CRACK GROWTH MODELLING IN PLAIN AND FIBRE–REINFORCED CEMENTITIOUS MATERIALS

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The K–superposition method is used to model stable crack growth in cementitious materials. With two simplified assumptions of a power law strain–softening characteristics and a linearized crack profile, stable fracture in either plain or fibre–reinforced cementitious materials can be readily modelled with K_R curves. The relationship between K_R–curve and the damaged zone where bridging stresses are transferred in accordance with the strain–softening law is emphasized. The advantages and drawbacks of the K–superposition method are also discussed.

INTRODUCTION

It is well recognized that stable fracture in cementitious materials is the main problem in fracture mechanics analyses. Various stable fracture models have been proposed, such as the fictitious crack model (Hillerborg et al (1) and Hillerborg (2)), the size effect law (Bazant and Oh (3) and Bazant and Cedolin (4)) and the two–parameter model (Jenq and Shah (5) and Jenq and Shah (6)), etc. Parallel to these fracture models, a simple but rigorous K superposition method (Foote et al (7), Foote et al. (8), Cotterell and Mai (9) and Cotterell and Mai (10)) has been developed for short fibre–reinforced cementitious materials. With the strain–softening law ($\sigma - \delta$) describing the stress–displacement relationship in a damaged zone, K_R curves can be readily determined for any given specimen geometries and loading configurations. However, it is found that different K_R curves exist for different specimens. Since the specific fracture energy G_f is closely related to the strain–softening characteristics, it may not be uniformly distributed across a broken specimen. Thus, any change in G_f inevitably influences the crack growth resistance K_R.

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In the present paper, K_R curves of both plain and short fibre–reinforced cementitious materials are discussed with special considerations given to the underlying mechanisms responsible for the crack growth resistance. While the critical crack tip opening displacement δ_c of a physical crack is the same as the fibre pull–out length for reinforced materials, it is not evident that δ_c has to be a constant for plain cementitious materials. The K_R curve and its relation with the parameters of the strain–softening law $\sigma - \delta$, particularly G_f, are also addressed.

MODELLING OF K_R CURVES

As described in the fictitious crack model, fracture propagation in a cementitious material is characterized by a progressive extension of a damaged zone or fracture process zone within which stresses are transferred in accordance with the strain–softening law of the material. The concept of a fictitious crack can be adopted to model the crack growth resistance curve $K_R(\Delta a)$. The extension of the fictitious crack is defined here as the crack growth Δa.

Plain Cementitious Materials

The maximum stress at the tip of a fictitious crack is limited by the tensile strength σ_{TS} of the material. Thus, no stress singularity can exist at the crack tip and the net stress intensity factor K_e must be zero. If K_a is the stress intensity factor due to the applied stress and K_r is that due to bridging stresses across the fracture process zone, then

$$K_e = K_a + K_r = 0. \qquad (1)$$

The crack growth resistance $K_R(\Delta a)$ is defined by:

$$K_R(\Delta a) = K_a = -K_r \qquad (2)$$

If both the strain–softening law $(\sigma - \delta)$ and the crack opening δ within the bridging zone are given, K_r and then $K_R(\Delta a)$ can be determined.

Fibre–Reinforced Cementitious Materials

For fibre–reinforced materials two K_R–curve models can be considered. In the first model the influence of the fibre bridging zone is predominant. In this case the fracture process zone in the matrix does not have to be included in the model and it can be sufficiently characterized by the intrinsic fracture toughness K_{IC}. Hence, similar to equation (1), at equilibrium fracture we have

$$K_e = K_a + K_r = K_{IC}. \qquad (3)$$

And the crack growth resistance is defined as:

$$K_R(\Delta a) = K_a = K_{IC} - K_r(\Delta a). \tag{4}$$

The relationship $\sigma - \delta$ between the bridging stress and the crack opening may be established in terms of the fibre pull–out process.

In the second model K_{IC} is insufficient to describe the matrix fracture process zone which therefore has to be included in the analysis. We assume here K_P is the stress intensity factor due to the matrix process zone. Thus, the net stress intensity factor taken at the tip of the fictitious crack including both the fibre bridging zone and the matrix fracture process zone must be zero, i.e.

$$K_e = K_a + K_r(\Delta a) + K_P(\Delta a) = 0. \tag{5}$$

$K_R(\Delta a)$ is then given by:

$$K_R(\Delta a) = K_a = -K_r(\Delta a) - K_P(\Delta a). \tag{6}$$

For practical purposes, however, the inclusion of K_P in the K_R curve analysis is not required as the experimental data do not allow a differentiation of the two models (10). It is convenient to present theoretical results in a nondimensional form where $\bar{K}_R$ and $\bar{a}$, the nondimensional crack growth resistance and crack length are defined by

$$\bar{K}_R = K_R/K_\infty \tag{7}$$

$$\bar{a} = a/(K_\infty/\sigma_{TS})^2 \tag{8}$$

where K_∞ is the plateau value of the crack growth resistance curve for a crack in an infinite plate.

