POLYNOMIAL EXPRESSIONS OF THE STRESS INTENSITY FACTOR FOR A CRACKED BOLT

J. Toribio*

This paper offers polynomial expressions of the stress intensity factor (SIF) for surface cracks in bolts subjected to tension and bending. SIF solutions are given along the crack front as a function of the ratio crack depth/bolt minor diameter and the crack aspect ratio. An analytical interpolation was carried out using FEM-results, to provide polynomial expressions. Values of the SIF for tension show that the influence of the crack aspect ratio is really important, and the SIF is higher either at the deepest point or at surface depending on such a ratio. For bending, all results fit in the same band, and the effects of both the crack shape and the position on the crack line are weaker.

INTRODUCTION

In the matter of cracked bolts, a great research effort has been made in recent years in this area, to improve the knowledge of their fracture behaviour (Toribio(1)). However, some difficulties have arisen because of the complex geometry of bolts: the 3D nature of the crack, causing the SIF change along the front, and the helical shape of the thread, which involves not only geometric difficulties but also problems in applying the load on the bolt.

There are really few published K-solutions for cracked bolts. James and Mills (2) presented an interesting review, although most of the solutions were calculated for cracks in non threaded bars (smooth cylinders). Only the K-solutions obtained by Reibaldi (3), and Nord and Chung (4) were computed using a threaded bar to model the geometry of the bolt, thus taking into account the effect of the thread on the stress intensity factor (SIF).

* Department of Engineering, University of La Coruña, E.T.S.I. Caminos, Pol. Sabón, Parc. 12-14, 15141 Arteixo, La Coruña, Spain.

This paper offers polynomial expressions of the SIF for semi-elliptical surface cracks in M8x1.0 bolts subjected to tension and bending. It is a development of the work that the author has been performing for the *European Space Agency* as a part of the Programme *"Damage Tolerance of Metallic Structures"*, whose basic results (Toribio *et al.* (5); Toribio (6)) were successfully applied to predict crack growth rates in metallic cracked bolts (Müller-Wiesner (7)).

F.E.M. MODEL

An ISO M8 x 1.0 bolt was analyzed with an elliptical crack in the open thread ground perpendicular to the bolt axis. The shape of the crack is semi-elliptical (Fig.1) with its center located on the surface of the inner cylinder and semi-axes *a* and *b*. Two aspect ratios (*Shallow crack*: a/b=0.2; *Circular crack*: a/b=1) and five crack depths were used (a/d = 0.1, 0.2, 0.3, 0.4 and 0.5) Two loading conditions were considered (see Fig.2): *Tension loading* (a), and *Bending moment* (b). Results were expressed in dimensionless form ($Y = K/\sigma/(\pi a)^{1/2}$) where σ is $4F/\pi d^2$ for tension and $32M/\pi d^3$ for bending.

The numerical computations were carried out by using the Finite Element Method (FEM) with singular quarter-point elements (Henshell and Shaw (8); Barsoum (9)). The SIF was computed by using a stiffness derivative technique (Parks(10); Hellen (11)). As reported previously (5,6), two modifications were made to improve the accuracy of the results: the displacement not only of the main node, but also of the quarter-point nodes located in the normal plane and the adjacent nodes placed in the crack line, avoiding both the change of the singularity and the crack curving.

NUMERICAL RESULTS

SIF versus crack depth is plotted at the crack center and crack surface in Fig. 3 for both shallow and circular cracks in tension. Fig. 4 shows the K-values at center and surface for both types of cracks in bending. K-values at the crack center are higher for shallow cracks subjected to tension loading (Fig. 3), and lower for circular cracks subjected to bending moment (Fig. 4). K-values at the crack surface are higher for circular cracks subjected to tension loading (Fig. 3), and lower for shallow cracks subjected to bending moment (Fig. 4). For shallow cracks (white symbols), the SIF is always higher at the crack center, and for circular cracks (dark symbols), always higher at the crack surface. This conclusion remains valid for all crack depths and both loading conditions (tension and bending) on the bolt.

