ON THE APPLICATION OF THE DUGDALE-PANASYUK MODEL IN ANALYSIS OF NON-STATIONARY CRACK MOTION.

A. Neimitz*

Kostrov's and Achenbach's approach is adopted to the analysis of the fast, non-stationary Dugdale-Panasyuk Mode III crack motion. The energy rate balance equation is applied as an equation of motion. Obtained results do not lead to the unique physical interpretation, give however a certain insight into the non-stationary crack propagation within elastic-plastic bodies.

INTRODUCTION

If a crack growth problem is approached analytically the mechanical fields must be determined for arbitrary crack motion and the proper growth criterion must be proposed in order to select the actual motion from the class of all dynamically admissible motions. Usually the growth criterion is constructed by comparing the mechanical quantity obtained at the continuum level from the analytical approach with so-called dynamic fracture thoughness, which is experimentally measured fracture resistance of the material. The problem of the fast crack motion has attracted the attention of many scientists for more than two decades. Among them the most significa progress has been reached by Yoffe [1], Broberg [2], Craggs [3], McClintock [4], Eshelby [5], Kostrov [6], Achenbach [7,8], Freund [9,10,11], Sleypan [12] and theirs coworkers. An excellent review article was published on the dynamic crack propagation by Freund [13] in 1986. At the time being only the fast crack motion in elastic bodies both stationary and non-stationary is fairly well understood. The crack motion in elastic-plastic materials is still an open problem although significant progress has been reached recently. The Dugdale-Panasyuk (D-P) model [14], [15] is an alternative which solved, can give a qualitative insight into fast crack motion phenomena in elastic-plastic materials.

* Mechanical Engineering Department, Technical University of Kielce, Al 1000 lecia PP 7, 25-314 Kielce, Poland.

For most performed experiments it was observed that dynamic fracture thoughness varies with crack-tip speed. At the continuum level there are, probably, two main reasons for the speed dependence: inertial resistance of the material to motion and strain-rate-dependent resistance of the material to deformation. As far the influence of inertia on fracture toughness versus the crack speed has been analysed with some success in the author's former articles for D-P crack moving with constant velocity (constant length of the D-P zone [16,17,18].) In the present article the above-mentioned analysis has been extended to the situation when crack-tip and D-P zone tip are moving with different velocity (it simulates acceleration/deceleration of the crack). The goal was to provide theoretical arguments to explain why the crack-tip has a tendency to propagate with constant-terminal velocity. As will be seen, the answer to this question is not unique and clear, indeed. The reason for this is probably, that inertia effect alone is not sufficient to explain this complex problem.

FRACTURE CRITERION

In order to discuss the D-P crack motion we adopt here results of Atkinson and Eshelby [19] and Freund's [20] analysis of the rate of energy balance of the moving cracks. The rate of energy flow out of the body through an arbitrary contour L surrounding the crack tip is equal to:

$$F = \int_L \left[\sigma_{ij}\, n_j\, \dot{u}_i + 0{,}5\left(\sigma_{ij}\, u_{i,j} + \varrho\, \dot{u}_i\, \dot{u}_i\right) v\, n_x\right] dx \quad \ldots\ldots\ldots(1)$$

where σ_{ij} are the stress tensor components, u_i are displacement vector components, n_j is unit vector normal to the contour L and v is velocity of the crack. F is related to the well-known energy release rate G_d through the relation:

$$F = vG_d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The relation (1) applied directly to the D-P crack, by selecting properly the shape of the contour L, simplifies to the form:

$$F = v\int_0^{\delta_c} \sigma(\delta)\, d\delta + \int_0^{r_p} \sigma(\delta)\, \frac{\partial \delta}{\partial t}\, dx_1 \quad \ldots\ldots\ldots\ldots\ldots (3)$$

where δ is the crack faces stretch within the D-P zone, δ_c is crack tip opening, r_p is the length of the D-P zone and t is time. For stationary moving crack Eq.3 assumes the well known relation:

$$F = vG_d = v\sigma_y \delta_c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

For dynamic case the σ_y and δ_c should be assumed or calculated from the dynamic analysis. Now one may postulate that during the motion the following relation is satisfied:

$$G_d = G_{dc} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where G_{dc} is material property and is assumed to be known. In order to use the equation of motion (5) for arbitrary motion one has to calculate displacements within the D-P zone $-\delta(t,x)$. This is the subject of the next chapter.

