STRAIN ENERGY RELEASE RATES AND RELATED MIXED-MODE CAUSTICS AT THE TIPS OF CURVILINEAR INTERFACE CRACKS

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By using Muskhelishvili's complex potentials method as well as by applying the method of conformal mapping a Hilbert-problem for a curvilinear interface crack located in the discontinuity area between two loaded half-planes with different elastic material properties has been solved. This solution allows the derivation of a closed form expression for the total energy release rate G at the crack tip as well as the calculation of strain energy release rates $G_{j,i}$ (j =1,2, i = I,II) related to each of the two sides of the material interface. Besides, the generating equations of the mixed-mode caustics at the tip of a curvilinear interface crack have been obtained by means of the complex potentials representing the solution of the associated Hilbert-problem. The influences of the curvature of the interface crack as well as of the different material properties on the shape of the caustics have been discussed for both cases of optical isotropy and anisotropy, respectively.

INTRODUCTION

Interface cracks represent elementary failure mechanisms arising especially in the high-fiber concentration range of fibrous composites subjected to mechanical and/or thermal loading. Besides, the appearance of branched crack systems consisting of a combination of curvilinear matrix and interface cracks has also been observed several times. Further, a layered medium can fail due to the delamination effect where interface cracks separate two layers from each other. Therefore, in the past a great deal of effort was focused on the problem of interface cracks. Owing to the importance and complexity of this mixed-mode fracture phenomenon a very large volume of literature has accumulated within the past three decades. A comprehensive survey of the state-of-the-art discussing also the oscillatory anomalies of the elastic stress and displacement fields near the tip of an interface crack as well as the establishment of an appropriate crack propagation criterion was given by Piva and Viola (1). Moreover, the quasistatic extension of straight and curved interface cracks, respectively, as well as the crack path prediction of extending thermal cracks in self-stressed multi-phase solids have been investigated by Herrmann (2) and Herrmann and Grebner (3,4). Furthermore, from a fracture mechanical point of view the relationship between the total strain energy release rate G at the tips of curvilinear interface cracks and the corresponding stress intensity factors, respectively, is of basic interest. Thus, several investigations have been performed in the past concerning the generalization of Irwin's formula (5) for the case of non-coplanar crack extension (6-11).

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Finally, stress intensity factors can be determined experimentally by means of the shadow optical method of caustics. By using the shadow optical method in transmission the appearance of so-called double caustics around the extending crack tip of a curvilinear thermal matrix crack could be observed because of the optical anisotropy of the matrix material (12,13) whereas the shadow optical method in reflection gives only single caustics. Comprehensive reviews about the shadow optical method of caustics have been given in the references (14,15).

FORMULATION AND SOLUTION OF A HILBERT-PROBLEM

In this paper, a semi-infinite curvilinear interface crack is considered located in the interface of two half-planes consisting of homogeneous, isotropic and linearly elastic materials with different elastic properties μ_j, κ_j (j = 1,2), cf. Fig. 1. By introducing the following boundary and continuity conditions, respectively, for the stresses and displacements

$$\{\sigma_n - i\sigma_{nt}\}_j = \begin{cases} \{p(t) - iq(t)\}_- & ; \quad t \in L' \\ \{p(t) - iq(t)\}_+ & ; \quad t \in L'' \end{cases} \quad ; \quad j=(1,2) \tag{1}$$

$$\{u_t + iu_n\}_1 = \{u_t + iu_n\}_2 \quad ; \quad t \in L'' \tag{2}$$

a mixed boundary value problem has been defined. Now by applying the mapping function $z = \omega(\zeta)$ a conformal mapping of the physical z-plane containing the semi-infinite curvilinear interface crack L' onto the mathematical ζ-plane containing the semi-infinite straight interface crack L' along the negative real ξ-axis can be performed.
Further, by using the complex potentials $\Phi_j(z)$, $\Psi_j(z)$; (j = 1,2) according to Muskhelishvili's theory (16) the boundary value problem (1), (2) has the following shape in the mapping plane, Herrmann and Meiners (11)

