TRANSFORMATION OF STRESS FREQUENCY IN CRITERIA FOR MULTIAXIAL RANDOM FATIGUE

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A frequency analysis of a number of linear and nonlinear criteria has been done with digital simulation and it has been shown how frequencies of stress state components transform while reduction of the multiaxial stress state to the uniaxial one. The criteria for which increase of frequency of the equivalent stress is large and unacceptable from the physical point of view have been shown.

INTRODUCTION

Bandwidth of frequency of the equivalent stress $\sigma_{eq}(t)$, influences a number of counted cycles and half-cycles, determines a value of cumulated damages and, as a consequence, determines fatigue life of materials. At the constant variance increase of frequency of the stress $\sigma_{eq}(t)$ leads to determination of lower fatigue life.

This paper contains an analysis of quantitative changes in bandwidth of frequency of the stress $\sigma_{eq}(t)$ according to nine mathematical models of multiaxial fatigue criteria for four stress states. The analysis concerns stationary and ergodic Gaussian processes.

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THE STRENGTH CRITERIA

Equations expressing the stress $\sigma_{eq}(t)$ according to the analysed criteria are the following:

- Criterion of the maximum normal stress in the fracture plane

$$\sigma_{eq}(t) = \hat{l}_1^2 \sigma_{xx}(t) + \hat{m}_1^2 \sigma_{yy}(t) + \hat{n}_1^2 \sigma_{zz}(t) + 2\hat{l}_1\hat{m}_1 \sigma_{xy}(t) + 2\hat{l}_1\hat{n}_1 \sigma_{xz}(t) + 2\hat{m}_1\hat{n}_1 \sigma_{yz}(t) \quad (1)$$

- Criterion of the maximum normal strain in the fracture plane

$$\sigma_{eq}(t) = \left[\hat{l}_1^2 (1+\nu) - \nu\right] \sigma_{xx}(t) + \left[\hat{m}_1^2 (1+\nu) - \nu\right] \sigma_{yy}(t) + \left[\hat{n}_1^2 (1+\nu) - \nu\right] \sigma_{zz}(t) + 2(1+\nu)\hat{l}_1\hat{m}_1 \sigma_{xy}(t) + 2(1+\nu)\hat{l}_1\hat{n}_1 \sigma_{xz}(t) + 2(1+\nu)\hat{m}_1\hat{n}_1 \sigma_{yz}(t) \quad (2)$$

- Criterion of the maximum shear stress in the fracture plane

$$\sigma_{eq}(t) = (\hat{l}_1^2 - \hat{l}_3^2)\sigma_{xx}(t) + (\hat{m}_1^2 - \hat{m}_3^2)\sigma_{yy}(t) + (\hat{n}_1^2 - \hat{n}_3^2)\sigma_{zz}(t) + 2(\hat{l}_1\hat{m}_1 - \hat{l}_3\hat{m}_3)\sigma_{xy}(t) + 2(\hat{l}_1\hat{n}_1 - \hat{l}_3\hat{n}_3)\sigma_{xz}(t) + 2(\hat{m}_1\hat{n}_1 - \hat{m}_3\hat{n}_3)\sigma_{yz}(t) \quad (3)$$

- Criterion of the principal maximum stress

$$\sigma_{eq}(t) = \sigma_1(t) \quad (4)$$

- Criterion of the extreme normal stresses

$$\sigma_{eq}(t) = \max\left\{\sigma_1(t) \; ; \; -\frac{\sigma_3(t)}{\varkappa}\right\} \tag{5}$$

- Criterion of the extreme normal strains

$$\sigma_{eq}(t) = \max\left\{(\sigma_1(t) - \nu[\sigma_2(t) + \sigma_3(t)]) \; ; \; -\frac{1}{\varkappa}(\sigma_3(t) - \nu[\sigma_1(t) + \sigma_2(t)]\right\} \tag{6}$$

- Criterion of the maximum shear stresses

$$\sigma_{eq}(t) = \sigma_1(t) - \sigma_3(t) \tag{7}$$

- Criterion of strain energy of distortion

$$\sigma_{eq}(t) = \{\sigma_1^2(t) + \sigma_2^2(t) + \sigma_3^2(t) - [\sigma_1(t)\sigma_2(t) + \sigma_2(t)\sigma_3(t) + \sigma_1(t)\sigma_3(t)]\}^{1/2} \tag{8}$$

- Criterion of total energy of distortion

$$\sigma_{eq}(t) = \{\sigma_1^2(t) + \sigma_2^2(t) + \sigma_3^2(t) - 2\nu[\sigma_1(t)\sigma_2(t) + \sigma_2(t)\sigma_3(t) + \sigma_1(t)\sigma_3(t)]\}^{1/2} \tag{9}$$

In criteria (1) ÷ (3) the fatigue fracture plane position is defined by the mean direction cosines of principal strains and stresses $\hat{l}_n$, $\hat{m}_n$, $\hat{n}_n$, (n = 1,2,3). It should be stated that the stress $\sigma_{eq}(t)$ according to equations (1) - (3) is linearly, and according to equations (4) - (9) nonlinearly dependent on stress $\sigma_{ij}(t)$, (i,j = x, y,z). In literature there are many examples of application of criteria (4) - (9) for description of multiaxial cyclic fatigue where static values of stresses are replaced by their amplitudes or ranges.

