THE BRAZILIAN DISK - A SPECIMEN FOR THE ASSESSMENT OF MULTIAXIAL FRACTURE CRITERIA FOR CERAMICS

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The failure probability of ceramic components depends on the Weibull parameters which can be determined by some standard test method such as four-point bending, and on a fracture criterion which allows the evaluation of the effect of multiaxial loading on the statistical distribution of the fracture stress. This fracture criterion has to be determined using a suitable multiaxial testing procedure. A characteristic stress is introduced to describe the local loading of the Brazilian disk and an example for the dependence of this quantitiy on the fracture criterion is given.

INTRODUCTION

The failure probability of ceramic components is determined by the probability of finding a flaw whose critical stress is exceeded by the applied stress. It is common practise to describe the natural flaws in a component in terms of fracture mechanics cracks. Hence, the critical stress corresponds to a critical crack size. In a multiaxial stress state the natural cracks are subjected to multimodal loading. An equivalent stress has to be defined in order to characterize the effect of different loading modes on the applied stress acting on a specific flaw. Various multiaxial fracture criteria have been proposed to calculate the equivalent stress (e.g. (1)- (5)).

It is well known that the failure probability of a ceramic component can be determined using Weibull statistics which is the mathematical equivalent of the weakest link model. Multiaxial stress states can be incorporated in the Weibull theory by calculating the probability that the equivalent stress of the most dangerous flaw exceeds the critical stress (6)-(9). Different multiaxial fracture criteria lead to different values for the failure probability at

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a given load level. Hence, a multiaxial test specimen has to be defined by which a fracture criterion suitable for a given ceramics material can be selected.

In this paper, the Brazilian disk test is analyzed where compressive forces are applied at opposing points on the circumference of a disk. The stress state of this specimen is fairly complicated with a tensile and a compressive component in the principal stress tensor. Hence the specimen is a promising candidate for multiaxial testing, as the discriminative power of such a test reaches its maximum for stress states containing principal stresses of opposite sign.

WEIBULL THEORY FOR MULTIAXIAL STRESS STATES

Natural flaws in ceramic materials can either be described as planar surface cracks or as planar embedded cracks. A multiaxial applied stress characterized by the principal stresses σ_1 , σ_2, σ_3 can be resolved in a stress σ_n normal to the crack plane and a shear stress τ parallel to the crack plane, where σ_n determines mode I loading of the crack and τ determines mode II and mode III loading.

Any failure criterion for multimodal fracture can be written in the form:

$$g(K_I,\ K_{II},\ K_{III}) \geq g_c \ , \tag{1}$$

where g_c is a critical value characterizing the materials resistance against unstable crack propagation; obviously, g_c is a function of the fracture toughness K_{Ic} and possibly of K_{IIc} and of K_{IIIc}. The equivalent mode I stress intensity factor, K_{Ieq}, is defined by the relation:

$$g(K_{Ieq},\ 0,\ 0) = g(K_I,\ K_{II},\ K_{III}) \tag{2}$$

and the corresponding equivalent stress σ_{Ieq} follows from:

$$K_{Ieq} = Y_I \cdot \sqrt{\pi a} \cdot \sigma_{Ieq} \ , \tag{3}$$

where Y_I is the correction factor for mode I loading.

The following failure criteria have been considered (10): coplanar energy release rate (1), maximum hoop stress factor (2), minimum strain-energy density (3), maximum non-coplanar energy release rate (4), and an empirical criterion of Richard (5). For negative normal stress σ_n, effective stresses have to be used to calculate K_I, K_{II}, and K_{III} (11):

$$\sigma_{n,eff} = \begin{cases} \sigma_n & \text{for } \sigma_n \geq 0 \\ 0 & \text{for } \sigma_n < 0 \ , \end{cases}$$
$$\tau_{eff} = \begin{cases} \tau & \text{for } \sigma_n \geq 0 \\ \tau - \mu|\sigma_n| & \text{for } \sigma_n < 0 \text{ and } \tau \geq \mu|\sigma_n| \\ 0 & \text{for } \sigma_n < 0 \text{ and } \tau < \mu|\sigma_n| \ . \end{cases} \tag{4}$$

where μ denotes a friction coefficient.

