COMPUTATIONS AND EXPERIMENTS ON THE INTEGRITY OF COMPONENTS CONTAINING PART-THROUGH CRACKS

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The theoretical-experimental examples presented here allow evalution of part-through surface and internal cracks not only under LEFM conditions, but also in elastic-plastic fracture (EPFM) analysis, fatigue and creep crack growth. The fatigue tests with tensile plates may be regarded as a suitable experimental verification of the numerical (FE) K-solution of Raju & Newman. Other solutions, such as K-values (for membrane stress loading and in bending), J integral and C* solutions were also investigated.

INTRODUCTION

The present paper shows that simplified models very often yield adequate information and are conservative when compared with models of real geometry and of actual service conditions in pressure vessels, pipes, nozzles, cylindrical bars and other machine elements. In particular, it appears, that the overall qualitative behaviour may safely be transferred from simple models to real situations. This point is very important in practice, since it is generally too expensive to perform full 3-D finite element calculations for many practical cases, so that only simplified analyses of elementary models can be made.

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1st EXAMPLE: SURFACE CRACK IN A TENSILE PLATE

In Fig. 1 and Table 1 a tensile plate is treated as a simplified model of a more complex structural configuration, e.g. a surface crack in a hollow cylinder.

Irwin´s equation for the stress intensity factor K of this configuration is:

$$K = \frac{\sigma\sqrt{\pi a}}{\Phi}\left(\left(\frac{a^2}{c^2}\right)\cos^2\varphi + \sin^2\varphi\right)^{1/4}, \tag{1}$$

where Φ is a complete elliptic integral of the second kind,

$$\Phi = \int_0^{\pi/2}\left(\sin^2\varphi + \left(\frac{a}{c}\right)^2\cos^2\varphi\right)^{1/2} d\varphi \; . \tag{2}$$

According to Finite Element (FE) computations of Raju & Newman the stress intensity factor K is

$$K = \frac{\sigma\sqrt{\pi a}}{\Phi} F\left(\frac{a}{c}, \varphi, \frac{a}{t}\right) \; , \tag{3}$$

where $c/W \leq 0.25$ and $c/h \leq 0.25$.

The correction factors

$$F\left(\frac{a}{c}, \varphi, \frac{a}{t}\right) = \frac{K \cdot \Phi}{\sigma\sqrt{\pi a}} \tag{4}$$

are given in the literature.

The theoretical-experimental procedure presented here and described in detail elsewhere (1,2) uses the readily available solutions of

Raju and Newman, Eq.(3) and Irwin, Eq.(1) for the fracture mechanics treatment of fatigue crack growth.

To calculate the growth of a crack, an iterative scheme with equidistant steps in the direction of the crack depth is used. The following operations are necessary to proceed from state (i-1) to state (i):

a) Calculation of the crack half length c_i:

$$c_i = c_{i-1} + (a_i - a_{i-1}) \left(\frac{a_{i-1}}{c_{i-1}}\right)^{m/2} \tag{5}$$

from the known crack depth a_i and the results of the previous step, c_{i-1} and a_{i-1}, using a material-dependent exponent m/2 ($2<m<5$, e.g. $m = 3$). Formula (5) is developed from equations established by Irwin, Eq.(1) and Paris, Ref.(2).

b) Calculation of ΔK_i from $\Delta\sigma$, c_i and a_i using the Finite Element solution, Eq.(3) or Irwin´s equation (1).

The fatigue experiments (1,2) with the tensile plates may also be regarded as a suitable experimental verification of the linear-elastic K-solution of Raju and Newman, Eq.(3). Other solutions are being investigated at present. These include surface cracks under pure bending.

It is planned that for elastic-plastic applications the present computer program will be updated using the recent Japanese basic FE results on J-integral.

The presented research work using a simplified model of a tensile plate was originally developed for a safety research program for nuclear power plants.

Subsequently, six cases of widely differing geometries were analysed using a finite element programme (ADINA). The geometries investigated included plates of various height and width, hollow cylinders and a nozzle of a pressure vessel.