SIMULATION CALCULATIONS

The digital simulation included:

- generation of six stationary and ergodic random components of the stress state tensor σ_{ij}, (i, j= x,y,z) with normal probability distribution and low-band frequency spectrum,
- calculations of σ_{eq} according to equations (1) - (9)
- calculations of power spectral density function for stresses σ_{ij} and σ_{eq} with FFT.

The random sequences of stresses σ_{ij} and σ_{eq} contained N=81920 values. Constants ν = 0.25 and $\varkappa$ = 0.644 were assumed as typical for cast iron. The fatigue fracture plane position is determined, according to paper (4) , by the following direction cosines

$$\hat{l}_1 = -0.17727, \quad \hat{m}_1 = 0.92392, \quad \hat{n}_1 = 0.33903$$
$$\hat{l}_3 = 0.93362, \quad \hat{m}_3 = 0.048894, \quad \hat{n}_3 = 0.35492$$

After calculations of autospectral and cross - spectral density functions of the generated stress state components their maximum frequencies $f_{max\,ij}$, (i,j = = 1,...,6) were determined; they form the matrix

$$\begin{bmatrix} 163 & 158 & 158 & 158 & 158 & 168 \\ 158 & 164 & 162 & 159 & 142 & 165 \\ 158 & 162 & 163 & 162 & 166 & 160 \\ 158 & 159 & 162 & 155 & 159 & 161 \\ 158 & 142 & 166 & 159 & 155 & 161 \\ 168 & 165 & 160 & 161 & 161 & 164 \end{bmatrix} \quad \text{Hz}$$

In a similar way the maximum frequencies of the equivalent stress $f_{max\,\sigma eq}$ were determined. In Table 1 there are the values of $f_{max\,\sigma eq}$ according to nine analysed criteria and four random states.

ANALYSIS OF THE CALCULATION RESULTS

From Table 1 it results that for linear criteria (1) - (3) $f_{max\,\sigma eq}$ = 160 Hz and the following inequality is right

$$f_{max\,\sigma eq} \leqslant \max_{ij} \{ f_{max\,ij} \}, \quad (i,j = 1,...,6) \tag{10}$$

For the nonlinear criteria (4) - (9) the mean value of

TABLE 1 - Maximum frequencies of the equivalent stress $f_{max\,\sigma\,eq}$ in Hz according to 9 criteria for four types of random stress state

	The stress state caused by			
Criter. Number	σ_{xx}	σ_{xy}	σ_{xx}, σ_{yy}, σ_{xy}	σ_{xx}, σ_{yy}, σ_{zz}, σ_{xy}, σ_{xz}, σ_{yz}
1	160	168	155	155
2	165	158	160	160
3	160	158	160	165
4	187	221	190	192
5	225	227	225	215
6	235	232	215	230
7	235	224	212	215
8	225	224	229	225
9	232	225	230	210

$f_{max\,\sigma\,eq}$=220 Hz and it means **that** mean increase of the frequency band by 37.5 % appears in relation to the frequency band of the stress state tensor (160 Hz). As it results from Table 1, the increase can reach even 46.9% ($f_{max\,\sigma\,eq}$=235 Hz). Thus the following inequality, opposite to (10)

$$f_{max\,\sigma\,eq} > \max_{ij} \{ f_{max\,ij} \}, \quad (i,j = 1,\ldots,6) \tag{11}$$

has been verified and the quantitative effect of change in width of the frequency band while applyind different nonlinear criteria for random fatigue has been shown. This increase is not acceptable from the physical point of view and means that there is a new important limitation for generalization of criteria for multiaxial cyclic fatigue to the range of random loading.

CONCLUSIONS

1. While reducing multiaxial random stress state to the equivalent uniaxial one with the linear fatigue criteria,frequency bands of the stress state components transform to the frequency band of the equivalent stress without enlarging its width. The

frequency band of the equivalent stress according to the nonlinear fatigue criteria widens in relation to that one for the random stress state tensor.

2. Any simple attempts of generalization of the criteria for cyclic or static loads, in which there are nonlinear combinations of the stress state components, to multiaxial random loads, meet some very important limitations, connected with changes in frequency structure of the equivalent stress.

SYMBOLS USED

$\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$	=	principal stresses (MPa)
ν	=	Poisson's ratio
$\varkappa$	=	material constant
FFT	=	Fast Fourier Transform
t	=	time (s)

REFERENCES

(1) Macha E., "Generalization of strain criteria of multiaxial cyclic fatigue to random loadings", Fortschr.-Ber.VDI Reihe 18, Nr 52, VDI -Verlag, Dusseldorf, 1988.

(2) Macha E., in: Advances in Fracture Research , Proc. of the 7th Inter.Conf. on Fracture ICF 7 Eds. K.Salama et al., Pergamon Press, Oxford, 1989, Vol.2, pp.1239 - 1247.

(3) Macha E., in: Simulation of Systems, L.Dekker Ed. North-Holland Publishing Company, Amsterdam, 1976, pp. 1033-1041.

(4) Macha E., "Mathematical models of the life to fracture for materials subjected to random complex stress systems", Scientific Papers of the Wrocław Technical University, No.41, Wrocław 1979 /in Polish/.