ON PATH-INDEPENDENT INTEGRALS FOR DYNAMIC FRACTURE

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Geometries with dynamically propagating cracks can be analysed with the use of path-independent integrals. Explicit calculation of these elastodynamic integrals in terms of dynamic stress intensity factors and the crack-propagation velocity reveals a dependence on the shape of the path inside the fracture-process zone. This fact is contrary to prior results presented in the literature. The differences, however, remain small for crack-propagation velocities up to 0.6 times the shear-wave speed.

INTRODUCTION

The concept of the energy release rate, based on the existence of a critical surface energy to initiate fracture, has found a wide range of application for crack-growth predictions in both static and dynamic fracture situations. As a generalization of the static J integral of Rice [1], Nishioka and Atluri [2] introduced elastodynamic integrals J_k' having the meaning of an energy release rate. These integrals are defined on an infinitesimal contour encircling the crack tip and lying entirely inside the fracture-process region. Expressions for arbitrary contours at remote positions are also presented in [2] and the elastodynamic integrals have been shown to be independent of the choice of these remote contours.

Nishioka [3] has claimed by numerical investigation that the integrals J_k' are independent of the integration contour inside the fracture-process zone. In the present paper it is shown by analytical evaluation for two distinct contours that the elastodynamic integral J_2' does depend on the infinitesimal path. The obtained results for the respective contours are compared with the results of [2, 3] and the differences are examined.

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ELASTODYNAMIC INTEGRALS

Consider a body of a linearly elastic material containing a dynamically propagating crack. The singular stress field only depends on the instantaneous crack-propagation velocity and not on the crack curvature as has been shown by Achenbach and Bažant [4] and Freund [5, Ch. 4]. Consequently, the model of a half-plane crack propagating at velocity c may be employed.

We introduce moving cartesian and polar coordinates x_i ($i = 1, 2, 3$) and r, θ with origin O attached to the crack tip. The compatibility equations, the momentum equations (including acceleration terms) and the constitutive relations have a singular solution for the stress components σ_{ij}. These stresses and the corresponding displacements u_i depend on the polar coordinates and the crack-growth speed c according to

$$\sigma_{ij} = \frac{K_M}{\sqrt{2\pi r}} f_{ij}^M(\theta, c) , \qquad u_i = \sqrt{\frac{2r}{\pi}} \frac{K_M}{\mu} g_i^M(\theta, c) , \tag{1}$$

where μ is the shear modulus and K_M are the dynamic stress intensity factors for mode M ($M = I, II, III$). The functions $f_{ij}^M(\theta, c)$ and $g_i^M(\theta, c)$ are well-known and given in e.g. [2] or [5, Ch. 4]. It is noted that the singular fields only apply near the crack tip and inside the region where the fracture process takes place.

Nishioka and Atluri [2] introduced the elastodynamic integrals

$$J'_k = \lim_{\varepsilon \to 0} \int_{\Gamma_\varepsilon} ((W + T)n_k - \sigma_{ij} n_j u_{i,k}) \, \mathrm{d}s \qquad (k = 1, 2), \tag{2}$$

where the accent is used to distinguish from the static integrals. Γ_ε is an infinitesimally small contour inside the fracture-process zone surrounding the crack tip and with outer normal vector n_k, see Fig. 1. The elastic energy density is $W = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}$ and the kinetic energy is $T = \frac{1}{2}\rho \dot{u}_i \dot{u}_i$ where ρ is the density of the material. The notation $_{,k}$ and the superposed dot indicate the derivatives with respect to x_k and time t, while the Einstein convention of summation over repeated indices is used.

For an arbitrary contour Γ and curves Γ_S along the crack faces, see Fig. 1, the following expression is obtained [2]

$$\begin{aligned} J'_k &= \lim_{\varepsilon \to 0} \left\{ \int_{\Gamma + \Gamma_S} ((W + T)n_k - \sigma_{ij} n_j u_{i,k}) \, \mathrm{d}s \right. \\ &\qquad \left. + \int_{A - A_\varepsilon} (\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}) \, \mathrm{d}A \right\} \qquad (k = 1, 2), \end{aligned} \tag{3}$$

where A is the domain bounded by $\Gamma + \Gamma_S$ and the crack faces, while A_ε is bounded by Γ_ε and the crack faces. The boundary of the difference $A - A_\varepsilon$

is the composed contour $\Gamma + \Gamma_S - \Gamma_\varepsilon$. The path independence of (3) can be verified by taking two contours Γ_1 and Γ_2 and subtracting the respective integral expressions. With the use of the internal equations, it is derived that the difference $J'_k|_{\Gamma_1} - J'_k|_{\Gamma_2}$ vanishes.

