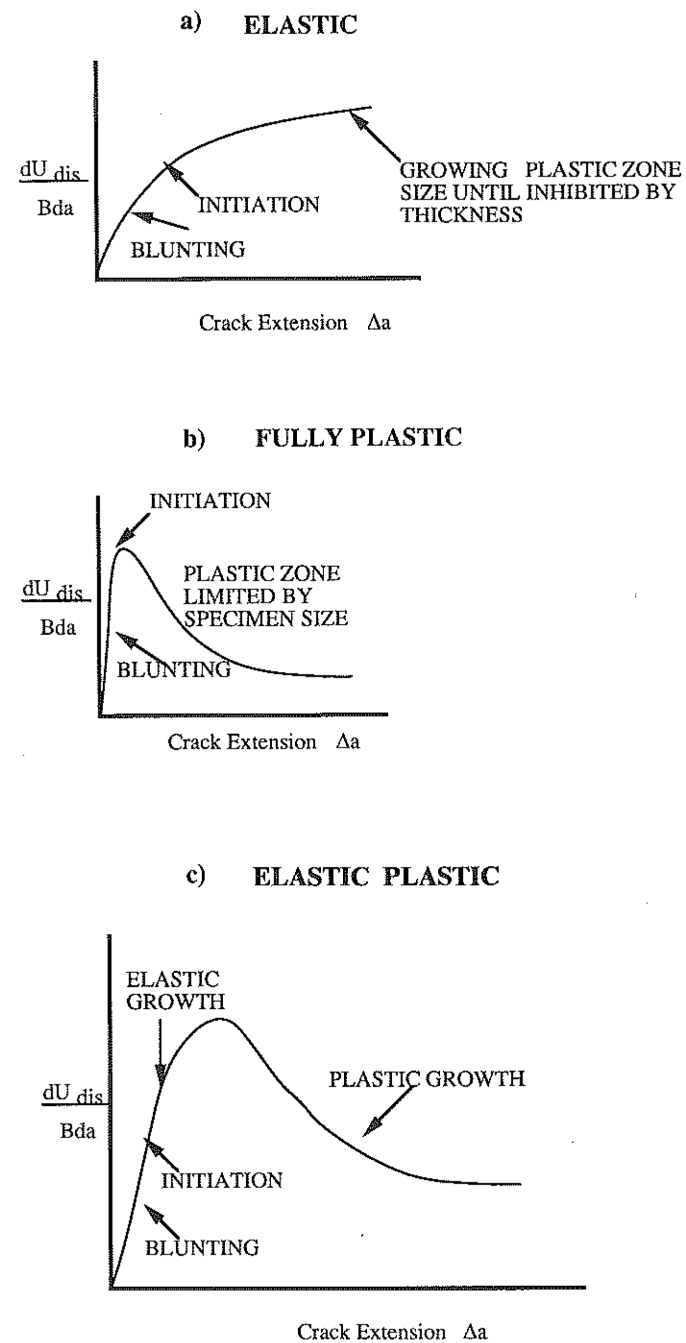

Fig 9 Schematic of $dU_{dis}/B\,da$ versus Δa for (a) elastic case, (b) typical fully plastic behaviour, and (c) elastic–plastic case

plausible physical meaning and value of J in equation (2a). After initiation, work absorbed naturally continues to increase with crack growth whereas the *WORK RATE* is clearly seen to decrease in the fully plastic case so that it now seems doubtful whether use of equation (2b), however modified to give 'J this' or 'J that' can be strictly justified.

The question seems to reduce to 'what is toughness'? Is it a 'work dissipation per unit cross-section' as implied by equation (2a), or is it a 'work dissipation rate $dU_{dis}/B\,da$' as used here. In expressing instability Griffith clearly implied the latter. The rate term is also relevant to instability in the epfm case and the dissipative side of the I energy rate balance, e.g., **(12)** p. 197 and worked example p. 413, could be much more readily expressed through dU_{dis} than through the various dJ terms used. Some of the present titanium tests that were taken to instability will be analysed in that way, elsewhere. However, measures such as Charpy energy or drop weight tear test energy relate to the quantity and not the rate; it is repeated that the quantity and the rate are proportional in lefm but not in epfm.

It is beyond the scope of this paper to argue such a fundamental question beyond suggesting that in principle both terms are separately relevant. Whichever is more useful in service, the present evidence suggests that the reducing curve of $dU_{dis}/B\,da$ is more amenable to scaling at least to other bend geometries and it has been shown in a paper to the 22nd National Symposium on Fracter Mechanics, Altanta, 1990, that an R-curve relevant to the same thickness for bending under lefm conditions of well contained yield, can be deduced from some epfm test data.

Conclusions

Tearing toughness data on a titanium alloy for a variety of sizes and geometries of three point bend type have been analysed in terms of the work dissipation rate, $dU_{dis}/B\,da$. For a series of thick pieces a unique curve that decreases with growth was obtained versus a normalised crack growth, $(\Delta a/b_0)$ (S/b_0), up to some 60 percent of the ligament, with a different, slightly higher, unique curve for a series of thinner pieces.

From such a unique curve for a given thickness, a rising R-curve can be deduced in terms of J_{dis} versus $\Delta a/b_0$ for a given value of S/b_0 and η. The term J_{dis} is not identical to J_m, the two being the same only if the elastic term is negligible. Although in the present tests tearing occurs in full plasticity the cumulative effect of the elastic term may be appreciable for large growths.

Apart from this different definition of the ordinate for an R-curve, the present data strongly supports the use of a scaled abscissa and that the concept of the reducing curves of $dU_{dis}/B\,da$ versus $(\Delta a/b_0)$ (S/b_0), being a direct extension of the lefm arguments, is more meaningful than the conventional rising R-curve. Examination of data for other materials is essential to confirm whether the unification offered by these concepts is particular to the results of (10) discussed in (6) and the present tests or whether there is indeed a more general relevance.

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