The failure probability of a ceramic component in an arbitrary stress state characterized by a stress level σ^* (e.g. the outer fiber stress in case of four point bending) is given by the value of the strength distribution, F_{σ_c}, at σ^*:

$$F_{\sigma_c}(\sigma_c = \sigma^*) = 1 - \exp\left(- \frac{\sigma^{*\,m}}{\sigma_0^{\,m}} \cdot \frac{1}{A_0} \int_A \frac{1}{\pi} \int_0^{\pi} \frac{\sigma_{Ieq}^{\,m}}{\sigma^{*\,m}} \, d\theta \, dA \right) \quad (5)$$

where θ denotes the angle of inclination of the crack plane in the coordinate system of the principal stresses, A_0 is the unit area and A is the surface area of the specimen. In Eq.(5) it is assumed that failure is caused by surface flaws. A similar formula is valid for volume flaws. Eq.(5) was derived in (6) - (9). A slightly different derivation was given in (10).

The equivalent stress in Eq.(5) is calculated according to a suitable fracture criterion. Eq.(5) can be cast into the form of a Weibull distribution by:

$$F_{\sigma_c}(\sigma_c = \sigma^*) = 1 - \exp\left(- \frac{\sigma^{*\,m}}{b^m} \right) \quad (6)$$

with the parameters b and m, where the notation

$$b = \sigma_0 \cdot \left(\frac{1}{A_0} \int_A \frac{1}{\pi} \int_0^{\pi} \frac{\sigma_{Ieq}^{\,m}}{\sigma^{*\,m}} \, d\theta \, dA \right)^{-1/m} \quad (7)$$

was used.

Hence, the distributions of the fracture stress σ^* obtained, e.g. for the four point bending test and a multiaxial test are characterized by Weibull distributions with identical values for the parameter m and values for the parameter b following from Eq.(7). The parameter σ_0 in Eq.(7) corresponds to the Weibull parameter b determined with specimens in a uniform equibiaxial stress state with a surface area equal to the unit area A_0 and is not determined in the experiments, but drops out if the fracture stress distribution for a multiaxial test is predicted from a four point bending test and vice versa.

BRAZILIAN DISK TEST FOR CERAMIC MATERIALS

An experimental determination of the parameters of the weakest link model Eq.(5) implies that a suitable fracture criterion is selected in addition to the Weibull parameters σ_0 and m. The applicability of a given fracture criterion depends on the characteristics of the natural flaws and therefore on the material.

A multiaxial test which has been used for quite some time in rock mechanics is the Brazilian disk test in which compressive forces are applied on opposing sides on the circumference of a circular disk (12). The stress tensor of the specimen is given by:

$$\sigma_{xx} = -\frac{2P}{\pi t} \cdot \left(\frac{x^2(R-y)}{(x^2+(R-y)^2)^2} + \frac{x^2(R+y)}{(x^2+(R+y)^2)^2} \right) + \frac{P}{\pi R t},$$

$$\sigma_{yy} = -\frac{2P}{\pi t} \cdot \left(\frac{(R-y)^3}{(x^2+(R-y)^2)^2} + \frac{(R+y)^3}{(x^2+(R+y)^2)^2} \right) + \frac{P}{\pi R t}, \quad (8)$$

$$\tau_{xy} = \frac{2P}{\pi t} \cdot \left(\frac{x(R-y)^2}{(x^2+(R-y)^2)^2} - \frac{x(R+y)^2}{(x^2+(R+y)^2)^2} \right),$$

where R is the radius of the disk, t is the thickness, and P is the applied load in y-direction. In the centre of the specimen, the ratio of the principle stress σ_1 to σ_2 is equal to -3, whereas high compressive stresses occur in the vicinity of the loading points. In realistic experimental conditions, the point forces are replaced by distributed surface loads, and the singularities at the loading points in Eq.(8) disappear.

