

# CONTROLLED CRACK PROPAGATION WITH EXTERNALLY BONDED LIGAMENTS

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The present work studies specimens with a pre-existing crack, where external ligaments are positioned in order to control the increase of the crack length; to this end, it simulates specimens with ligaments by means of idealized geometrical shapes; using finite elements, it determines the stress and strain fields as well as the stress intensity factor. This study deals with the influence of ligament position, size and hardness on the value of the stress intensity factor and the instability of the cracked specimen. Our findings are compared to already existing experimental results.

## INTRODUCTION

Construction technology today uses external ligaments to stop crack propagation in large-scale constructions such as aircraft, ships, bridges etc. This intervention aims at stopping further propagation of the crack. The technique reduces crack-tip stress intensity factors or imposes residual compressive stresses. Experimental methods, such as photoelastic caustic on the cracked specimens, with externally bonded ligaments, have been presented in the past.

In this work, the above problem is approached theoretically by means of the finite element method; we consider cracked specimens where we examine the influence of the externally bonded ligament. The ligament is considered similar to a material having greater stiffness than the specimen material. We study the variation of the stress intensity factor due to the position, size and hardness of the liga-

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ment. Strain energy density theory (1,2) also studies the crack instability of cracked specimens with ligaments. Conclusions of this analysis are confirmed by experimental findings referred to in paper (3).

### FAILURE CONSIDERATION

The strain energy density theory has made assumptions for the role of peaks and valleys of the function of strain energy density, for stability of a system. In the location of a crack tip the notation  $[(dW/dV)_{\min}^{\max}]_L$  stands for the maximum of the many local minima of  $dW/dV$ . In the single coordinate system the notation  $[(dW/dV)_{\min}]_G$  stands, equally, for global values of  $dW/dV$ . Let  $\ell$  be the distance between points, L and G, along the predicted trajectory of crack propagation and  $\Delta(dW/dV)$  be the difference between the values of  $dW/dV$  at points L and G. Then, the quantity  $S_{LG} = \ell \Delta(dW/dV)$  may be used as a measure of mechanical system instability. More details of this method can be found in references (1,2).

### PROBLEM AND METHOD OF APPROACH

We studied single edge-cracked rectangular plate specimens, with externally bonded ligaments in the direction of crack propagation, subjected to tension (Fig. 1a). The dimensions of the specimen are: length  $2H=20$  cm, width  $W=10$  cm and thickness  $B=0.2$  cm. For a crack length of  $a_0=1$  cm, we considered rectangular ligaments, of dimensions  $h \times b=2$  cm  $\times$  1 cm and 4 cm  $\times$  1 cm and thickness  $c=0.2$  cm, facing the crack tip at such positions as to have,  $a/w=0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$  and  $0.9$ . The specimens were supposed to be made of plexiglass with Young's modulus  $E=3.4 \times 10^3$  MPa, Poisson ratio  $\nu=0.33$ , tensile strength  $\sigma_{\max}=85$  MPa and fracture toughness  $K_{Ic}=1200$  kN m<sup>-3/2</sup>. We considered three different Young's moduli,  $E_a$ , for the ligaments such that  $E_a/E=2, 10$  and  $24$ .

For the case of the plate, as shown in Figure 1a, the stress field and stress intensity factors are computed by application of the finite element program (APES). This computer program incorporates the 12-noded quadrilateral isoparametric elements, allowing for cubic displacement fields and quadratic stress and strain fields within each element. To determine stress intensity factors, enriched elements are employed, having the elastic singular solution superimposed so that a corner node corresponds to a crack tip.

Due to existing symmetry the analysis is made for only half of the plate. The idealization of the half plate is displayed in Figure 1b. The size and position of the external ligaments on the grid pat-

tern were determined from the description of its corresponding finite elements; their mechanical behaviour was identified with that of superimposed bonded plates. The composite stiffness was computed from the elastic moduli and the thicknesses of the specimen and the ligament.

Using the program, we also calculate the strain energy density  $dW/dV$  and, further, trace the contours of  $dW/dV$  on the finite elements of the idealization by means of plotter. These findings make it possible to calculate accurately the position and magnitude of the stationary values of  $dW/dV$ , and the instability parameter  $S_{LG}$ . The above quantities can be used to predict the crack trajectory and instability of the crack path and crackings of a mechanical system.