$$\tilde{\Phi}_1^+(\xi) + \tilde{\Omega}_1^-(\xi) = \tilde{\Phi}_2^-(\xi) + \tilde{\Omega}_2^+(\xi) \quad ; \quad \xi \in \tilde{L} \tag{3}$$

$$\frac{1}{2\mu_1}\{\kappa_1\tilde{\Phi}_1^+(\xi) - \tilde{\Omega}_1^-(\xi)\} = \frac{1}{2\mu_2}\{\kappa_2\tilde{\Phi}_2^-(\xi) - \tilde{\Omega}_2^+(\xi)\} \quad ; \quad \xi \in \tilde{L}'' \tag{4}$$

with the definitions

$$\tilde{\Omega}_j(\zeta) = \overline{\tilde{\Phi}}_j(\zeta) + \frac{\omega(\zeta)}{\omega'(\zeta)}\overline{\tilde{\Phi}}'_j(\zeta) + \frac{\overline{\omega}'(\zeta)}{\omega'(\zeta)}\overline{\tilde{\Psi}}_j(\zeta) \quad ; \quad (j = 1,2) \tag{5}$$

After some transformations the equations (3), (4) lead to the following Hilbert-problem

$$\tilde{\Phi}_1^+(\xi) + \tilde{\Phi}_2^-(\xi) = 0 \quad ; \quad \xi \in \tilde{L}' \tag{6}$$

$$\tilde{\Phi}_1^+(\xi) - g^{-1}\tilde{\Phi}_2^-(\xi) = 0 \quad ; \quad \xi \in \tilde{L}'' \tag{7}$$

with $$g = \frac{\kappa_1\mu_2 + \mu_1}{\kappa_2\mu_1 + \mu_2} \quad ; \quad \beta = \frac{\ln g}{2\pi} \tag{8}$$

The asymptotic near-tip solution of the Hilbert-problem (6), (7) is given by the complex potentials in the mapping plane

$$\tilde{\Phi}_j(\zeta) = K_j \zeta^{-\left(\frac{1}{2}+i\beta\right)} \quad ; \qquad (j = 1,2) \tag{9}$$

with the definitions of the complex stress intensity factors

$$K_j = K_{j,I} - iK_{j,II} \quad ; \quad (j = 1,2) \tag{10}$$

CALCULATION OF STRAIN ENERGY RELEASE RATES

By using Irwin's modified crack closure integral in the mapping plane the total strain energy release rate G for a straight interface crack can be determined according to, cf. reference (11)

$$G = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} \int_{\xi=0}^{\Delta a} \mathrm{Re}\{\, i\,[\sigma_n(\xi,0) - i\sigma_{nt}(\xi,0)]\,[\Delta u_t(\xi,\Delta a) - i\Delta u_n(\xi,\Delta a)]\}\, d\xi \tag{11}$$

By introducing the expressions for the complex stress vector and the complex crack opening displacement vector, respectively

$$\sigma_n - i\sigma_{nt} = (1+g)K_1 \xi^{-\left(\frac{1}{2}+i\beta\right)} \quad ; \quad \xi \in \tilde{L}'' \tag{12}$$

$$\Delta(u_t - iu_n) = \left\{\frac{\kappa_1+1}{\mu_1} + \frac{\kappa_2+1}{\mu_2}\right\} \frac{\overline{K}_1}{1+2i\beta} \left(e^{-i\pi}|\xi|\right)^{\frac{1}{2}+i\beta} \quad ; \quad \xi \in \tilde{L}' \tag{13}$$

into the equation (11) a closed form expression for the total energy release rate G has been obtained

$$G = \frac{\pi}{2} g \left\{\frac{\kappa_1+1}{\mu_1} + \frac{\kappa_2+1}{\mu_2}\right\} K_1 \overline{K}_1 \tag{14}$$

Furthermore, by splitting the integral (11) into two parts four separate energy release rates Gi,j (j=1,2; i=I,II) associated with the two sides of the material interface as well as with the corresponding fracture modes can be derived by some lengthy calculations, Noe (17).