ANALYTICAL EXPRESSIONS

Starting from these results, an interpolation was performed to provide useful analytical expressions to be used in fatigue life prediction. The Y-solutions can be written in the form of a second-degree polynomial depending on the dimensionless crack depth a/d:

$$Y(a/b, a/d) = A_o(a/b)+A_1(a/b)*(a/d) + A_2(a/b)*(a/d)^2$$

where A_o, A_1 and A_2 are first-degree polynomials which depend on the crack aspect ratio (a/b). For each loading condition (tension and bending) and for the two relevant points of the crack (center and surface), the expressions for A_o, A_1 and A_2 are the following:

– TENSION LOADING, CENTER:

$A_o = 1.0155 - 0.2375\ (a/b)$
$A_1 = -0.584 + 0.015\ (a/b)$
$A_2 = 6.45575 - 3.34875\ (a/b)$

– TENSION LOADING, SURFACE:

$A_o = 0.4695 + 0.8225\ (a/b)$
$A_1 = 0.37775 - 1.47875\ (a/b)$
$A_2 = -\ 0.16025 + 2.94625\ (a/b)$

– BENDING MOMENT, CENTER:

$A_o = 0.89375 - 0.36375\ (a/b)$
$A_1 = -\ 0.55925 + 0.36625\ (a/b)$
$A_2 = 2.379 - 1.88\ (a/b)$

– BENDING MOMENT, SURFACE:

$A_o = 0.6535 - 0.0925\ (a/b)$
$A_1 = -\ 1.14875 + 1.55875\ (a/b)$
$A_2 = 3.028 - 1.855\ (a/b)$

CONCLUSIONS

1. K-solutions for a cracked bolt subjected to tension and bending were obtained for various crack shapes and crack depths. The bolt was modelled as a threaded bar and an improved stiffness derivative technique was used to compute the SIF.

2. K-values for both tension and bending are higher at the crack center of shallow cracks and the crack surface of circular cracks.

3. Present values of the SIF for tension show that the influence of the crack aspect ratio *a*/*b* is very important, and the SIF is higher either at the deepest point or in surface depending on such a ratio.

4. For bending moment, all results fit in the same band, and the effects of both the crack shape and the position on the crack line are weaker in this case.

5. An interpolation was performed with the numerical results, to provide analytical expressions useful for fatigue life prediction. Such expressions are second-degree polynomials on the crack depth, and their coefficients are linear expressions on the crack aspect ratio.

ACKNOWLEDGEMENTS

This work was supported by ESA under subcontract R 1190.3131 E between MBB-ERNC and the Materials Science Department of the Polytechnical University of Madrid. The author gratefully acknowledges this support as well as the encouragement of Dr. Henriksen (ESA-ESTEC) and Dr. Müller-Wiesner (MBB-ERNO). In addition, the author would like to thank the *Colegio de Ingenieros de Caminos, Canales y Puertos-Demarcación de Galicia*, Spain, for the financial support of technical trips during the final stages of this work.

REFERENCES

(1) Toribio, J., 4th Progress Meeting (Phase II) of the ESA-Programme 'Damage Tolerance of Metallic Structures', Noordwijk, The Netherlands, 1989.
(2) James, L.A., and Mills, W.J., Eng. Fract. mech., Vol. 30, 1988, pp. 641-654.
(3) Reibaldi, G.G., 'Advances in Fracture Research -ICF6' (Edited by R. Valluri et al.), Pergamon, Oxford, pp. 1177-1183, 1984.
(4) Nord, K.J., and Chung, T. J., Int. J. Fracture, Vol. 30, 1986, pp. 47-55.
(5) Toribio, J., Sánchez-Gálvez, V., Astiz, M.A., and Campos, J.M., Eng. Fract. Mech., Vol. 39, pp. 359-371.
(6) Toribio, J., Int. J. Fracture, Vol. 10, 1992, pp. 367-385.
(7) Müller-Wiesner, D, Technical Note TN-0T217-014/90, MBB-ERNO, Bremen, Germany, 1990.
(8) Henshell, R.D. and Shaw, K.G., Int. J. Num. Meth. Eng., Vol. 9, 1975, pp. 495-507.
(9) Barsoum, R.S., Int. J. Num. Meth. Eng., Vol. 10, 1976, pp. 25-37.
(10) Parks, D.M., Int. J. Fracture, Vol. 10, 1974, pp. 487-502.
(11) Hellen, T.K., Int. J. Num. Meth. Eng., Vol. 9, 1975, pp. 187-207.

