BEHAVIOR OF R.C. BEAM CROSS SECTIONS UNDER UNIDIRECTIONAL CYCLIC BENDING MOMENTS

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If a reinforced concrete beam cross section is subjected to unidirectional cyclic bending moment, a dissipative phenomenon can occur. Some experimental tests have been carried out to verify the reliability of a Fracture Mechanics model which interprets the above-mentioned phenomenon. Different concrete curing conditions (in air or steam) are considered. The energy dissipated during the cyclic loading and unloading processes can be calculated from the hysteretic loops described in the moment-rotation diagram.

INTRODUCTION

The behavior of a reinforced concrete beam cross section subjected to unidirectional cyclic bending moment is examined. If the maximum value of the bending moment is higher than twice the bending moment of steel plastic flow and lower than that of concrete fracture, a dissipative phenomenon occurs [1-5], that is to say, the moment-rotation diagram shows hysteretic loops with energy dissipation [6, 7].

This phenomenon has been analyzed through experimental tests and the results are presented in the following. Some simply reinforced concrete beams have been subjected to unidirectional cyclic loading by means of "four point bending test". A through-thickness edge crack has been obtained in the stretched part of the centreline cross section by means of either a disk saw 2 mm thick or a sheet-steel 1 mm thick drowned in the beam casting.

Different concrete curing conditions have been considered (in air or steam). The moment-rotation diagram is shown for each case examined and the energy dissipation can be calculated from the hysteretic loops described during the cyclic loading and unloading processes. It can be remarked that the above-mentioned loops shift toward right-hand side in the moment-rotation diagram when the crack depth increases, as it is predicted by the theoretical model [3, 5].

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HYSTERETIC BEHAVIOR

If the reinforced concrete beam cross section in Fig. 1 is loaded with a maximum bending moment M greater than the bending moment M_p of steel plastic flow [1] and then is unloaded, the concrete element compresses the steel reinforcement. Then the bending moment of plastic shake-down M_{SD} is calculated as the lowest bending moment for which the above-mentioned steel compression is equal to F_p [2], where F_p is either the force of yielding $f_y A_s$ (f_y = steel yield strength; A_s = steel area) or the force of pulling-out, when the latter is lower than the former:

$$M_{SD} = 2\, M_p = 2\, F_p b\, [1/2 - h/b + r(\xi)] \, , \tag{1}$$

where $r(\xi)$ = function of the relative crack depth ξ, with $\xi = a/b$.

The moment-rotation diagram presents hysteretic loops with energy dissipation for $M_{SD} < M < M_F$ (Fig. 2), the unstable concrete fracture bending moment M_F being obtained by equalling the stress-intensity factor expression [8] to the concrete fracture toughness K_{IC}:

$$M_F = [(K_{IC}\, b^{3/2}\, t)/Y_M(\xi)] + F_p b\, \{1/2 - h/b + [Y_F(\xi)/Y_M(\xi)]\}, \tag{2}$$

where $Y_F(\xi)$ and $Y_M(\xi)$ are functions of the relative crack depth [2]. The slope of the hardening line in Fig. 2 is [1]:

$$1/\lambda_{MM} = (b^2 tE)/(2 \int_0^{\xi} Y_M^2(\xi)\, d\xi) \, , \tag{3}$$

where E is the Young modulus of concrete. Following this, the energy dissipated in each cycle is equal to the area 1-2-3-4 [2]:

$$\text{work/cycle} = M_{SD}\, \lambda_{MM}\, (M - M_{SD}) \, , \tag{4}$$

where M_{SD} and λ_{MM} are obtained from eqns (1) and (3).

If the relative crack depth increases, the hardening line obtained from eqn (3) becomes more inclined (Fig. 3) and the hysteretic loops shift toward right-hand side in the moment-rotation diagram [3, 5].

EXPERIMENTAL RESULTS

The test samples of the present experimental investigation are reinforced concrete beams with t = 20 cm, b = 30 cm, h = 3 cm, total length ℓ = 220 cm and span s = 180 cm (Figs 1 and 4). The longitudinal steel reinforcement consists of two bars (with diameter Ø = 8 mm) in the stretched part and two bars (Ø = 8 mm) for stirrup anchorage in the upper edge. The spacing of closed stirrups (Ø = 6 mm) is 10 cm near the external supports, while there is no transverse shear reinforcement in the central zone of the beams (Fig. 4). A through-thickness edge crack with depth a = 3 cm has been obtained in the stretched part of the centre-line cross section by means of either a disk saw 2 mm thick or a sheet-steel 1 mm thick drowned in the beam casting.