THE STRESS AND DISPLACEMENT FIELDS

The procedure introduced by Kostrov [6] and Achenbach [8] to the analysis of the stress and displacement fields around fast moving Mode III crack in elastic bodies has been directly adopted by Achenbach and Neimitz [16] to the D-P crack. Stresses ahead of the crack tip can be calculated from the formula:

$$\tau(\xi,\eta) = \frac{1}{\pi}\frac{1}{[\eta - N(\xi)]^{1/2}}\int_{\xi}^{N(\xi)} \frac{f(\xi,\eta)\,[N(\xi)-u]^{1/2}}{\eta - u}\,du \,.. \quad (6)$$

or in the x,s coordinate system:

$$\tau(x,s) = \frac{1}{\pi}\frac{\left(1-\frac{dX}{ds}\right)^{1/2}}{[x - X(s)]^{1/2}}\int_{0}^{X(s)} \frac{f(v,s-X(s)+u)}{[X(s)-v]^{1/2}}\,dv \,\ldots\ldots\ldots \quad (7)$$

where most of the notation is depicted in Fig.1; $s = c_T t$, c_T is the shear wave speed, $f(\xi\cdot\eta)$ is defined below in Eq.9. The crack faces opening w can be easily calculated from the relation:

$$w(\xi_1,\eta_1) = -\frac{1}{\mu\pi(2)^{1/2}}\int_{K(\eta_1)}^{\xi_1}\frac{d\xi}{(\xi_1-\xi)^{1/2}}\int_{\xi}^{\eta_1}\frac{f(\xi,\eta)\,d\eta}{(\eta_1-\eta)^{1/2}} \quad (8)$$

The relations (6-8) together with Fig. 1 are a "receipt" to solve various problems for Mode III fast moving cracks. The "key" to the solution of the D-P crack is a proper definition of the $f(\xi,\eta)$ or $f(x,s)$ function. Here it is assumed that:

$$f(x,s) = -\tau_o(x)\, H[L(s) - x]\, H(s) + \tau_f\, H[x - T(s)]\, H[L(s) - x]\, H(s) \qquad (9)$$

where: $\tau_o = (2\pi r)^{1/2}\, K_{III}(L)$ .... (10)

represents shearing stress in the plane y = 0 prior to t = 0, K_{III} (L) is an instantaneous static stress intensity factor, H [--] is Heaviside's function and τ_f is the stress within D-P zone. As a first step in the analysis it was assumed that τ_f is constant, independent of the crack speed. We simply neglect here the strain-rate-dependent resistance of the material to deformation. Indeed, it is a very strong assumption.

Displacement within the moving D-P zone can be calculated from the equation:

$$w(\xi^o, \eta^o) = \frac{1}{\mu\pi 2^{1/2}}\left[-\int_{K(\eta_1)}^{\xi^o} \frac{d\xi}{(\xi^o - \xi)^{1/2}} \int_{T(\xi)}^{\eta^o} \frac{\tau_f\, d\eta}{(\eta^o - \eta)^{1/2}} + \int_{K(\eta_1)}^{\xi^o} \frac{d\xi}{(\xi^o - \xi)^{1/2}} \int_{\xi}^{\eta^o} \frac{\tau_o\, d\eta}{(\eta^o - \eta)^{1/2}}\right] \qquad (11)$$

Notation is depicted in the Fig. 1b. The basic problem in evaluating this integral is the fact that the trajectory of the crack should be known prior to the calculations since it enters limits of inegration of the inner integrals. When the crack-tip speed is constant the unknown slope of the trajectory, being a reciprocal of the crack-tip speed enter the final solution. Exact solutions for the accelerating or decelerating D-P cracks are not possible. Nevertheless, we will take advantage of one of the former results [18] when it was shown that the length of the D-P zone is changing with the changing crack-tip speed and we will approximate acceleration/deceleration by different constant speeds of the leading and trailing edges of the crack. In such situation the following relations hold:

$$T(\xi) = \eta = \xi\,\frac{m_T + 1}{m_T - 1} = \xi\,\frac{1 + \beta_T}{1 - \beta_T} \qquad (12)$$

$$\beta_T = \frac{V}{c_T} = \frac{1}{m_T} \qquad (13)$$

$$K(\eta_1) = \xi^\circ - (2)^{1/2} \phi \frac{1}{1+\beta_T} \qquad (14)$$

where: $\phi = -x$, $\eta^\circ - \varepsilon \xi^\circ = \frac{(2)^{1/2}}{1-\beta_T} (r - \phi)$ .... (15)

and: $\varepsilon = (1+\beta_T)/(1-\beta_T)$.

