THE NONLINEAR BEHAVIOUR OF PMMA

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In this paper the creep and recovery behaviour of PMMA is described. From the uniaxial tensile creep and recovery tests it is found that PMMA displays linear viscoelastic behaviour below around 25MPa and nonlinear viscoelastic behaviour above this stress level. In the nonlinear range, the behaviour can also be divided into two subranges. The first range (about 25MPa to 37.5MPa) can be described by separation of variables ($\varepsilon = f(\sigma)h(t)$), and the second range (above 37.5MPa) is too strongly nonlinear to be predicted by separation of variables which means the effect of stress and time can not be uncoupled ($\varepsilon = F(\sigma,t)$).

INTRODUCTION

Linear viscoelastic theory was fully described in 1953 [1]. Since then the theory has been often used in practice. However, the linear theory is limited to low stress levels, but some plastics show a nonlinear viscoelastic behaviour even at low stress levels or short time *etc.*. Our aim is to establish a constitutive relationship for the nonlinear viscoelastic behaviour in order to be able to predict the stress and strain field in high stressed regions and apply this constitutive relationship to FEM calculation.

Up to now the literature has been studied and the models, which were proposed earlier by other investigators, have been described [2]. According to the classification by Ward [3], these models are divided into three groups, namely, *the engineering models, the rheological models* and *the molecular models*. In order to describe the material behaviour of PMMA in high stress field during short time the attention is focused on the rheological models, because the other two groups have

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a limited application range such as the engineering models do not provide a general representation for creep, recovery and behaviour under complicated loading programs *etc.*.

The rheological models include several models but finally the Schapery model [4] is taken as the starting point to describe nonlinear viscoelastic behaviour of PMMA on the basis of the creep and recovery tests. In essence, the Schapery model is a general single integral representation of viscoelasticity. Other models such as the Boltzmann Superposition Principle and the Leaderman theory *etc.* can be seen as the special cases of the Schapery model. When stress is treated as an independent state variable, a creep formula under uniaxial stress σ can be expressed in the Schapery model as

$$\varepsilon(t) = g_0 D_0 \sigma + g_1 \int_{0^-}^{t} \Delta D(\psi - \psi') \frac{d g_2 \sigma}{d\tau} d\tau \tag{1}$$

where $D_0 = D(0)$ is the initial value of compliance, $\Delta D = D(\mathrm{t}) - D_0$ is the transient component of compliance *in the linear range* and ψ and ψ' are the so-called reduced-times defined by

$$\psi = \int_0^t \frac{d\zeta}{a_\sigma[\sigma(\zeta)]} \quad , \quad \psi' = \int_0^\tau \frac{d\zeta}{a_\sigma[\sigma(\zeta)]} \quad . \tag{2}$$

The material properties g_0, g_1, g_2 and a_σ are functions of stress. In the linear range $g_0 = g_1 = g_2 = a_\sigma = 1$. Thermodynamically the changes in g_0, g_1 and g_2 reflect the third and higher order dependence on the Gibb's free energy on the applied stress; a_σ arises from the similar high-order effect in both entropy production and free energy. Physically, g_0 can describe the initial nonlinear behaviour, g_1 the behaviour when the initial creep strain and the initial recovery strain are not identical. Another advantage is that the Schapery model can take into account other effect factors such as temperature, moisture and so on by introducing the effect factors in the material functions if needed.

EXPERIMENTAL

The tensile creep and recovery tests are carried out on injection moulded PMMA specimens with a cross section $4 \times 10\mathrm{mm}^2$. The raw material (ICI grade, Diakon MG102/CMG 302) was provided by ICI company. At room temperature, the specimens stand 20 minute creep at constant tensile force, and then recover for 40 minutes. The step loading and unloading rates are 25kN/s. The creep tests have been carried out at stress levels from 10MPa to 50MPa.

RESULTS AND DISCUSSIONS

From Fig. 1 and Fig. 3 it is noticed that PMMA displays obvious linear range (around below 25MPa) and nonlinear range (above 25MPa). Above 37.5MPa, the nonlinear behaviour of PMMA is strongly dependent on the combined effect of stress and time, e.g., the influence of stress and time on the material behaviour can not be uncoupled at high stress levels.

