THEORETICAL AND NUMERICAL ANALYSIS OF CRACKED WELDED PRESSURE VESSEL

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Theoretical and finite element analysis of the cracked welded pressure vessel have been performed regarding the problem of path independency of J integral for the bi-material body (i.e. for the welded structure) and for the thin shells (i.e. pressure vessel). It has been shown that only under specific conditions Rice's J integral is path independent (cylindrical pressure vessel with an axial crack and fusion line of weldment positioned parallel to the crack). In all other cases one should use the generalized form of J integral which is due to some additional terms path independent and can be identified as the energy release rate due to unit quasi-static crack growth. Such an integral expression is evaluated applying the finite element method for an appropriate example.

INTRODUCTION

It has been recently shown by Sedmak, Jarić and Berković (1-4) that J integral, as given by Rice (5), is not path independent when applied for thin shells, unless the crack is positioned along a generatrix of cylindrical shell. It is also well-known that J integral loses its path independency in the case of a bi-material body, unless the crack is parallel to the material boundary, as shown by Gurtin and Smelser (6), Miyamoto and Kikuchi (7) and Sedmak, Sedmak and Ogarević (8). In both cases the generalized form of J integral are defined, being path independent unconditionally.

Investigations of the crack growth behaviour in welded pressure vessel require sophisticated experimental techniques such as direct J integral measurements, developed by Read (9). Nevertheless, it is obvious that both thin shells and bi-material problems are involved. Therefore, the aim of this paper is to investigate path dependency problem of J integral for bi-material thin shell in respect to the direct measurement evaluation technique.

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J INTEGRAL FOR BI-MATERIAL THIN SHELLS

Starting point in this analysis is the J integral for thin shells, which is given here in its final form, since all other details are given elsewhere, (1-4):

$$J = \int_{\Gamma}\left(W\delta_1^{\alpha} - \mathbf{N}^{\alpha}.\frac{\partial \mathbf{u}}{\partial \theta^1}\right)n_{\alpha}ds - \int_{D}\left(W\delta_1^{\alpha} - \mathbf{N}^{\alpha}.\frac{\partial \mathbf{u}}{\partial \theta^1}\right)B_{\alpha}^1 N_1 da + \int_{S^-} W^- m_1 ds - \int_{S^+} W^+ m_1 ds \tag{1}$$

where W stands for the strain energy density, $\mathbf{n}=n_{\alpha}\theta^{\alpha}$ for the unit normal to the integration path Γ, $\mathbf{N}^{\alpha}$ for the membrane forces, $\mathbf{u}$ for the displacement vector, θ^{α} for the convective coordinates, α=1,2; B_{α}^1 for the components of the second metric tensor, N_1 for the component of the unit normal to the shell middle surface along the coordinate θ^1, m_1 for an analogous quantity for the crack face, as shown in Fig. 1, together with the integration area D and paths along the crack surfaces $S^{\pm}$.

As in the original formulation of J integral (5), such an integral expression is path independent not only for the homogeneous body, but also for the heterogeneous one, provided that heterogeneity is confined to an axis perpendicular to the crack. In that case integral expression 1 can be identified with the energy release rate due to unit quasi-static crack growth, using the same arguments as Gurtin (10).

Now let us consider the general case of a cracked bi-material body, schematically shown in Fig. 2, with a material boundary not being parallel to the crack. Applying the integral expression 1 for two paths shown in Fig. 2, Γ_1 and Γ_2, one can obtain:

$$J = \int_{\Gamma_1}\left(W\delta_1^{\alpha} - \mathbf{N}^{\alpha}.\frac{\partial \mathbf{u}}{\partial \theta^1}\right)n_{\alpha}ds - \int_{D_1}\left(W\delta_1^{\alpha} - \mathbf{N}^{\alpha}.\frac{\partial \mathbf{u}}{\partial \theta^1}\right)B_{\alpha}^1 N_1 da + \int_{S^-} W^- m_1 ds - \int_{S^+} W^+ m_1 ds \tag{2}$$

$$0 = \int_{\Gamma_2}\left(W\delta_1^{\alpha} - \mathbf{N}^{\alpha}.\frac{\partial \mathbf{u}}{\partial \theta^1}\right)n_{\alpha}ds - \int_{D_2}\left(W\delta_1^{\alpha} - \mathbf{N}^{\alpha}.\frac{\partial \mathbf{u}}{\partial \theta^1}\right)B_{\alpha}^1 N_1 da \tag{3}$$