### DISCUSSION OF RESULTS

Figure 2 presents the variation of the normalized stress intensity factor  $K_I^a/K_I$  versus the position of the ligament when its dimensions are  $h \times b = 2 \text{ cm} \times 1 \text{ cm}$ , and for  $E_a/E = 2, 10$  and  $24$ .  $K_I^a$  and  $K_I$  are the stress intensity factors for the specimen with and without ligament, respectively. We see that the decrease of stress intensity factor is drastic when the ligament is positioned close to the crack tip. Increasing the ligament hardness by 10 to 24 times does not give the analogous decrease of the stress intensity factor.

Figure 3 presents the variation of the normalized stress intensity factor versus the position of the ligament, when  $E_a/E = 10$ , if the dimensions of ligament are  $h \times b = 2 \text{ cm} \times 1 \text{ cm}$  and  $4 \text{ cm} \times 1 \text{ cm}$ . We observe drastic decrease of stress intensity factor when the ligament is positioned close to the crack tip and its length  $h$  is increased.

In Figure 4, the normalized stress intensity factor is plotted against the ratio  $E_a/E$ , when the dimensions of the ligament are  $h \times b = 4 \text{ cm} \times 1 \text{ cm}$ , for its positions  $a/w = 0.2, 0.3$  and  $0.5$ . We observe that, for a crack length  $a_0 = 1 \text{ cm}$  and a position of the ligament beyond the middle of the specimen width, the hardness of the ligament has no influence at all.

Figure 5 shows the variation of parameter of instability  $S_{LG}$  as a function the position of the ligament, when its dimensions are  $h \times b = 4 \text{ cm} \times 1 \text{ cm}$ , for  $E_a/E = 2, 10$  and  $24$ .

In Figure 6 the contours of the strain energy density on the region between the crack tip and the ligament, for  $E_a/E = 24$  are plotted. The ligament has dimensions  $h \times b = 4 \text{ cm} \times 1 \text{ cm}$  and is posi-

tioned at a distance 3cm from the crack tip. It is observed that the point G where the minimum value of the strain energy density  $(dW/dV)_{\min}$  appeared, is not on the axis of the crack. Considering both the point G and the shape of the contours it is concluded that the trajectory of the crack will deviate from the straight line (2). There is a probability of the crack path by-passing the ligament and breaking the specimen completely. We arrive at the same conclusion when the ligament is much harder than the specimen's material and is located far from the crack tip.

### CONCLUSIONS

From this study, the following general conclusions can be reached:

1. The ligaments are most effective when positioned close to the initial crack tip.
2. The length (h) of the ligament drastically reduces the stress intensity factor for positions close to the initial crack tip.
3. For ligament hardness greater than that of the specimen, we have no corresponding influence on the crack stopping ability.
4. For ligament positions beyond the middle of the specimen ( $a/w=0.5$ ) the situation is not altered no matter what the ligament hardness.
5. With increasing ligament hardness there is a corresponding increase in the probability of the crack path deviating from a straight line.

The above conclusions are in good agreement with the corresponding experimental results (3).

### REFERENCES

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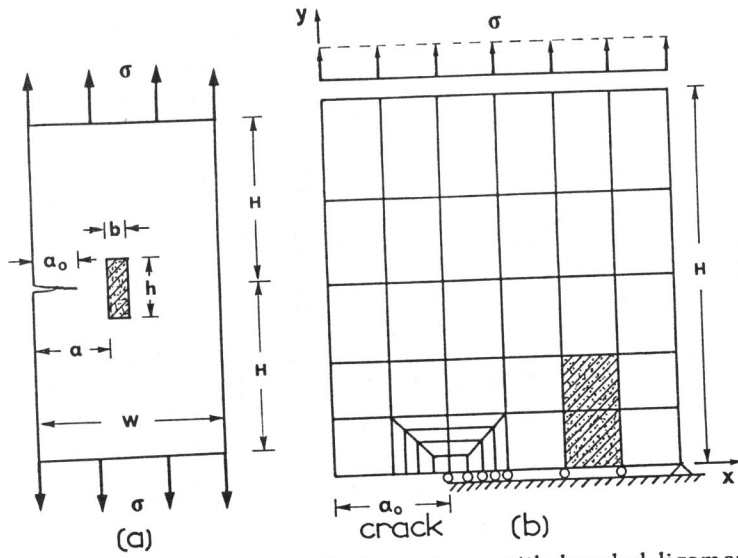


Figure 1 (a) Geometry of cracked specimen with bonded ligament. (b) Grid pattern for one-half of the plate.