From Eq.(5), a characteristic dimensionless stress σ_{ch} can be defined which describes the contribution of each point in the specimen to the failure probability:

$$\sigma_{ch} = \left(\frac{1}{\pi} \int_0^{\pi} \frac{\sigma_{Ieq}^{\ m}}{\sigma^{*\,m}} \, d\theta \right)^{1/m} . \quad (9)$$

Figures 1 and 2 show the distribution of σ_{ch} on the Brazilian disk for two different values of the friction coefficient μ. The stress distribution Eq.(8) was modified at the loading point according to (12) in order to avoid the singularities occuring for $y = R$ in Eq.(8). In Figure 1, μ is equal to one, and the areas of high compressive stresses in the vicinity of the loading point yield very high values of the characteristic stress σ_{ch}. This means that fracture of the specimen is likely to be initiated from flaws which are located close to the loading point. Figure 2 shows that in the limit of very high values of μ, the maximum of σ_{ch} is shifted to a point on the specimen axis well below the loading point and that the gradients of σ_{ch} are much less pronounced, because σ_{Ieq} vanishes for very high compressive stresses.

DISCUSSION

The distribution of the characteristic stress shows that the most critical point of the Brazilian disk test is the vicinity of the loading point for materials which do not have a very high resistance against compressive loading. As this area is also very susceptible to errors in the experimental loading conditions, great care has to be taken in the design of a suitable testing fixture. Other experimental problems are buckling of thin disks and superimposed bending stresses caused by deviations of the specimen axis from the vertical loading axis. These problems are currently under investigation.

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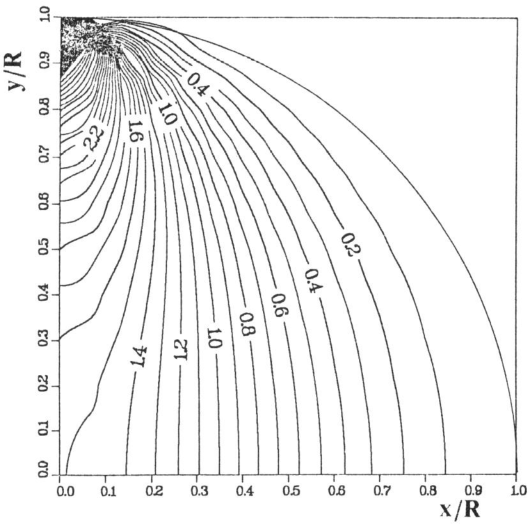

Figure 1: Distribution of the characteristic stress σ_{ch} in the Brazilian disk; fracture criterion: energy release with friction coefficient $\mu = 1$ for compressive normal stresses.

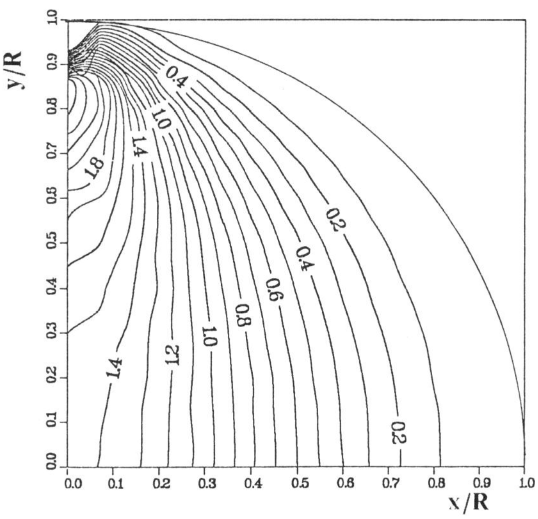

Figure 2: Distribution of the characteristic stress σ_{ch} in the Brazilian disk; fracture criterion: energy release with $\sigma_{leq} = 0$ for compressive normal stresses.