After integration, the formula for the displacement within the D-P zone can be written in the form:

$$w(x^\circ, s^\circ) = \frac{4}{\pi} \frac{\tau_f}{\mu} - \left\{ \frac{r_p^{1/2} \phi^{1/2}}{\vartheta} - \frac{\phi^{1/2}}{2\vartheta} \left[r_p - \phi \frac{\beta_L - \beta_T}{1+\beta} \right]^{1/2} + \right.$$

$$\left. + \frac{r_p - \phi}{2\lambda} \ln \left[\left(1 + \frac{\varepsilon \phi}{r_p - \phi} \right)^{1/2} - \left(\frac{\varepsilon \phi}{r_p - \phi} \right)^{1/2} \right] \right\} \qquad (16)$$

where: $\vartheta = (1+\beta_L)^{1/2} (1-\beta_T)^{1/2}$, $\lambda = (1-\beta_T^2)^{1/2}$ and the relation between the length of the D-P zone and stress intensity factor, following from the stress analysis [17] was also used:

$$r_p = \frac{\pi}{8} \left(\frac{K_{III}}{\tau_f} \right)^2 (1-\beta_T) \qquad (17)$$

NON-UNIFORM EXTENSION OF THE CRACK

Equation (16) obtained in the previous paragraph can be directly applied in the equations of motion (5), (4) and (3).

Evaluation of the first term in (3) leads to the relation:

$$F - v \frac{K_{III}^2}{2\mu\tau_f} \left(\frac{1-\beta_T}{1+\beta_L} \right)^{1/2} \left[2 - \left(\frac{1+\beta_T}{1+\beta_L} \right)^{1/2} \right] + \int_0^{r_p} \tau_f \frac{\partial \delta}{\partial t} dx_1 \qquad (18)$$

The time derivative of the crack faces opening within the D-P zone is equal:

$$\frac{\partial \delta}{\partial t} = A \left\{ \frac{2B}{r_p^{1/2}} \phi^{1/2} - B \left[\frac{\phi}{r_p - (1-\varepsilon)\phi} \right]^{1/2} + 2C \left[\ln \left(1 - \frac{\varepsilon \phi}{r_p - \phi} \right)^{1/2} - \right. \right.$$

$$\left. \left. - \left(\frac{\varepsilon \phi}{r_p - \phi} \right)^{1/2} \right] - C + C \frac{}{[r_p - (1-\varepsilon)\phi] - [r_p \varepsilon \phi - \varepsilon (1-\varepsilon) \phi^2]^{1/2}} \right\} \qquad (19)$$

where:

$$A = c_T \frac{2}{\pi} \frac{\tau_f}{\mu}, \qquad B = \frac{(\beta_L - \beta_T)}{\vartheta}, \qquad C = \frac{(\beta_L - \beta_T)}{\lambda},$$

$r_p = r_{po} + s(\beta_L - \beta_T)$ and r is a length of the D-P zone prior to $s=s^*$ when $\beta_L=\beta_T$. When relation (19) is substituted to (3) it leads to an unexpected result. Both numerical and analytical integration gives at upper limit of integration infinite values for energy release rate unless both velocities β_T and β_L are equal.

This unexpected result can be interpreted in various ways. Firstly, one can either reject the D-P model as an unreasonable one or the adopted method of calculation. Secondly, the assumed stress distribution within the D-P zone or assumed strain-rate-independent resistance of the material to deformation might be an oversimplification of the physical situation that leads to the obtained singularity. Thirdly, the obtained result can be in some sense equivalent to the result or lack of the result presented by several authors e.g. Goodier and Field [21] for the energy rate balance of the quasi - statically moving Dugdale crack. Finally, one can interpret the obtained results as a tendency of the crack to move with the constant, say terminal velocity and indication that the crack tip speed changes in a discontinuous way. It has already been suggested by other authors e.g. Freund [13] or Knauss and Ravi-Chander [22] who reported that crack reaches its terminal velocity in a period less than 5 μs. Similar results were presented by Theocaris and Milios [23]. Another argument supporting the above conclusion is that the first term in Eq.3 reaches a maximum value for $\beta_T = \beta_L$. Further research on the above problem is now in progress.