It is emphasized that the *same* contour Γ_ε near the crack tip must be chosen for both integration paths. This fact is not as clearly mentioned in [2], but the dependence on Γ_ε can become significant. Since the integral J'_1 is not influenced by the infinitesimal contour, see [3], only the effects on J'_2 are further investigated below.

EXPLICIT CALCULATION

Dilatational and shear waves play an important role in dynamic fracture. The respective wave speeds are equal to $c_d = c_s\sqrt{2(1-\nu)/(1-2\nu)}$ and $c_s = \sqrt{\mu/\rho}$ with ν being Poisson's ratio [2, 5]. The following velocity-related parameters are introduced

$$\beta_1 = \sqrt{1-(c/c_d)^2}\,, \qquad \beta_2 = \sqrt{1-(c/c_s)^2} \tag{4}$$

and also the Rayleigh function

$$D = D(c) = 4\beta_1\beta_2 - (1+\beta_2{}^2)^2\,. \tag{5}$$

For the evaluation of the integral J'_2 as defined by (2), a rectangular contour is chosen inside the fracture-process zone surrounding the crack tip, see Fig. 2. After the substitution of the singular stress and displacement fields (1), the limit for $\delta \to 0$ is taken first and then for $\varepsilon \to 0$. This calculation results in

$$\begin{aligned} J'_2 \;&=\; -\frac{(\beta_1-\beta_2)(1-\beta_2{}^2)\,K_I K_{II}}{\mu D^2} \times \\ &\times \left(\frac{(\beta_1+\beta_2)\left[4\beta_1\beta_2+(1+\beta_2{}^2)^2\right]}{2\sqrt{\beta_1\beta_2}} - 2(1+\beta_2{}^2)\right). \end{aligned} \tag{6}$$

On the other hand, evaluation of (2) for a circular contour yields [2]

$$\begin{aligned} J'_2 \;&=\; -\frac{(\beta_1-\beta_2)(1-\beta_2{}^2)\,K_I K_{II}}{\mu D^2} \times \\ &\times \left(\frac{(2+\beta_1+\beta_2)\left[4\beta_1\beta_2+(1+\beta_2{}^2)^2\right]}{2\sqrt{(1+\beta_1)(1+\beta_2)}} - 2(1+\beta_2{}^2)\right). \end{aligned} \tag{7}$$

It is obvious that expressions (6) and (7) do not coincide. This finding is at variance with the conclusion of Nishioka [3] who asserted on the basis of a numerical investigation that the elastodynamic integrals J'_k would be independent of the infinitesimal contour inside the process region.

DISCUSSION OF RESULTS

The results for rectangular (6) and circular contours (7) are plotted in Fig. 3 for a Poisson's ratio of 0.30. The integrals have been normalized to their values for zero crack-growth speed c.q. the value for static fracture $J_2 = -(1-\nu)/\mu \cdot K_I K_{II}$. Although the curves are close to each other, the relative difference between both expressions increases rapidly when the Rayleigh-wave velocity $c_R = 0.9274\, c_s$ is approached, see Fig. 4.

On the other hand, this deviation is not more than 1% (or 5%) for crack-propagation velocities up to $0.65\, c_s$ (or $0.80\, c_s$). Consequently, both formulas (6) and (7) are equivalent and may be used interchangeably. The numerical errors in the computed values of the integrals in [3] are probably in the same range as these small deviations. In addition, only velocities up to $0.6\, c_s$ have been investigated in [3]. The combination of these two facts may explain why the dependence on the shape of the infinitesimal contour was not detected and also the subsequent erroneous conclusion of path invariance of the elastodynamic J_2' integral.

The analysis above leads to the following conclusions:

1. Contrary to J_1', the elastodynamic integral J_2' is dependent on the shape of the small contour inside the fracture-process zone.

2. The deviations between the expressions for rectangular and circular contours remain relatively small for low crack-propagation velocities.

3. For crack-growth speeds approaching the Rayleigh-wave velocity, the differences become increasingly significant and further investigation of the integral J_2' is required.

