F.E. CALCULATIONS OF THE CRACK EXTENSION FORCE IN BOTH ELASTIC AND PLASTIC NOTCH FIELDS

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Two efficient finite element techniques have been developed to evaluate plane strain stress intensity factors for samples of arbitrary shape. One of the methods is a simplification of the "change in stored elastic energy" calculation; the other is an improvement of the "crack closure method". Computations are made for standard test specimens and the accuracy of both these methods is confirmed by comparing the results with available analytical solutions.

The methods are further applied to calculate the crack extension force for the nucleation and growth of cracks within residual stress fields at the root of notches.

INTRODUCTION

Various numerical finite element (F.E.) procedures have been used in fracture mechanics to evaluate the stress intensity factor, $K(a)$, or the crack extension force $G(a)$ for specimens of different geometries. Some of these methods are purely mathematical such as the computation of the stress field and stress gradient method (1), and the superposition methods (2). Others are based on physical concepts, like the compliance calibration (3), the contour integral and energy methods (4-8), and the crack closure integral method (9). Some of these procedures require rather complex and tedious analysis, and in some cases they have been made unnecessarily laborious. In this paper, two straightforward methods are proposed to calculate the crack extension force for cracks with various types of notches in specimens of different geometries. These respectively are, the improved compliance method (based on the change in stored elastic energy), and the improved virtual work method (based on the internal stress or force fields ahead of a crack tip). Furthermore, the application of the virtual work method has been extended to investigate the problem of the nucleation of cracks at the root of notches containing residual stresses.

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CALCULATION OF G_I AND K_I BY THE CHANGE IN STORED ELASTIC ENERGY

The existing calculations of stress intensity factors by energy methods (7-8) are based on computing the changes in the elastic energy stored in a sample with crack length by adding, over the whole sample volume, the energy changes at each element of the F.E. discretization. This procedure is unnecessarily laborious and a simpler and more direct method is described below.

Specified forces F_i are applied at the boundaries of the sample of uniform thickness, t, and these forces are maintained constant whilst a crack, length a, is enlarged by increments Δa. At each crack length increment the displacements Δu_i of the points where the constant forces act are calculated. The product $F_i(\Delta u_i/t\Delta a)$ is the work per unit thickness done by the forces F_i as the crack length increases by Δa and it is equal to twice the increase in the elastic energy, per unit thickness, stored in the body. The crack extension force is then given by:

$$G = \frac{1}{2t} \lim_{\Delta a \to 0} F_i \; (\Delta u_i / \Delta a) \qquad (1)$$

This method of calculation has been used to obtain values of $G_I(a)$ and $K_I(a)$ relevant to specimens for which the solutions are known and others with new shapes and geometries.

In these F.E. calculations we have used linear strain elements of quadrilateral type (8-noded isoparametric) to model both a simulated crack zone, along a prescribed length, and the load application region. 6-noded triangular elements (LSE) were used in the remainder of the specimen, at regions away from the crack tip and the load application region, in order to reduce the total nmber of elements in a mesh. See figure 1. The fine mesh used to simulate the crack zone increases the accuracy of the calculation in this critical region. Referring to figure 1, the calculation of G is based on the following finite element representation.

$$G(a) = \frac{P_{11}}{2t} \lim_{\Delta a \to 0} (u_{11,i+2} - u_{11,i}) \qquad (2)$$

The change in the nodal displacement at the point of load application is the only variable needed to calculate $G(a)$. Hence only a limited amount of data needs to be generated in a typical F.E. analysis.

CALCULATION OF $G_I(a)$ AND $K_I(a)$ BY AN IMPROVED VIRTUAL WORK METHOD

The modified crack closure integral method of Rybicki and Kanninen (9), although very efficient, is limited to one type of finite element, namely the constant strain type. Bochholz and Meiners (10--11) introduced a modified version extending its use to the 6-noded isoparametric elements (LSE). In their method however, displacements at the crack tip (u_i) are related to forces acting ahead of the crack tip (F_{i+1}). In the present method, we have made a further improvement by calculating the virtual work accurately, since the nodal forces at the tip of the crack (F_i) are multiplied by the corresponding displacements (Δu_i) of the same nodal point. Referring to figure 2 the calculation of $G(a)$ by this improved method is based on the finite element representation.

$$G(a) = \frac{1}{t} \lim_{\Delta a \to 0} \frac{1}{2\Delta a} [F_{y,i}(a)\Delta u_{y,i}(a) + F_{y,i+1}(a)\Delta u_{y,i+1}(a)] \tag{3}$$