REFERENCES

(1) Yoffe,E.H., Phil. Mag., Vol.42., 1951, pp.739-750.

(2) Broberg, K.B., Archiv. für Physik, Vol.18., 1960, pp. 159-192.

(3) Craggs, J.W., J. Mech. Phys. Solids, Vol. 8., 1960, pp. 66-75.

(4) Mc Clintock, F.A. and Sukhatme, S.R. J. Mech.Phys. Solids, Vol. 8., 1960, pp. 187-193.

(5) Eshelby, J.D., J. Mech. Phys. Solids, Vol. 17., 1969, pp. 177-199.

(6) Kostrov, B.V., PMM, Vol. 30., 1966, pp.1241-1248.

(7) Achenbach, J.D., Mechanics Today, Vol.1., 1972, pp. 1-57.

(8) Achenbach, J.D. and Dunayevsky, V., J. Mech. Phys. Solids, Vol. 29., 1981, pp. 283-303.

(9) Freund, L.B., J. Mech. Phys. Solids, Vol.21., 1973, pp. 47-61.

(10) Freund, L.B., J. Mech. Phys. Solids, Vol.21., 1973, pp. 47-61.

(11) Freund, L.B., "Dynamic crack propagation" in The Mechanics of Fracture, AMD, Vol.19., Ed.F.Erdogan, ASME, 1976, pp. 105-134.

(12) Sleypan, L.I., Mekhanika Tverdogo Tela, Vol.11., 1976, pp. 126-134.

(13) Freund, L.B., "Dynamic crack propagation in solids" in Computational Methods in Mechanics, Vol.2., Ed. S.N. Atluri, North-Holland, 1986, pp. 85-120.

(14) Dugdale, D.S., J. Mech. Phys. Solids, Vol.8., 1960, pp. 100-104.

(15) Panasyuk, V.V., Dopovidi Akademii Nauk Ukrainskoi RSR, Vol.9., 1960, pp. 1185-1192.

(16) Achenbach, J.D. and Neimitz, A., Engineering Fracture Mechanics, Vol. 14., 1981, pp. 385-395.

(17) Neimitz, A., Archiv, of Mechanics, Vol. 33., 1981, pp. 815-827.

(18) Neimitz, A., Engineering Transactions, Vol.29, 1981 , pp. 629-640.

(19) Atkinson, J.C. and Eshelby, J.D., J. Fracture Mech. Vol. 4., 1968, pp. 3-8.

(20) Freund, L.B., J. Elasticity, Vol.2., 1972, pp.341-349.

(21) Goodier, J.N. and Field, F.A., "Plastic energy dissipation in crack propagation" in Fracture of Solids, Eds.D.D.Drucker,J.J.Gilman,1963,pp.103-118.

(22) Knauss, W.G. and Ravi-Chander, K., Engineering Fracture Mechanics, Vol.23., 1986, pp. 9-20.

(23) Theocaris, P.S. and Milios J., Int. Journ. of Fracture, Vol. 16., 1980, pp. 31-50.

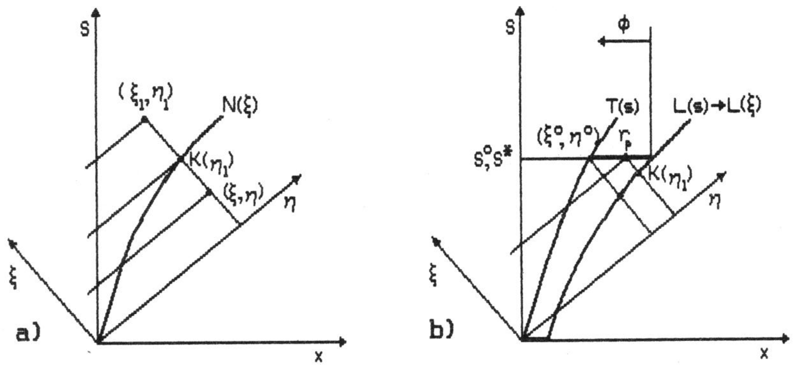

Fig.1. Scheme of the crack trajectories