A MODEL OF CUMULATIVE FATIGUE DAMAGE

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INTRODUCTION

In this paper the concept of the cumulative fatigue damage which accounts for the previous damage history and loading order effect is formulated through a phenomenological approach. The development is an extension of the model recently proposed by Kujawski and Ellyin (1).

DESCRIPTION OF THE CONCEPT

Let us assume that the relative increase of damage dD/D, is a linear function of the relative increase of cycles, dn/n, i.e.

$$\frac{dD}{D} = f(\psi;p) \frac{dn}{n} \tag{1}$$

where $f(\psi;p)$ is a damage function which contains a controlling damage variable ψ and material properties p, as a parameter. The controlling damage variable can be, for example, the range of stress, strain or input of energy per cycle of loading. The damage parameter,p, and its change which may occur as a result of the fatigue cycling was associated with apparent fatigue limit.

Integrating Eq. 1 for any ψ and p= const and noting that for $n=N_f$, $D=D_f$ then dimensionless damage $d = D/D_f$ at any stage is given by

$$d = \left(\frac{n}{N_f}\right)^{f(\psi;p)} \tag{2}$$

where N_f is the number of cycles to failure at ψ and d=0 for the virgin material and d=1 at fracture.

The damage curves for two different ψ's are shown schematically in Fig. 1. From Fig.1 the concept of

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the cumulation of damage can be easily deduced.

In the case of two-stage loading the above analysis gives the following relation

$$\left(\frac{n_1}{N_{f1}}\right)^{f(\psi_1;p) \,/\, f(\psi_2;p)} + \frac{n_2}{N_{f2}} = 1 \qquad (3)$$

DISCUSSION

Specific forms of the damage function, $f(\psi;p)$, and discussion with reference to earlier investigations can be found in Kujawski and Ellyin[2]. A particular form of the damage function is the logarithm of the hysteresis energy per cycle.

Predictions of the proposed method are compared in Fig.2 with typical results of the two-stage loading reported by Miller and Zachariah[3].

A fairly good agreement is observed between the predicted values and the experimental data.

REFERENCES

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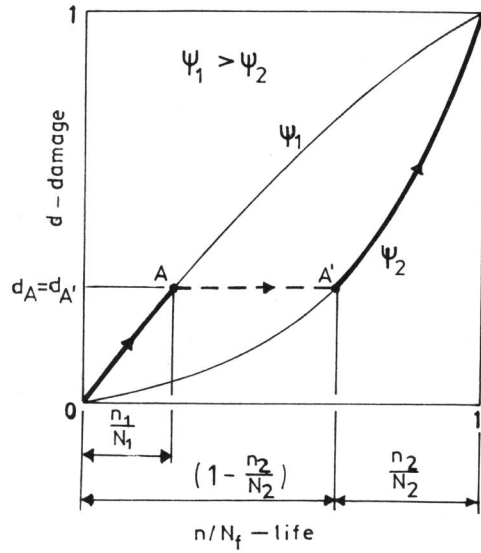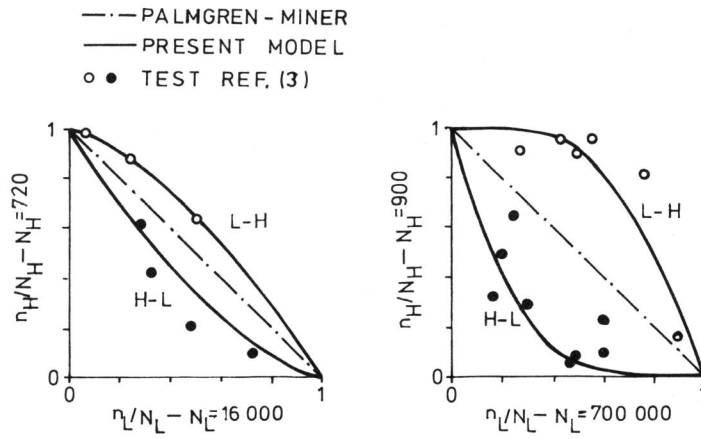
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Figure 1 Schematic of normalized damage curves

Figure 2 Test data Ref.(3) and the present model