The evaluation of K_R curves by the K–superposition method relies on the strain–softening law $\sigma - \delta$ and the crack opening profile within the damaged zone (either the fracture process zone or the fibre bridging zone depending on whether plain or fibre–reinforced cementitious materials are considered). It is difficult to measure the crack opening and the $\sigma - \delta$ law experimentally. However, two assumptions can be made to simplify the K_R analysis. Firstly, a linear crack face profile can be assumed for many cementitious and ceramic materials ((7), Rodel et al (11), Steinbrech et al (12)) since the crack opening displacement is normally very small during crack growth. Secondly, a simple ($\sigma - \delta$) power law is assumed, i.e.

$$\sigma = \sigma_{TS}\left[1 - \frac{\delta}{\delta_c}\right]^n \tag{9}$$

where δ_c is the critical crack tip opening displacement of the physical crack in the case of plain cementitious materials, or the fibre pull–out length for reinforced materials. The specific fracture energy G_f is given by

$$G_f = \int_0^{\delta_c} \sigma d\delta = \frac{\sigma_{TS}\,\delta_c}{n+1} \tag{10}$$

The selection of the exponent n is somewhat arbitrary. However, it is shown that insofar as

$$\frac{\sigma_{TS}}{n+1} = \frac{G_f}{\delta_c} = \text{constant} \tag{11}$$

for a given material, experimental K_R curves can be equally well fitted by different sets of (σ,n) or (G_f,δ_c). For fibre–reinforced cementitious materials, a simple linear stress–displacement relationship $(n = 1)$ is sufficient. Therefore, with the two abovementioned assumptions the K superposition method provides a simple and powerful tool in stable fracture analyses of both cementitious and ceramic materials.

Nondimensionalized K_R curves of a short fibre–reinforced material calculated from equation (4) are shown in Figs. 1 to 3. A steady plateau value of K_R can be achieved with the double–cantilever–beam (DCB) specimens, which is independent of specimen depth. However, results of notched bend (NB) specimens show that for small beam depths K_R increases rapidly as the crack approaches the back face. The maximum K_R can be higher than the plateau value for a large beam depth. The apparent high values of K_R of small NB specimens do not necessarily suggest more energy is consumed at the back face since a unique strain–softening law has been assumed in the calculation. In other words, the specific fracture energy G_f is

TABLE 1 – Properties of asbestos/cellulose–reinforced mortar composite

a) Composite properties		
Young's modulus (E)		6 GPa
Crack resistance at initiation (K_{IC})		1.9 MPa$\sqrt{m}$
Plateau value of crack resistance (K_∞)		5 MPa$\sqrt{m}$
b) Fibre properties		
	Asbestos	Cellulose
Aspect ratio	80	135
Fibre length	2 mm	3.5 mm
Volume fraction	0.08	0.07
Bond strength (τ)	≈ 2.0 MPa	0.88

assumed uniformly distributed over the ligament. Experimental crack growth resistance curves obtained for an asbestos/cellulose mortar sheet (Mai et al (13)) for NB specimens of different depth are shown in Fig. 4. The composite and fibre properties of the mortar sheet are given in Table 1. The bond strengths τ shown in the table were not measured directly, but selected to give reasonable agreement with the experimental fracture strength. The fibres in the mortar sheet, which was manufactured by the Hatschek process, were not randomly aligned and the efficiency factor η was estimated to be 0.31. The values of K_∞ and K_{IC} given in Table 1 were chosen empirically so that the resistance curves for the largest specimens gave the best fit with the experimental data. The theoretical resistance curves for the smaller specimens agree well with the experimental data giving confidence in the approximate method outlined above.

DISCUSSION

In notch bend and compact tension specimen, the fictitious crack model predicts that K_R increases without limit as the remaining ligament decreases to zero if the stress–displacement curve is assumed to be unique. This aspect of the fictitious crack model is an artifact of assumptions that, while true for large ligaments, are invalidated for small ligaments. If the stress–displacement curve were unique then G_F would be independent of specimen size. However, it has been shown that G_F depends on the ligament size (Hu and Wittmann (14) and Hu (15)). The reason for G_F being smaller for small ligaments is twofold. Firstly when the remaining ligament is small compared with the fracture process zone width, there will be a narrowing of the process zone. If after Bazant (3,4), it is assumed that the strain to completely crack the process zone is a constant then a narrowing of the process zone will cause a decrease in the displacement across the zone. This decrease in δ_c implies a decrease in G_F. Another contribution to a decrease in G_F comes from prior damage of the cementitious material as it undergoes compression ahead of the crack tip. When the ligament is large the compression stresses are relatively small, but with small ligaments the compressive stresses can be high. The effect of prior damage is likely to be a decrease in the maximum strength of the cementitious material that will again reduce G_F.

The above arguments apply mainly for unreinforced cementitious materials. With fibre reinforced materials the fracture process zone of the matrix is comparatively small and unimportant. As far as fibre bridging is concerned, the fictitious line crack is an accurate model even if the ligament is small. Also since most of the work of fracture comes from fibre pull–out, any prior damage due to compression of the matrix will be unimportant.