where expression 3 does not contain the line integrals along $S^{\pm}$ and equals zero because the integration path Γ_2 does not contain the crack. Taking both expressions into account one can write the following integral expression:

$$J = \int_{\Gamma}\Big(W\delta_1^{\alpha} - \mathbf{N}^{\alpha}\cdot\frac{\partial \mathbf{u}}{\partial\theta^1}\Big)n_{\alpha}ds - \int_{D}\Big(W\delta_1^{\alpha} - \mathbf{N}^{\alpha}\cdot\frac{\partial \mathbf{u}}{\partial\theta^1}\Big)B_{\alpha}^1 N_1 da -$$
$$+ \int_{S^-} W^- m_1 ds - \int_{S^+} W^+ m_1 ds \;\; - \int_{l}\Big([W]\delta_1^{\alpha} - \Big[\mathbf{N}^{\alpha}\cdot\frac{\partial \mathbf{u}}{\partial\theta^1}\Big]\Big)n_{\alpha}ds \qquad (4)$$

where l stands for the boundary between two different materials and [ ] for the jump function across this boundary. Using Gurtin's approach (6,10) both path independency and physical interpretation of the integral expression 4 can be proved, but that is beyond the scope of this paper. We only emphasize here that the second, third and fourth integral expressions are thin shell "correction" terms and the fifth one eliminates the material boundary contribution.

Having in mind conclusions given in references (1-4) for the thin shell J integral, and in (6-8) for the J integral for bi-material body, it is obvious that Rice's J integral (the first term in the expression 4) is path independent only for the cylindrical shell with an axial crack positioned parallel to the material boundary.

Finally, one should notice that expression 4 can not be applied for the asymmetrical problems. In such a case J integral should be considered as a vector quantity with the components J_{β}, as follows:

$$J_{\beta} = \int_{\Gamma}\Big(W\delta_{\beta}^{\alpha} - \mathbf{N}^{\alpha}\cdot\frac{\partial \mathbf{u}}{\partial\theta^{\beta}}\Big)n_{\alpha}ds - \int_{D}\Big(W\delta_{\beta}^{\alpha} - \mathbf{N}^{\alpha}\cdot\frac{\partial \mathbf{u}}{\partial\theta^{\beta}}\Big)B_{\alpha}^1 N_1 da -$$
$$+ \int_{S^-} W^- m_1 ds - \int_{S^+} W^+ m_1 ds \;\; - \int_{l}\Big([W]\delta_{\beta}^{\alpha} - \Big[\mathbf{N}^{\alpha}\cdot\frac{\partial \mathbf{u}}{\partial\theta^{\beta}}\Big]\Big)n_{\alpha}ds \qquad (5)$$

with β=1,2. Unfortunately, the physical meaning of J_2 component is so far not clearly understood, limiting our analysis to the symmetrical problems.

FINITE ELEMENT METHOD APPLICATION AND RESULTS

The finite element method of thin shell J integral evaluation is already described in reference (4). Therefore, we shall give here only an outline of the procedure, with some details about the material boundary term evaluation.

The regular meshes of quadrilateral isoparametric elements were used, with the singularity modeled by the Richardson extrapolation method. Material nonlinearity has been employed in order to solve elastic-plastic problem using von Mises criterion of plastic flow.

Calculation of integral expression 4 has been performed using specially written post-processor, based on already determined displacement and stress field. Methodology is completely analogous to the one already described in reference (4) and is in fact extended in order to incorporate the fifth integral term in expression 4. An example of the mesh used is shown in Fig. 3, together with the appropriate integration paths.

Cylindrical welded pressure vessel has been chosen for finite element analysis. As it is shown in Fig. 3, an axial crack is positioned perpendicular to the fusion lines between the base and weld metal. For such configuration both surface integral (the second term in expression 4) and line integrals along the crack surface (the third and fourth terms) become zero. On the other hand, the integral around fusion line (the fifth term) does not vanish for this position of the crack against the fusion line.

The difference in material properties of the base and weld metal becomes evident only when plastic deformations are reached, as shown in Table 1. Therefore, as far as elastic analysis in this example is performed, Rice's J integral is still path independent, as it is shown in Fig. 4, where the results for both elastic and elastic-plastic analysis are presented. The results are shown for the complete integral expression 4 (denoted by ∘) and for the first term only (denoted by □). As it can be seen, the difference becomes evident as soon as plastic deformations occur, confirming the path independency of integral expression 4.