The reinforcement is steel FeB 38 K with yield strength equal to 373 N/mm^2. The concrete was subjected to either air curing or steam curing at atmospheric pressure. The concrete compressive strength is assumed to be equal to the mean of the results obtained on four cubic specimens with side 10 cm: 63.77 N/mm^2 for air curing and 68.38 N/mm^2 for steam curing. The longitudinal elastic modulus E of concrete is determined by means of three prismatic specimens with base 10x10 cm: the average value is about 34.3 kN/mm^2 for both curing conditions. The concrete fracture energy G_F is evaluated by means of an empirical equation proposed in [9] and, therefore, the critical stress-intensity factor according to the expression $K_{IC} = \sqrt{G_F E}$ results to be 60.16 $N/mm^{3/2}$ for air curing and 61.65 $N/mm^{3/2}$ for steam curing.

On the basis of the geometrical and mechanical properties of the beams under consideration, the bending moment of plastic shake-down for $\xi = 0.1$ is $M_{SD} = 2\ M_P = 13.04$ kNm, from eqn (1), and the bending moment of unstable concrete fracture for the same relative crack depth is $M_F = 24.42$ kNm for air curing and $M_F = 24.88$ kNm for steam curing, from eqn (2).

The test beams were subjected to unidirectional cyclic loading by means of "four point bending test" (Fig. 5) so that the centre-line cross section was loaded by a maximum bending moment M equal to 9.81 kNm for 5 cycles and equal to 16.68 kNm for other 20 cycles. Therefore, a phenomenon without energy dissipation was expected to occur in the first 5 cycles since the closed curve described in the moment-rotation diagram during each loading cycle degenerates into a segment when $M_P < M \leq M_{SD}$, while a dissipative phenomenon was theoretically predicted for the last 20 cycles because $M_{SD} < M < M_F$ (Fig. 2).

The bending moment $\mathcal{M}$ against rotation φ diagram is plotted for the centre-line cross section of each sample tested (Fig. 6). The samples were marked in the following way:

(N01) air curing; sheet-steel drowned in the beam casting and then removed before the testing;
(N02) air curing; crack by means of the disk saw;
(N03) air curing; sheet-steel drowned in the beam casting;
(V01) steam curing; crack by means of the disk saw;
(V02) steam curing; sheet-steel drowned in the beam casting.

On the basis of the recordings by the transducers mounted on the beam sides (Fig. 5), the final relative crack depth after the experimental testing appears to be about 0.7 for N01 and N02, 0.5 for N03 and V01, 0.4 for V02. Therefore, according to eqn (3), the ratio between the hardening slope for the generic beam tested and the hardening slope for N01 is about 1 for N02, 3.8 for N03 and V01, 7.4 for V02. The experimental diagrams in Fig. 6 confirm such predictions.

CONCLUSIONS

(1) The rigid-linear hardening behavior predicted by the model [1-5] appears to be experimentally confirmed, in relation to both the bending moment of steel plastic flow and the hardening line slope.
(2) A phenomenon without energy dissipation is theoretically predicted and experimentally confirmed for $M_P < M \leq M_{SD}$, since the closed curve described in the moment-rotation diagram during each loading cycle degenerates into a segment.
(3) On the contrary, a dissipative phenomenon occurs for $M_{SD} < M < M_F$ and, when the relative crack depth increases, the hysteretic loops shift toward right-hand side.
(4) The crack growth for R.C. beam cross sections appears to depend on both the concrete curing condition and the method utilized to obtain the initial notch.

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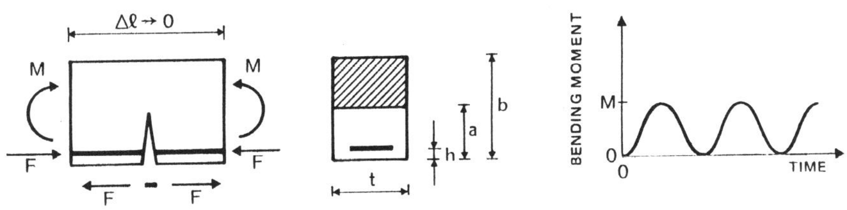


Fig. 1. Cracked reinforced concrete beam cross section, subjected to unidirectional cyclic bending moment.

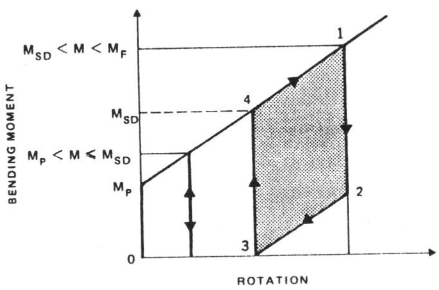


Fig. 2. Non-dissipative phenomenon for $M_P < M \leq M_{SD}$ and dissipative phenomenon for $M_{SD} < M < M_F$.