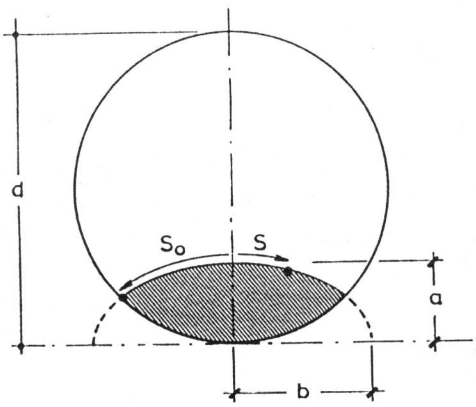

Figure 1. Crack shape.

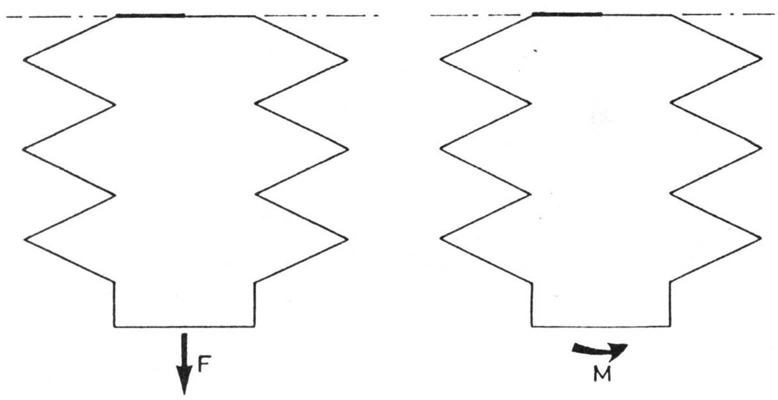

Figure 2. Loading conditions: (a) tension, (b) bending.

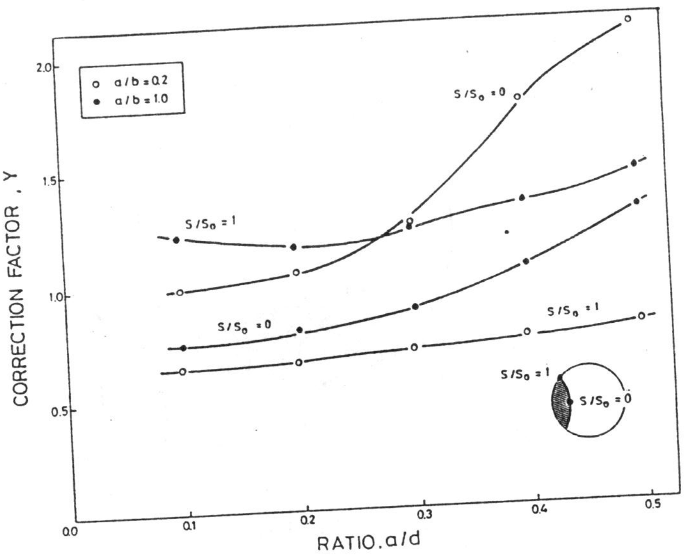

Figure 3. K-values for tension.

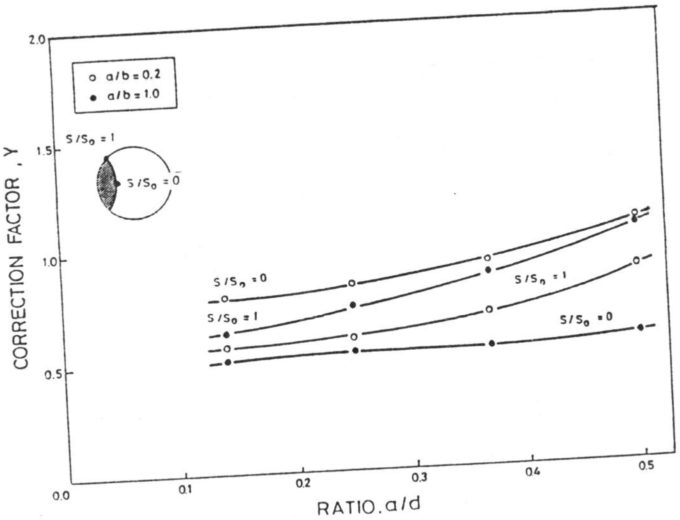

Figure 4. K-values for bending.