Results

The two procedures described above have been used to obtain the crack extension force and stress intensity factors for specimens of different geometries. The results are compared in figure 3 with known analytical solutions due to Newman (12-13) for the case of a sharp crack in a standard ASTM compact tension specimen (figure 3a) and for a crack at a circular central notch in a large plate (figure 3b). The accuracy of both these methods seems to be excellent and because of its simplicity the stored elastic energy method has also been used to obtain K_I and G_I curves for a variety of specimen geometries, some of them designed to give suitable variations of K_I with crack length (14-15). The results of one such application are illustrated in figure 4. This figure shows the functions $K_I(a)$ for a sharp crack and a crack emanating from a notch in a standard C.T. specimen of width W and thickness B. Two solutions corresponding to two different notch tip radii, $R = 0.03W$, and $R = 0.0625W$, are plotted in this figure.

RESIDUAL STRESSES AT A NOTCH ROOT AFTER PRE-COMPRESSION

The residual stresses generated at the root of a blunt notch with a radius of $0.03W$ in a compact tension specimen of mild steel after being subjected to pre-compressive load, sufficient to cause local yielding, have been calculated by finite element elasto-plastic solutions based on the tangent stiffness method. The crack tip zone was discretized using 8-noded isoparametric elements (LSE) with a mesh size of $5 \times 10^{-4} W$. The remainder of the mesh consisted of both 8-noded and 6-noded elements.

The equivalent stresses and strains were calculated using the Van-Mises yield criterion and the Prandtl-Reuss flow rule. The mild steel used had a tensile yield stress of σ_y = 288 MPa and its strain hardening behaviour was modelled by four linear regions with appropriate equivalent slopes, up to a plastic strain of 0.2.

The residual stress components ahead of the notch following the application of pre-compressive loads of 8, 10 and 12 kN are plotted in figure 5, where we note the large tensile component of the residual stress at the notch root. There is associated with the residual stress state, a local stored elastic energy which can be released if a crack opens at the notch; this provides local crack extension force which can be easily calculated.

Local crack extension force

The magnitude and range of the local crack extension force due to the residual stresses after pre-compressions of 8, 10 and 12 kN have been calculated using the improved virtual work method described above. The only stresses present in this calculation are those of the residual stress field of figure 5, the external loads being zero. Each time the crack is extended by Δa, the residual stresses are recalculated and used in the new crack increment. The results of the calculation are shown in figure 6.

We note that the local crack extension force, G_L has a very short range but a large finite value for crack length tending to zero. This is in contrast to the crack extension force arising from the externally applied loads (also shown in the same figure) which is zero for crack length tending to zero. It is therefore energetically possible that if G_L exceeds the fracture energy (or the corresponding K_I exceeds the fracture toughness) a crack may nucleate at the root of the notch on unloading from compression (16). However this crack would not grow beyond the range of the local G_L without an external load. Even if a crack does not nucleate spontaneously on unloading from pre-compression, the residual driving force will add up to any crack extension force produced by subsequent tensile loading. It would not be strictly correct to add the local and the external crack extension forces directly because a tensile load may cause further plastic yield at the notch tip and this would reduce and redistribute the residual stresses. This effect should be included in a proper and accurate calculation.

We have demonstrated that it is possible to calculate the effect of residual stress on the nucleation of cracks at blunt notches using F.E. methods and we believe that with these calculations it should be possible to understand and quantify the reduction in fracture toughness produced by pre-compressive loads in notched components (16-17).

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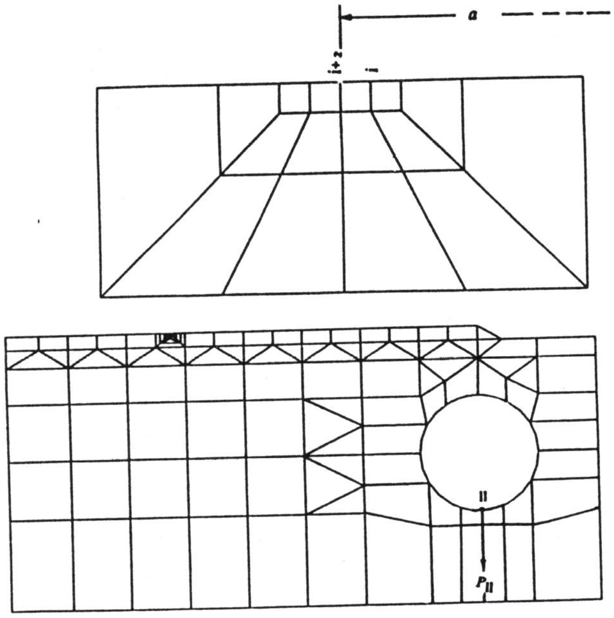