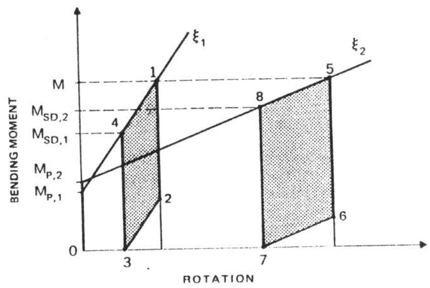


Fig. 3. Crack growth from ξ_1 to ξ_2.

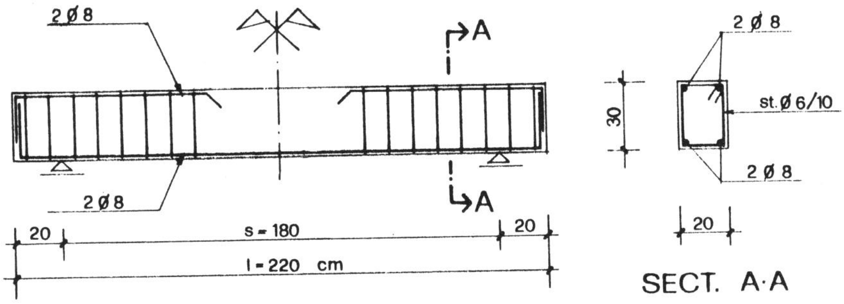


Fig. 4. Test reinforced concrete beam geometry.

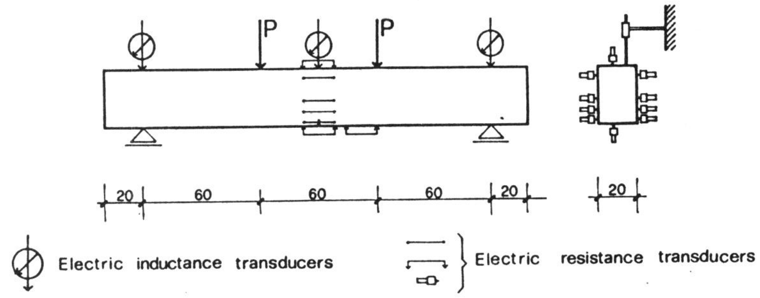
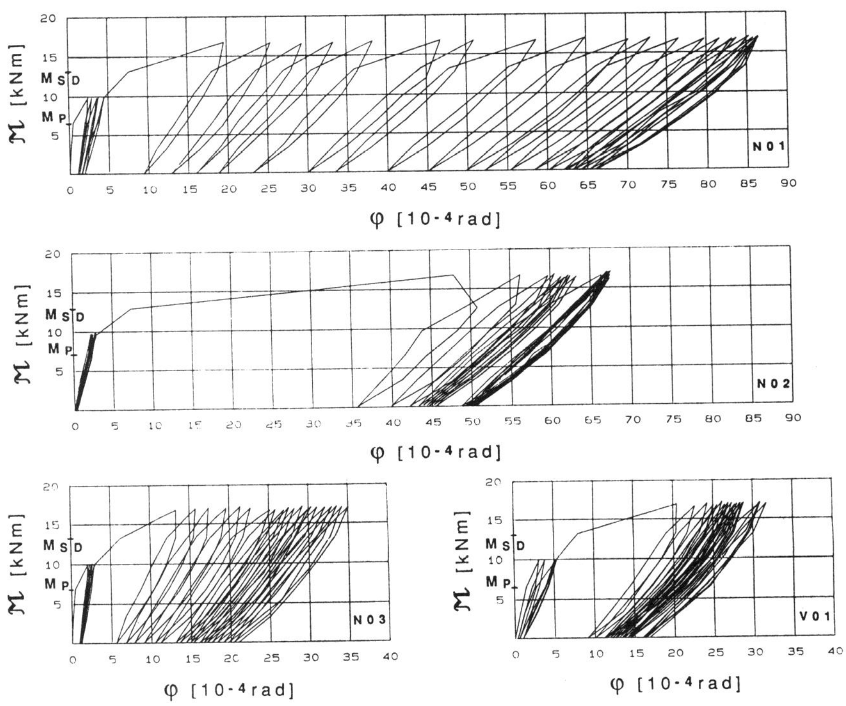


Fig. 5. Unidirectional cyclic loading (four point bending test).



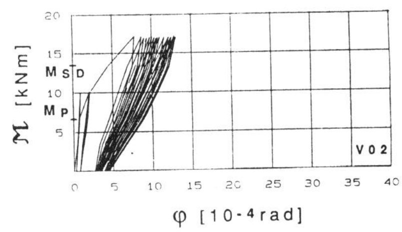


Fig. 6. Bending moment against rotation for 5 beams. The theoretical values M_P and M_{SD} are also reported.