T INTEGRAL

S. Yin[x]

As it is well-known, the energy consumed in crack expanding in the elasto-plastic medium for a unit length is $\frac{\partial U}{\partial a}$; for the sake of convenience we will call this value T.

$$T = -\frac{\partial U}{\partial a} \tag{1}$$

If an object is regarded as a unit thickness, U can be defined as

$$U = \int_A W dA - \int_{ot} T_1 U_1 \ ds \tag{2}$$

where W is the strain energy density, and Ct is the part of area A focused by force T_j.

It is generally known that the crack expanding force in linear elastic materials is

$$G_1 = -\frac{\partial U}{\partial a} = \frac{k+1}{8\mu} K_I^2 \tag{3}$$

where μ is the shear modulus, ν the Poisson rate, for plain stress $\kappa = \frac{3-\nu}{I+\nu}$,; plain strain $\kappa = 3 - 4\nu$, and K_I is the stress strength factor.

From (1) and (a) we can say that T integration is in fact a development of G. The constitutive equation for nonlinear elasticity is the only one to be choosen to determine elastic-plastic deformation if the plastic deformation theory is available.

To nonlinear elastic materials, based on the thinking and consideration of virtual work [1], the above T integration could be recognized as the apparent crack expanding force, the equation for virtual work becoming the generalized T integration. Thus, (1) could be taken as an energy balance for the crack body, and T integration is used to indicate the fracture energy for the length unit of crack expanding.

x North China Institute of Hydropower, Handan City, Hebei Province, China

So as to make easier the physical concept of w in (2), ($w^p + w^e$) can be used to replace w, and w^e and w^p are respectively the densities of the elastic strain energy [2], and of the plastic strain energy, thus the formula becomes:

$$T = \frac{\partial}{\partial a}\left[\int_0 T_1 U_1 ds - \int_A W^e dA - \int_A W^p dA\right] \quad (4)$$

and we have to nonlinear elastic materials:

$$W^e = \int_0^{\varepsilon_{ij}^e} \sigma_{ij}\, d\varepsilon_{ij} \quad (5)$$

The relation of complete strain and separate strain is:

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p \quad (6)$$

The above relations, through interferencing and calculating on balance equation, Green formula, and the principle of virtual work, finally we have:

$$T = \int_r W^e dx_2' + \int_A \sigma_{ij} \frac{\partial \varepsilon_{ij}^p}{\partial x_1} dA - \int_r T_1 U_{1,1} ds \quad (7)$$

The nature of T integration has nothing to do with the choice of the way of integration, which is clearly shown in the following demonstration.

We may take an enclosed loop B contained in a plastic zone as an example, in consideration of the principle of virtual work, thus we have:

$$\int_B \vec{T} \frac{\partial \vec{u}}{\partial x_1} ds = \int_A \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_1} dA \quad (8)$$

and along the enclosed loop, we have

$$L_B = \oint_B \left[W_e dx_2 - T_1 U_{1,1} ds\right] = \int_A \left[\frac{\partial W^e}{\partial x_1} - \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_1}\right] dA \quad (9)$$

The above formula - to which the differential relationship between Hook's law and complex function is applied - will give:

$$L_B = \int_A \left[\sigma_{ij} \frac{\partial \varepsilon_{ij}^e}{\partial x_1} - \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_1}\right] dA = -\int_A \sigma_{ij} \frac{\partial \varepsilon_{ij}^p}{\partial x_1} dA = -\int_{AP} \sigma_{ij} \frac{\partial \varepsilon_{ij}^p}{\partial x_1} dA \quad (10)$$

If such a result is applied, the enclosed loop (see Figure 2) is $\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4$, and let A' be the plastic zone contained in Γ_1, A_3 that one is Γ_3 and A'^{-3} the plastic zone between Γ_1 and Γ_3, and $A_p' = A_p^3 + A_p'^{-3}$, therefore:

$$T_\Gamma = \oint [W^e dx_2 - T_1 U_{1,1} ds] = -\int_{A_p}^{1-3} \sigma_{ij} \frac{\partial \varepsilon^p_{ij}}{\partial X_1} dA \quad (11)$$

Attention should be paid to the fact that on the enclosed loop Γ_2 and Γ_4 are the free surfaces of the crack, without any force on them, i.e. $T_1 = 0$, making zero the second term on the left half of T integral formula, and it is in direction of X_1 axle, $dx_2 = 0$, making zero the first term; therefore the integration of Γ_2 and Γ_4 are both zero, and the direction of Γ_1 and Γ_3 are opposite, thus

$$T_\Gamma = \int_{\Gamma_1} [W^e dx_2 - T_1 U_{1,1} ds] - \int_{\Gamma_3} [W^e dx_2 - T_1 U_{1,1} ds] = -\int_{A^{1-3}_p} \sigma_{ij} \frac{\partial \varepsilon^p_{ij}}{\partial X_1} dA$$

and $A_p^{1-3} = A_p^1 - A_p^3$, therefore we can have (12)

$$\int_{\Gamma_1} [W^e dx_2 - T_1 U_{1,1} ds]_p - \int_{\Gamma_3} [W^e dx_2 - T_1 U_{1,1} ds]$$