When the Schapery model is used, the linear compliance must be determined first. According to the literature survey and the suggestion of Schapery, the power law of time is applied to describing compliance due to its practical use and simplicity. It is expressed as $D=D_0+D_1t^n$ where D_0, D_1 and n are material constants. According to the tests, the parameters are determined

$D_0=3.0885\times10^{-2}$	$D_1=5.7593\times10^{-4}$	$n=0.26$

when the units of strain and stress are % and MPa, respectively. For the creep under constant stress, the Schapery model is reduced as $\varepsilon=k+pt^{0.26}$ where $k=D_0\sigma g_0$ and $p=D_1\sigma g_1g_2/a_\sigma^{0.26}$. From Fig. 2, it is observed that at high stress levels (above 37.5MPa) the behaviour can not be described accurately enough. In fact, n strongly depends on the stress level and related with stress by $n=-0.655+0.0244\sigma$ (unit of σ is MPa) if the stress is higher than 37.5MPa. In other word, the power law of time is unsuitable above 37.5MPa. Therefore other description of compliance must be applied when the stress level is higher than 37.5MPa.

From the creep data, the isochronous curves are presented in Fig. 3. It is clear that the initial strain ε_0 $(=\varepsilon(0))$ is linearly proportional to the stress σ until stress around 42.5MPa that is higher than the linear viscoelastic range. Therefore the initial strain displays linear behaviour. It is also noticed that the initial modulus is about 3237MPa. In the same figure, it is also obvious that at lower stress levels the strain caused only by creep $\Delta\varepsilon_c^{**}$ at 20 minutes is very small in comparison with the initial strain and about 15% of the initial strain at the same stress levels.

The creep strain $\varepsilon_c(t)$ and the recovery strain $\varepsilon_r(t)=\varepsilon(t)-\varepsilon(t-t_a)$ $(t\geq t_a^+)$ are compared in the same figure where t_a is creep time. In figure 4, it is clear that in the nonlinear range the recovery strain is larger than the creep strain. This deviation increases with the stress level.

When the initial creep strain ε_0 (e.g. the jump of strain at $t=0$) is compared with the initial recovery strain $\varepsilon_r(t_a^+)$ (e.g. the jump of strain at $t=t_a$), it is observed that at very lower stress levels they are equal and in the higherstress levels, the

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[**] The subscript c and r indicate creep and recovery respectively.

initial creep strain is smaller than the initial recovery strain. This can be described by parameter g_1 in the Schapery model.

CONCLUSIONS

From the uniaxial tensile creep and recovery tests it is found that

1. PMMA displays linear viscoelastic behaviour below around 25MPa and nonlinear viscoelastic behaviour above this stress level. About from 25MPa to 37.5MPa the material behaviour can be described by separation of variables and about above 37.5MPa it is strongly nonlinear and can not be predicted by separation of variables.
2. The initial loading strain shows linear behaviour until 42.5MPa, but is not identical with initial unloading strain in the nonlinear range.
3. When the Schapery model is applied, the power law of time is unsuitable for describing compliance when stress is higher than 37.5MPa. Other description of compliance is necessary, which is under investigation. Further the Schapery model is expected to be used in biaxial case.

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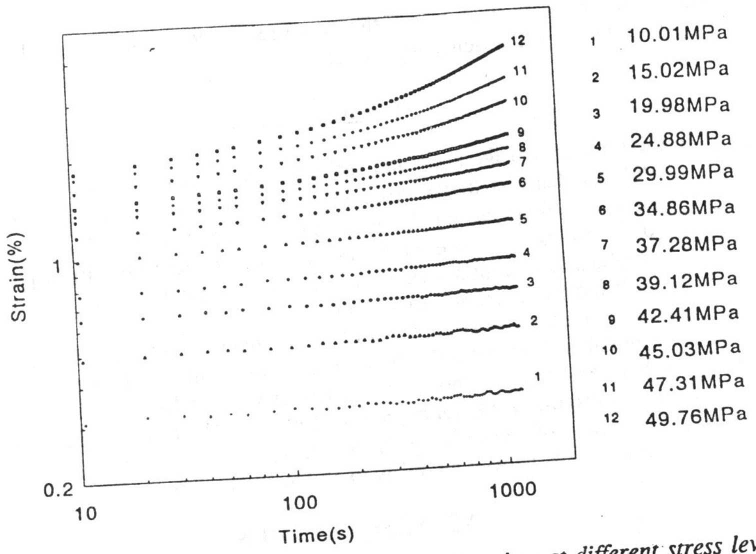

Fig. 1 Double-logarithm curves of strain against time at different stress levels.