Hence, the above approximate method should be used with caution in unreinforced cementitious materials if the ligament is small. However, for fibre reinforced cementitious materials the above method is accurate for even small ligaments.

CONCLUSIONS

K_R curves for cementitious materials can be calculated if G_F and either σ_{TS} or δ_c is known by simple superposition using the fictitious crack model, assuming that the fictitious crack profile is linear. Accurate description of the stress–displacement curve, apart from maintaining the specific work of fracture equal to G_F is not necessary.

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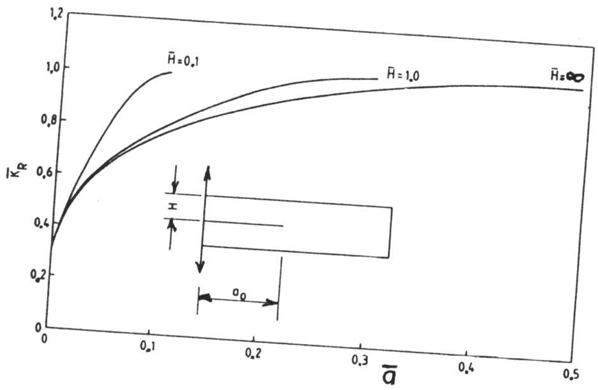

Figure 1 Nondimensional crack growth resistance curves for DCB specimens for $a_o/H = 3.0$

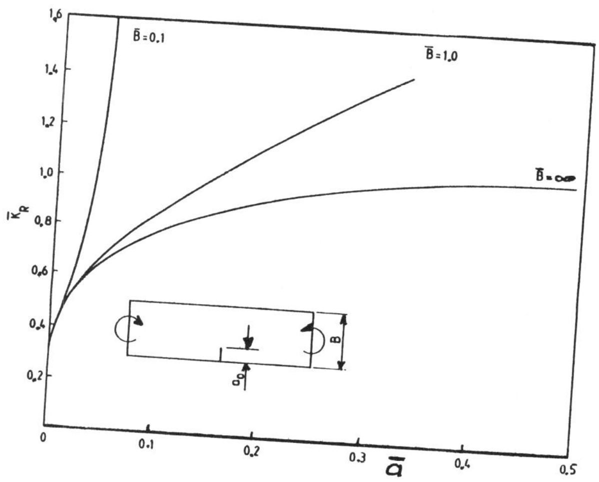

Figure 2 Nondimensional crack growth resistance curves for NB specimens for $a_o/B = 0.3$

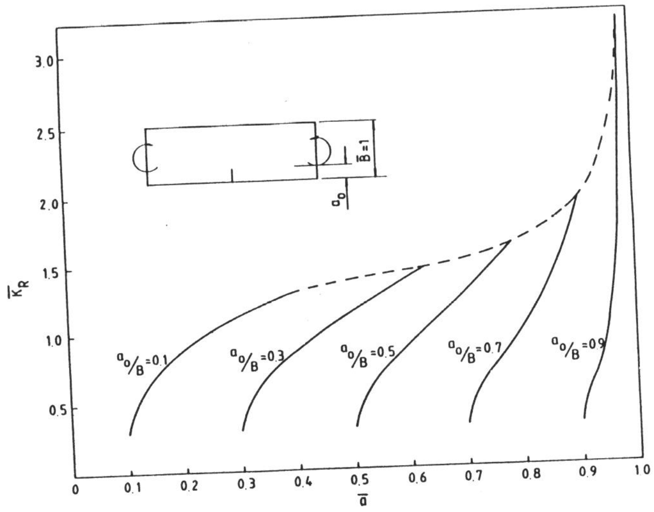

Figure 3 Nondimensional crack growth resistance curves for NB specimens for $\bar{B}$ = 1

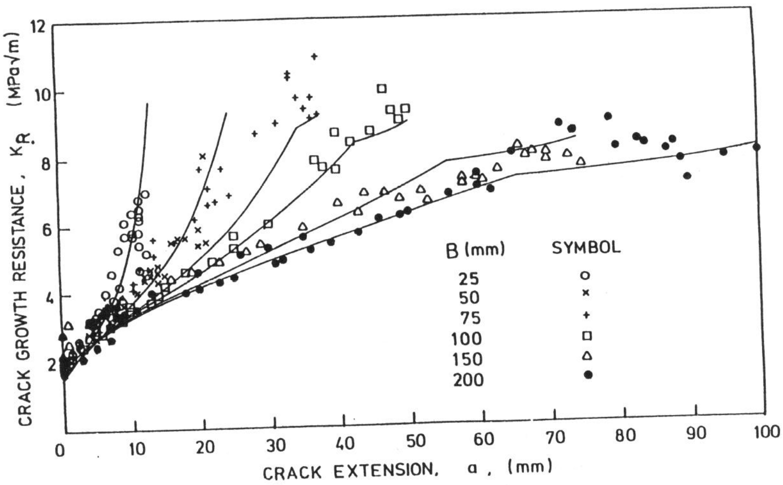

Figure 4 Experimental crack growth resistance curves for asbestos/cellulose mortar NB specimens a_o/B = 0.3