The results of this analysis showed only very small deviations in the K-values caused by the different geometries. Applying standard engineering criteria it was concluded that the transfer of K-results from the plate to a cylinder and from a cylinder to a nozzle was straightforward. Consequently, the emphasis in this work rests on a much simpler model of a "plate".

2nd EXAMPLE: CIRCUMFERENTIAL CRACK IN A HOLLOW CYLINDER

In analysing a real damage situation, the model of Fig. 2 was used; however, in this case the method used varied substantially from that in the previous approach.

Considering the point of view of a practical design engineer or even of a research engineer in the industry (Prodan), the importance of a close cooperation with a university approach, be it in mechanical engineering or material science (Radon) appears to be selfevident. The model of Fig. 2 was developed from a basic university research work, Ref.(3,4).

It is expressed in a very simplified form, acceptable to a practicing design engineer.

The necessary formulas are shown in Eq.(7-9) and the correction factor f_{Iz} is shown graphically in Fig. 3

$$\sigma_n = \frac{F_z}{\pi[R_a^2-(R_i+a)^2]} \tag{7}$$

$$K_I = \sigma_n\sqrt{\pi a}\; f_{IZ} \tag{8}$$

$$K_{II} = K_{III} = 0 \tag{9}$$

CONCLUSION

Two examples of simplified stress intensity factor calculations for components of relatively complicated shape, containing surface cracks were presented. Research programs in the nuclear and heavy engineering industries were discussed and relevant references quoted (1-5).

The authors dedicate this research work to the memory of Mrs. Judit Prodan.

REFERENCES

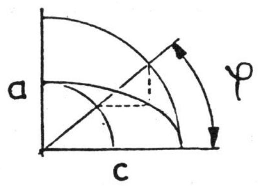
(1) Prodan, M. and Radon J.C., "Special Computations and Experiments on the Surface Crack Growth", Proc. Symposium on Surface Cracks, Reno (NV)/USA, 25th April 1988, ASTM STP, in print.

(2) Prodan, M., "A Fracture Mechanics Approach for the Evaluation of the Fatigue Crack Growth of Surface Cracks", (in German), PhD thesis No. 7374, ETH Zurich, Switzerland, 1983.

(3) Nied, H.F. and Erdogan, F., "The Elasticity Problem for a Thick-Walled Cylinder Containing a Circumferential Crack", Int. J. Fracture, Vol 22, pp. 277-301, 1983.

(4) Theilig, H. and Nickel, J., "Spannungsintensitätsfaktoren", VEB Fachbuchverlag, Leipzig, 1987.

(5) Atluri, S.N. (Editor), "Computational Methods in the Mechanics of Fracture", North-Holland, Amsterdam, New York, Oxford, Tokyo, 1986.



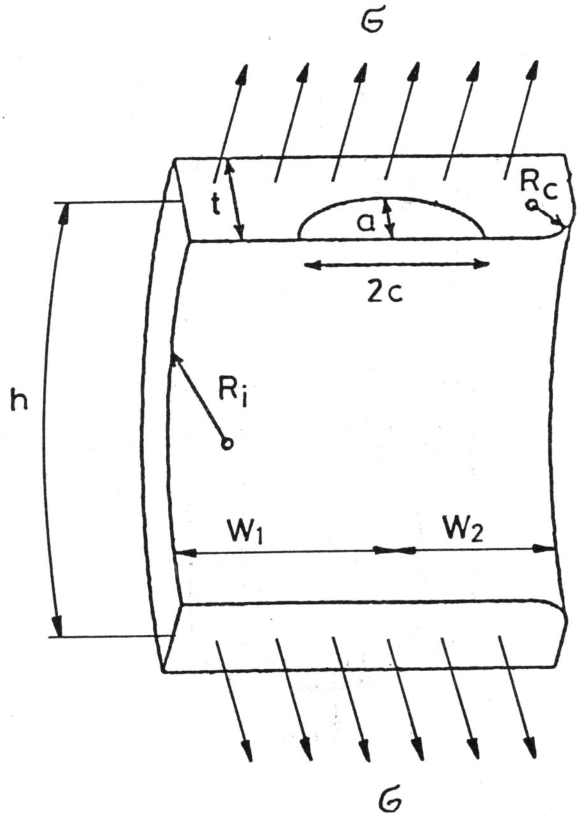


Figure 1 Geometry of a simplified model (shell→plate) and loading

TABLE 1 - Geometric data in mm, tensile σ-range ($\Delta\sigma$) in N/mm^2 (see Fig. 1)

a	c	$W_1=W_2$ W	h	t	R_i	R_c	Tensile σ-range
2-20	7-23	$\geq$4c	$\geq$4c	20	∞	0	250