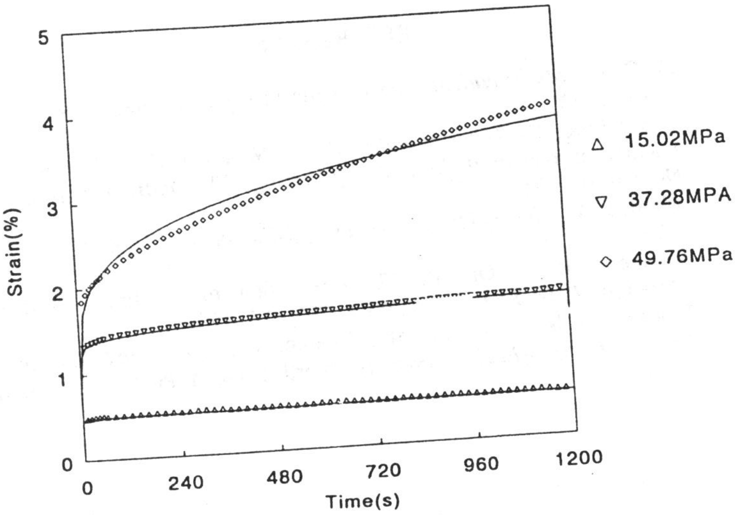

Fig. 2 Description of power law of time with constant exponential n=0.26.

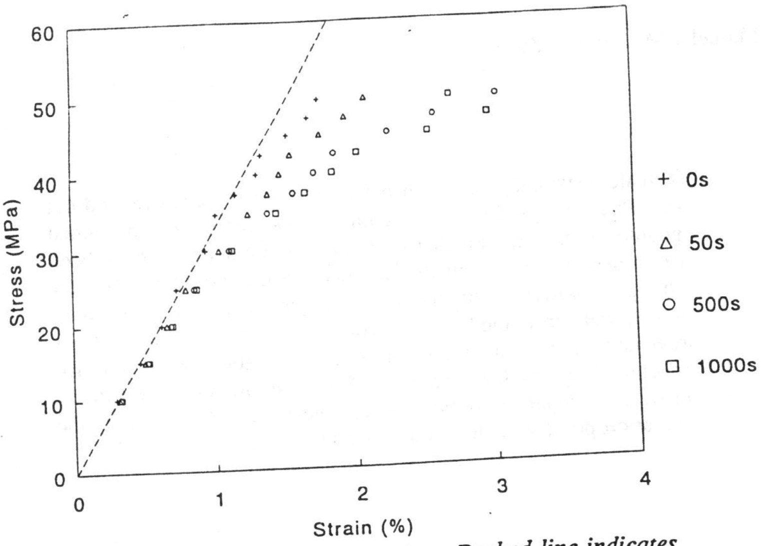

Fig. 3 Isochronous curves at different time. Dashed line indicates the linear elastic behaviour of initial strain.

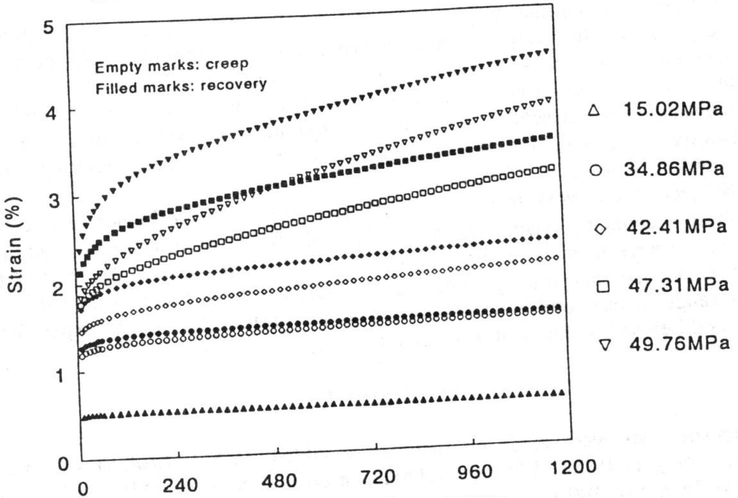

Fig. 4 Comparison between creep and recovery. t_a *is creep time and it is 20 minutes in these tests.*