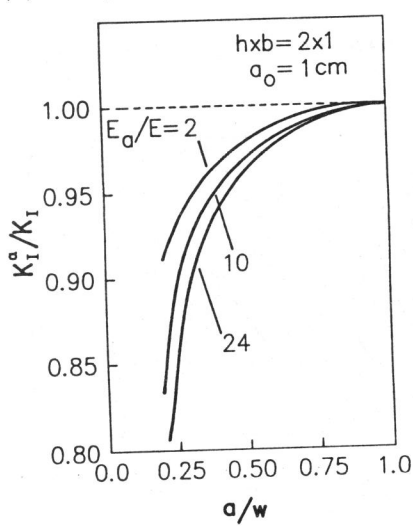


Figure 2 Normalized SIF versus ligament position.

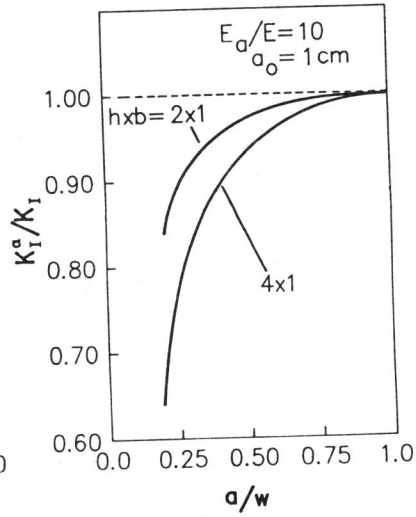


Figure 3 Normalized SIF versus ligament position.

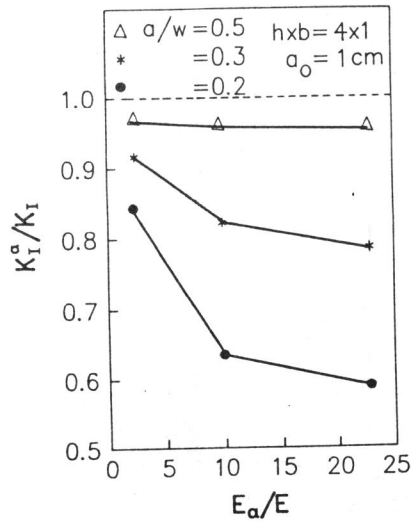


Figure 4 Normalized SIF versus of ratio stiffness  $E_a/E$ .

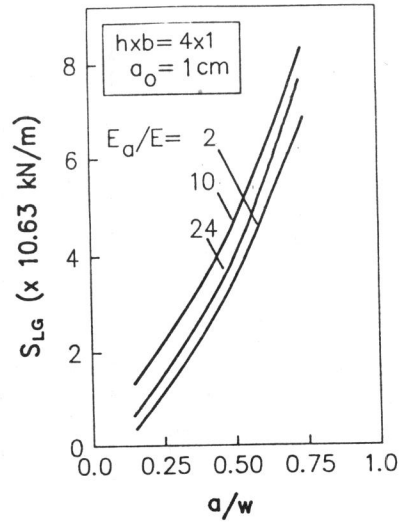


Figure 5 Variation of instability parameter with ligament position.

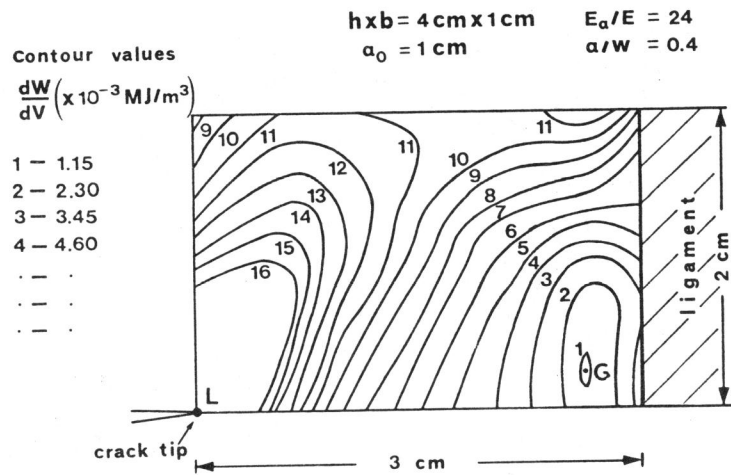


Figure 6 Strain energy density contours on the region in front of the crack tip.