Figure 1

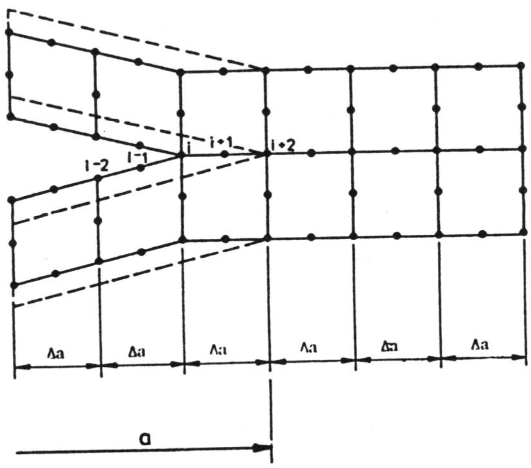
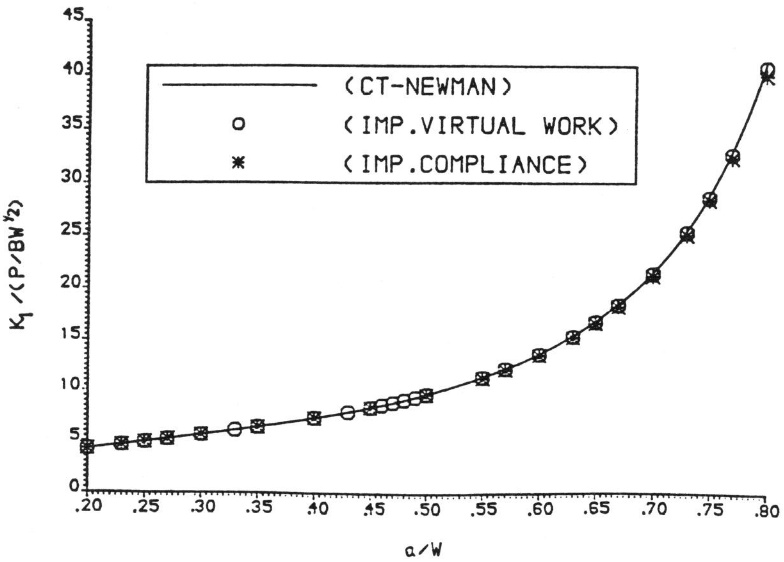


Figure 2


(CT-NEWMAN)
(IMP.VIRTUAL WORK)
(IMP.COMPLIANCE)
K_I /(P/BW$^{1/2}$)
a/W
0
5
10
15
20
25
30
35
40
45
.20
.25
.30
.35
.40
.45
.50
.55
.60
.65
.70
.75
.80

(a)

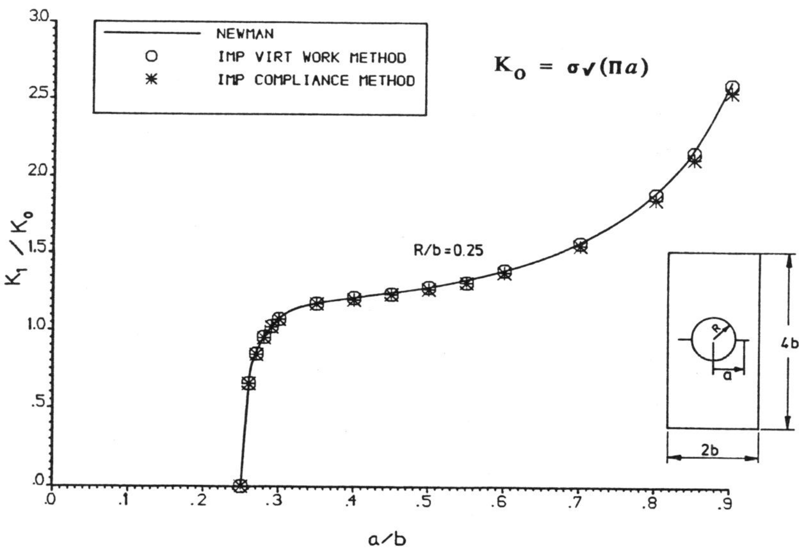

NEWMAN
IMP VIRT WORK METHOD
IMP COMPLIANCE METHOD
$K_o = \sigma\surd(\Pi a)$
R/b=0.25
K_I / K_o
a/b
4b
2b
a
R
.0
.5
1.0
1.5
2.0
2.5
3.0
.1
.2
.3
.4
.5
.6
.7
.8
.9

Figure 3 (b)