TABLE 1 - Base and weld metal material properties

	Yield strength (MPa)	Hardening coefficient	Young's modulus (GPa)	Poisson ratio
Base metal	800	0.02	210	0.3
Weld metal	600	0.05	210	0.3

Finally, let us notice that other examples, e.g. circumferential crack in an axial weldment of cylindrical pressure vessel, could be used for further analysis, in order to verify other aspects of theoretical predictions given in this paper.

DISCUSSION AND CONCLUSIONS

- Cracked welded pressure vessel behaviour can be very complex problem because of various influencing parameters, such as residual stresses, heterogeneity in heat-affected-zone and local geometrical misalignments (due to welding), making an experiment inevitable.

- Direct J integral measurement has been proved to be one of the most suitable experimental technique for an evaluation of crack behaviour in complex structures, such as welded pressure vessels. Nevertheless, as it has been shown here, J integral is not path independent, unless the crack is positioned along the generatrix of a cylindrical shell and the fusion lines are parallel to the crack. In all other cases generalizations of the J integral are necessary.

Theoretical and numerical analysis of the generalized integral expression is available, but on the other hand it is not possible (or at least we do not see how) to modify the existing experimental procedure, such as direct measurement of J integral, in order to take thin shells and bi-material effects into account. Therefore, it is obvious that only complete analysis, including experimental, theoretical and numerical methods, can provide reliable estimation of crack growth behaviour in the welded pressure vessel.

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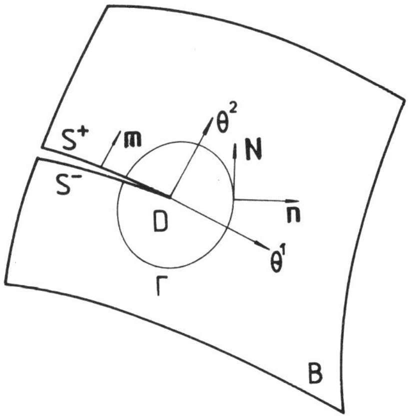

Figure 1 Thin shell with an edge crack

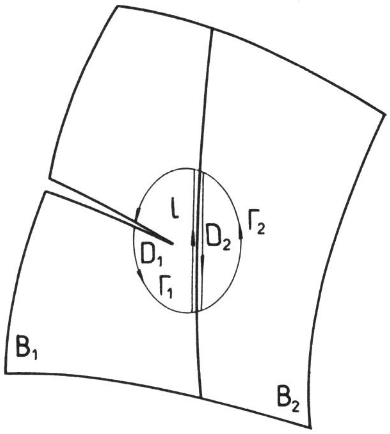

Figure 2 Integration domains for bi-material thin shell

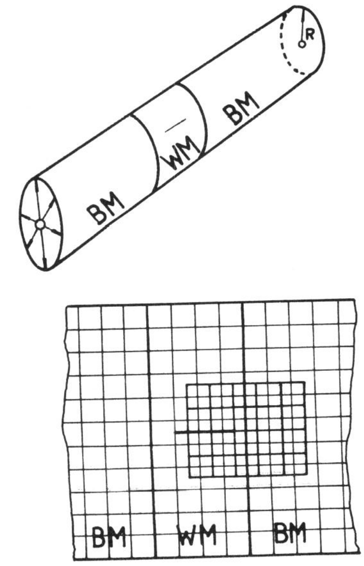

Figure 3 Welded cracked vessel with FE mesh

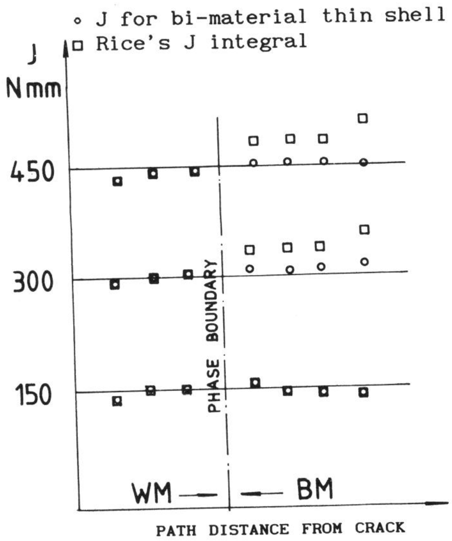

Figure 4 J integral vs. path distance