$$= -\{ \int_{A^1_p} \sigma_{ij} \frac{\partial \varepsilon^p_{ij}}{\partial x_1} dA - \int_{A^3_p} \sigma_{ij} \frac{\partial \varepsilon^p_{ij}}{\partial x_1} dA \}$$

$$= -\int_{A^1_p} \sigma_{ij} \frac{\partial \varepsilon^p_{ij}}{\partial x_1} dA + \int_{A^3_p} \sigma_{ij} \frac{\partial \varepsilon^p_{ij}}{\partial x_1} dA \quad (13)$$

from which we get

$$\int_{\Gamma_2} [W^e dx_2 - T_1 U_{1,1} ds] + \int_{A^1_p} \sigma_{ij} \frac{\partial \varepsilon^p_{ij}}{\partial x_1} dA$$

$$= \int_{\Gamma_3} [W^e dx_2 - T_1 U_{1,1} ds] + \int_{A^3_p} \sigma_{ij} \frac{\partial \varepsilon^p_{ij}}{\partial x_1} dA \quad (14)$$

Thus the T relativeness of the integral value and the choice of the integrating approach in (7) has been proved.

For a further demonstration the nature and the applied scope of T integration, we are able to make a comparison of T integration and the commonly used J integration. In T integral formula both the elastic part and the plastic part are represented. And we have all known that for ideal elastic materials, the elastic strain $\varepsilon^p_{ij} = 0$; the elastic strain energy W stored in loading upon the material is equal to elastic strain energy W^e uncharged in unloading from the material, i.e. $W = W^e$, therefore (7) becomes:

$$T = \int_\Gamma [W^e dx_2 + \int_A \sigma_{ij} \frac{\partial \varepsilon^p_{ij}}{\partial x_1} dA - \int_\Gamma T_1 U_{1,1} ds]$$

$$= \int_\Gamma W dx_2 - \int_\Gamma T_1 U_{1,1} ds \quad (15)$$

so to say that T = J.

This means that J integration only fits into ideal elastic materials, while T integration fits into elastic-plastic materials; J integration can only pretest the cracking state of cracks, while T integration can also pretest the sub-critical expanding of cracks and the whole process of the instable expanding, too.

It can also be seen from the comparison of calculation and testing results in Figure 3, that there is a better fit between their tendencies and values.

REFERENCES

[1] Broeh, D., Elementary Engineering Fracture Mechanics, Noordhoff International Publishing, Leyden, 1974.

[2] Yin Shuangzeng, The Restriction of J Integration and a Further Study on it, Science Journal, China, No. 2., 1988.

[3] J.R. Rice and G.F. Rosengren: J.Mech.Phys., Solids, Vol. 16., 1968.

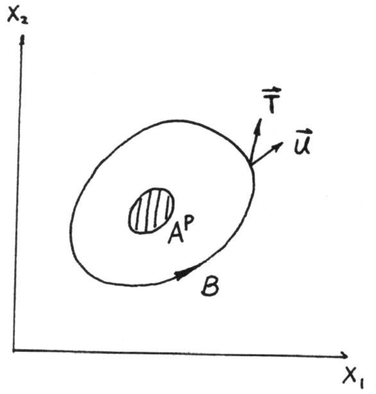

Figure 1

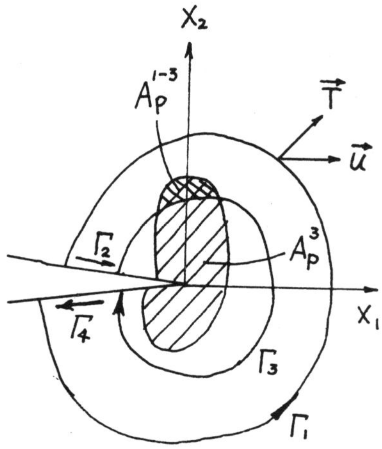

Figure 2

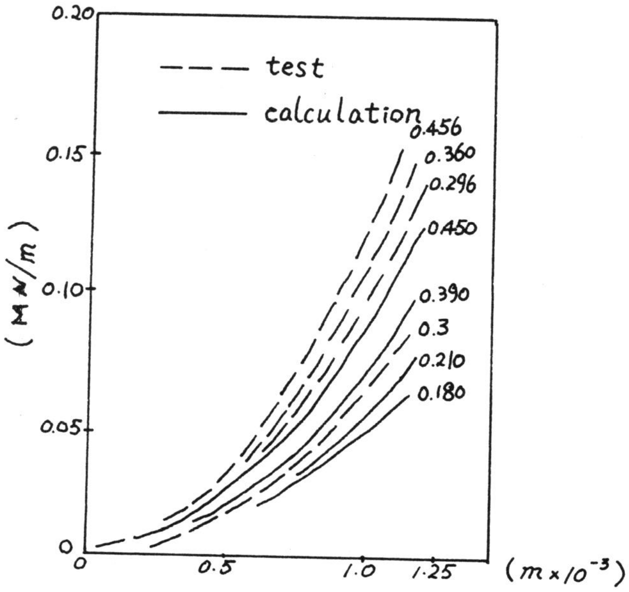

Figure 3