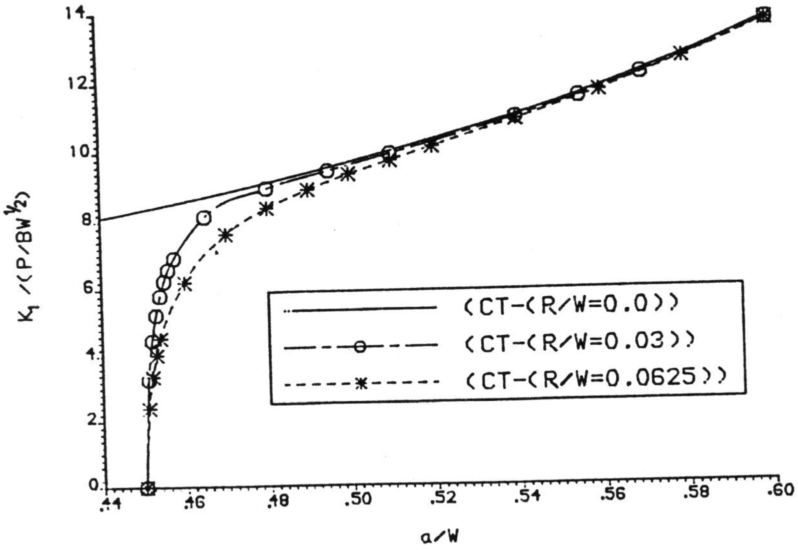


Figure 4

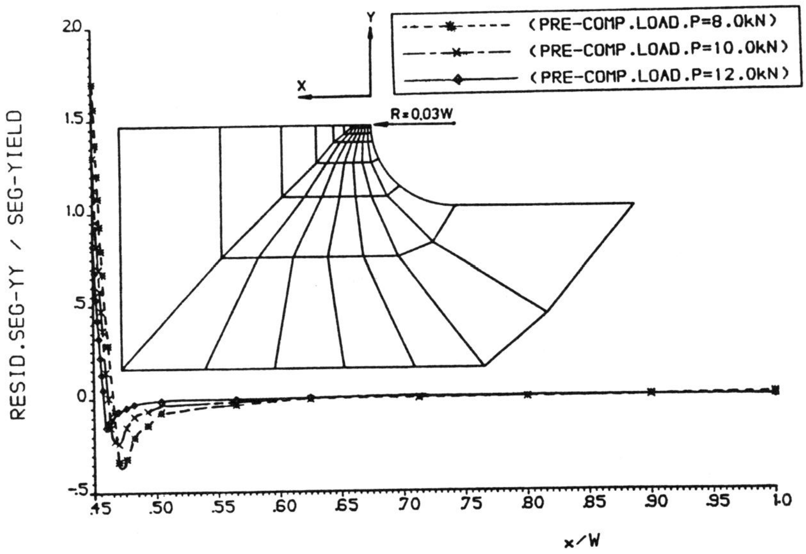
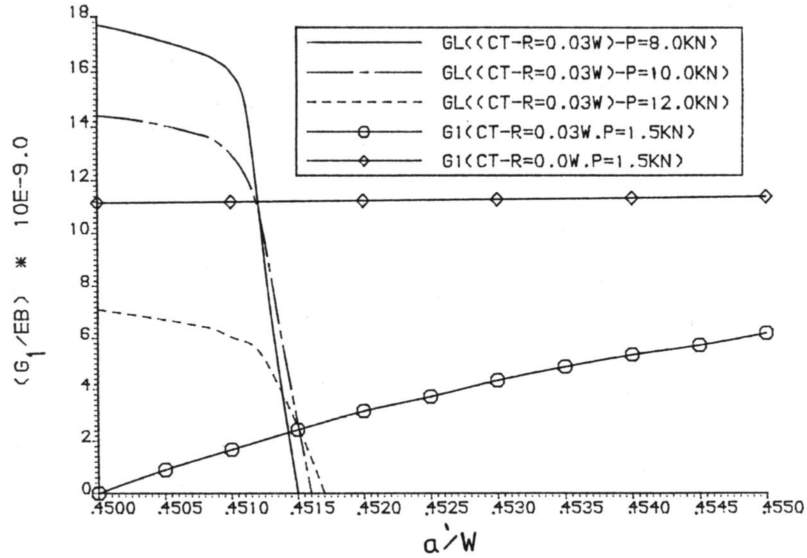


Figure 5



GL((CT-R=0.03W)-P=8.0KN)
GL((CT-R=0.03W)-P=10.0KN)
GL((CT-R=0.03W)-P=12.0KN)
G1(CT-R=0.03W.P=1.5KN)
G1(CT-R=0.0W.P=1.5KN)
(G1/EB) * 10E-9.0
a'/W
0
2
4
6
8
10
12
14
16
18
.4500
.4505
.4510
.4515
.4520
.4525
.4530
.4535
.4540
.4545
.4550

Figure 6