MIXED-MODE CAUSTICS AT THE TIPS OF CURVILINEAR INTERFACE CRACKS

The general mapping equation for a shadow spot generated in an image plane by convergent light penetrating a transparent two-phase composite structure containing an interface crack reads as follows

$$\underline{w}_{1,2} = m\{\underline{r} - C_{1,2}\,[\nabla(\sigma_1+\sigma_2) \pm \lambda\nabla(\sigma_1-\sigma_2)]\} \tag{15}$$

with the definitions of the mapping scale $m = z_2/(z_o+z_2)$ and the corresponding constants for transmitted and reflected light, respectively

$$C_{1,2} = \frac{z_0 B c_{1,2}}{m} \quad ; \qquad \text{transmission} \tag{16a}$$

$$C_{1,2} = \frac{z_0 B v_{1,2}}{m E_{1,2}} \quad , \quad \lambda = 0 \quad ; \quad \text{reflection} \tag{16b}$$

where z_o, z_2 mean special distances of the experimental set-up of the shadow optical method, B is the thickness of the specimen, c and λ are the shadow optical constant and the so-called anisotropy constant, respectively. The initial curve of the caustic around the crack tip is then a singular function of the mapping function (15). The caustic itself is the image of this curve at which the latter can be obtained from the condition

$$J = \frac{\partial(x', y')}{\partial(r, \phi)} = 0 \tag{17}$$

By assuming an interface crack of a parabolic shape and by using the mapping function

$$z = \omega(\zeta) = \zeta + \frac{1}{2} i \rho \zeta^2 \tag{18}$$

where ρ is the curvature of the crack the corresponding solution of the Hilbert-problem in the physical z-plane reads

$$\Phi_j(z) = K_j \left(z - \frac{1}{2} i \rho z^2 \right)^{-\left(\frac{1}{2} + i\beta\right)} \quad ; \quad (j = 1,2) \tag{19}$$

By introducing the equations (19) into equation (15) and by considering the specialization m=1 as well as plane stress conditions the following equations for the caustics around the crack tip are obtained, cf. (17)

$$w_{1,2} = r_{1,2} \left\{ \exp(i\phi) + \frac{R_1 R_2}{R_3} \exp\left(i \left[\beta(\ln R_1 + \ln r) - \frac{3}{2}\gamma_1 + \gamma_2 - \eta + \omega + \frac{3}{2}\phi \right] \right) \right\} \tag{20}$$

with the definition of the initial curve of the caustic

$$r_{1,2} = R_1^{-1} \left\{ (K_I^2 + K_{II}^2)_{1,2} (1 + 4\beta^2) \right\}^{\frac{1}{5}} (2C_{1,2} R_3)^{\frac{2}{5}} \exp\left[\frac{2}{5} \beta(\phi - \gamma_1) \right] \tag{21}$$

The figures 2-5 show the results of the evaluation of equation (20). Thereby Fig. 2 gives the mixed-mode caustics around the tip of a curvilinear interface crack at both sides of a material interface of a two-phase composite structure (Araldite B/Steel) in case of optical isotropy of the material. Further, Fig. 3 demonstrates the influence of curvature and material inhomogeneity on the mixed-mode caustics around the tip of a curvilinear interface crack where only one side of the material interface as well as optical isotropy of the material have been considered. Moreover, the figures 4 and 5 show the caustics around the tips of a straight as well as a curvilinear interface crack in the most general case of optical anisotropy for one side of the material interface.
Besides, it should be mentioned that the method of caustics can be applied in transmission as well as in reflection at which in the latter case cracked opaque model materials can also be handled. The experimental set-up of the shadow optical method in reflexion has then be based on an appropriate coating of the surface of the opaque material, for instance by a thin silver layer. In this case the shadow spots and associated caustics around the tip of a curvilinear interface crack in a mechanically and/or thermally loaded two-phase composite structure can also be simulated by means of a finite element calculation based on the displacement vector field around the crack tip, Ferber and Herrmann (18).