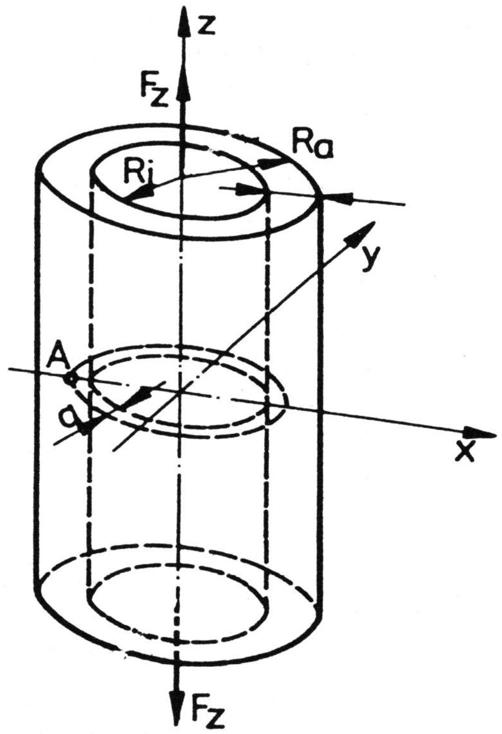


Figure 2 A circumferential crack on the inner surface of a hollow cylinder loaded by tension F_z

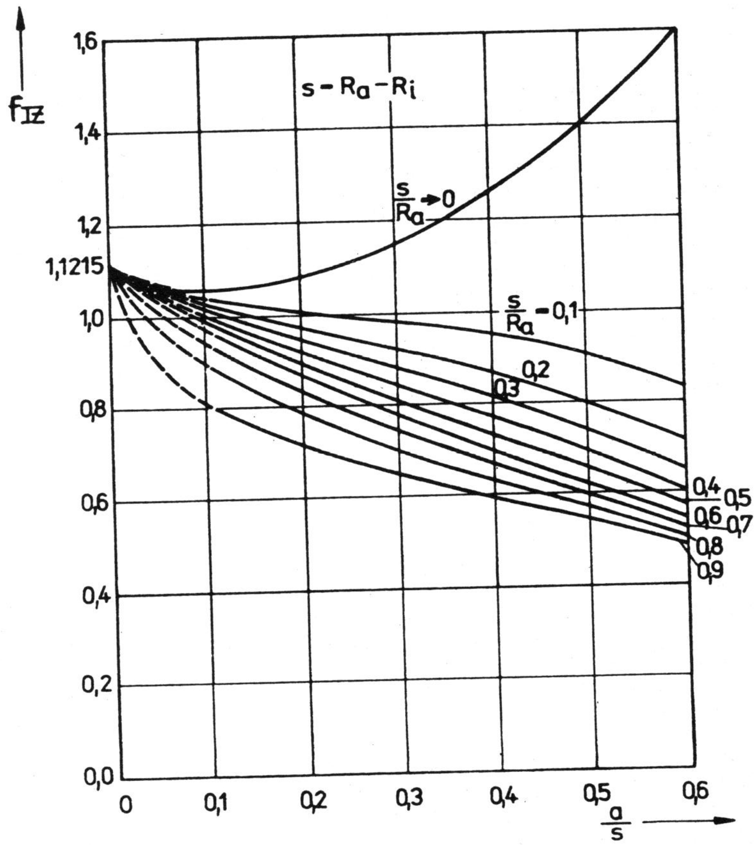


Figure 3 Correction factors f_{Iz} from Ref. (3,4) for Eq. (8)