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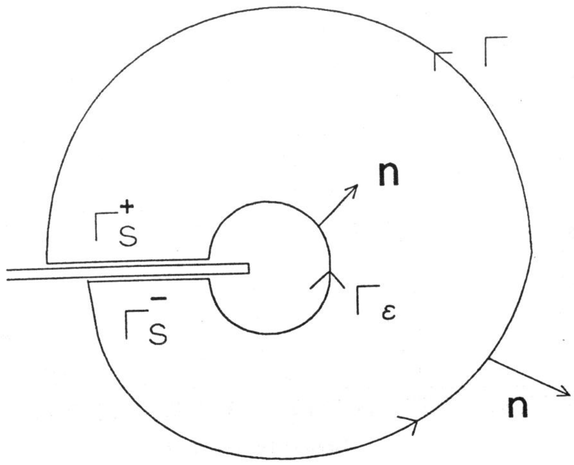

Figure 1. Integration contours surrounding the crack tip.

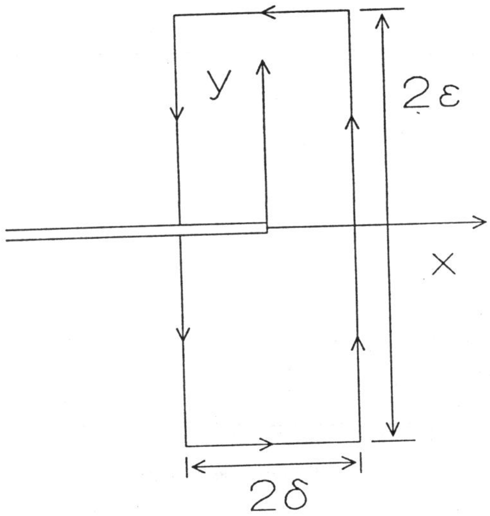

Figure 2. Contour for evaluation of J_2'.

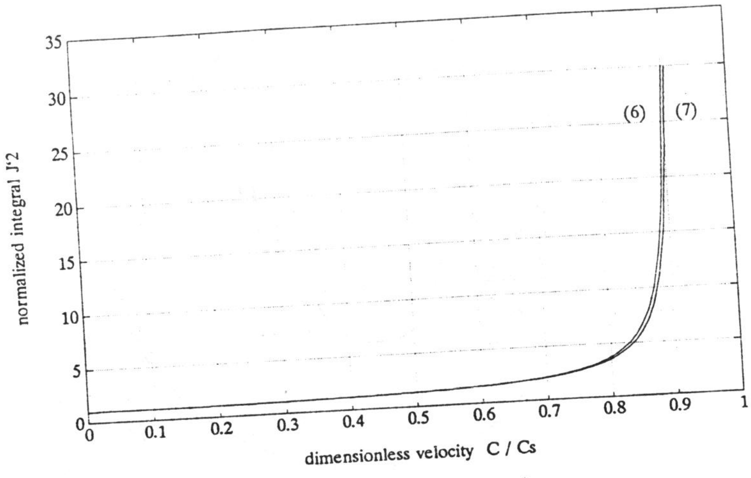

Figure 3. Variation of expressions (6) and (7) for integral J_2'.

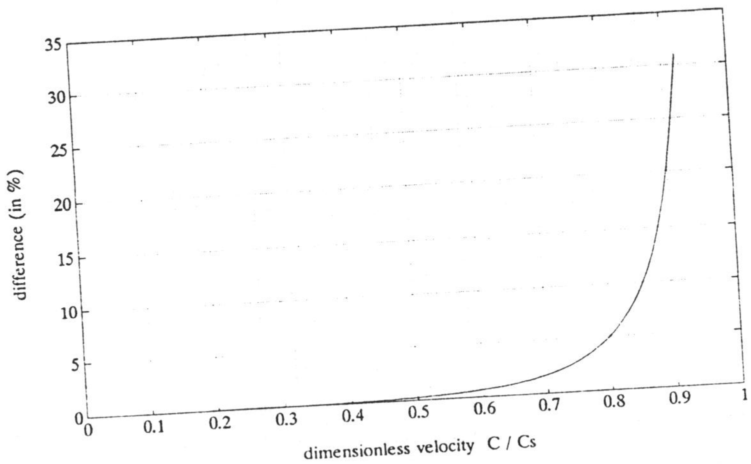

Figure 4. Relative difference between expressions (6) and (7).