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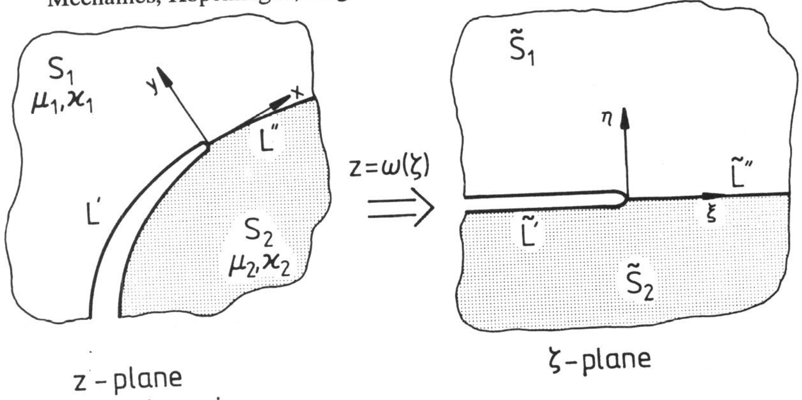
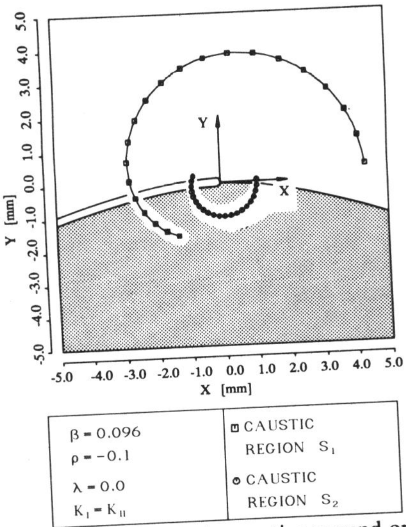


Fig. 1: Conformal mapping



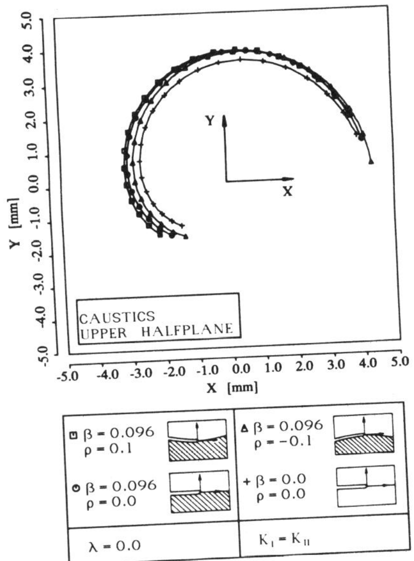
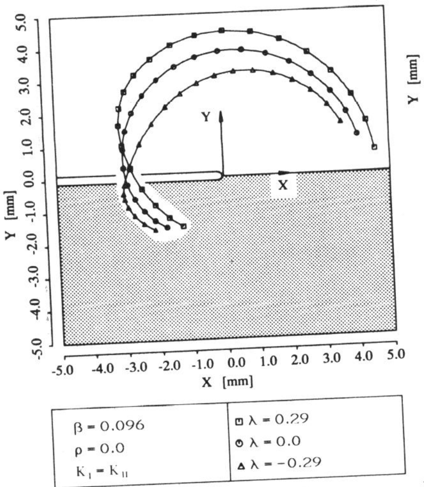


Fig. 2: Mixed-Mode caustics around on interface crack tip, optical isotropy
Fig. 3: Influence of curvature and material inhomogenity an mixed-mode caustics



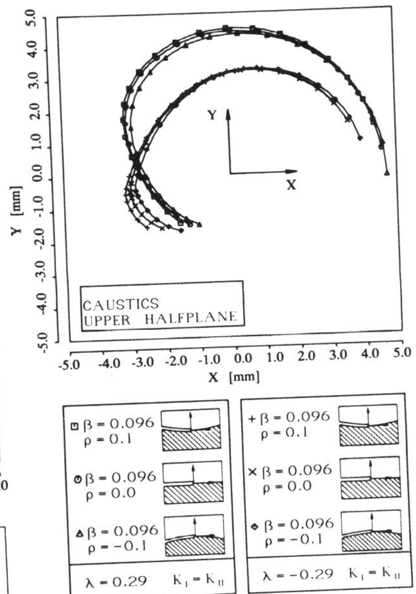


Fig. 4: Straight interface crack Fig. 5: Curvilinear interface crack
Fig. 4-5: Influence of curvature and optical anisotropy on mixed-mode caustics