LINEAR AND NONLINEAR FATIGUE DAMAGE AND CUMULATION

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SYNOPSIS

In this article an attempt to establish a damage evolution model based on linear fracture mechanics is described. A computer program using the model and the crack propagation results under two amplitude stress levels can take into account the retardation effect due to an existent plastic zone at the crack front.

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I INTRODUCTION

Generally speaking, the growth of the fatigue damage of a member submitted to any action of variable amplitude may be interpreted, from a physical point of view, as being the loss of its mechanical properties liable to lead to fracture after a certain number of cycles.
The governing rule of this cumulative fatigue damage is based on two essential laws or relationships :
- the damage law for a cyclic variation of stresses with constant amplitude, and
- the cumulative damage law for various levels of cyclic variation of the stresses.
After reminding some of the definitions and assumptions that govern the above mentioned laws, the paper aims
- to determine the governing assumptions of both the damage and the cumulative damage laws inherent in the expression of a crack

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growth law, and
- to work out a model of damage growth based on the fracture linear mechanics, which takes into account the retardation effect of the crack extension.

By means of the results of fatigue tests with two stress amplitudes, a calculation program allows to optimize the parameters of the model related to the mechanical properties of the test pieces.

II DAMAGE AND DAMAGE CUMULATION /1/, /2/, /3/

The law of the damage growth describes the velocity of the damage factor $\mathcal{D}$ as a function of the number of cycles N. This velocity, da/dN, does generally not depend on the frequency of stresses in the case of high number of cycles fatigue and varies in the same direction as the amplitude variation of the stresses $\Delta\sigma$.

Nevertheless, the definition of the damage factor leads itself to several interpretations. In any case, this factor defines the structure state as follows :

$\mathcal{D} = \Delta_0$ - undamaged (intact) structure

$\Delta_r > \mathcal{D} > \Delta_0$ - partially damaged structure

$\mathcal{D} = \Delta_r$ - completely damaged structure (collapse)

with Δ_r and Δ_0 - two parameters resulting from the physical or conventional definition of the collapse criterion and of the intact state of the structure. They are usually taken as 1 and 0 respectively.

II.1 Physical definitions of the damage law

The loss of strength properties of a mechanical member submitted to fatigue may be interpreted by one of several damage indicators. Each indicator consists of the ratio of a parameter that has been adopted to describe the existent state of the member to another parameter corresponding to the definition of the member collapse. It is according to the choice of these parameters, which may be either local or global, that the damage law is defined.

Global parameters are intended as parameters related to the member loading, such as the stress variation $\Delta\sigma$, the number of applied cycles n, etc.

Local parameters are intended as parameters concerned with local stresses and strains in the member, or even variation of crack length.

Among the definitions for global parameters, there is the following one:

$$\mathcal{D} = \Delta_0 + (\Delta_r - \Delta_0)\ \frac{n}{N} \qquad (1)$$

This is the definition given by the Miner's cumulative damage rule, with:
n - a data relative to the actual loading state and
N - a data relative to the collapse.

Among the definitions for local parameters, there is :

$$\mathcal{D} = \Delta_0 + (\Delta_r - \Delta_0)\left(\frac{a - a_0}{a_f - a_0}\right)^l \qquad (2)$$

with a - crack length in actual state of loading,
a_0 and a_f - crack lengths corresponding to the intact and the damaged states respectively,
l - material dependent parameter.

One should note that any local interpretation of the damage possesses a pattern which provides a global interpretation, and conversely. For instance, the taking into account of a damage law such as (2) has a pattern that leads to a damage law as follows:

$$\mathcal{D} = f(\frac{n}{N})$$

where $f(\frac{n}{N})$ being the nonlinear function of $\frac{n}{N}$.

II.2 Physical definitions of the damage cumulation law

The "damage cumulation" is intended as the law that governs the summation of partial damages that characterize the deterioration of the material or of the member submitted to each cycle of stress variation. Besides the dependence of the damage laws, the damage cumulation laws depend on a certain "memory effect" into the material that generates a nonlinear relation between partial damages; in other words:

$$f\ (D_1,\ D_2,\ \ldots D_n) = 0$$

where D_1, D_2, $\ldots D_n$ - factors of partial damages,

when $f(D_1,\ D_2,\ \ldots D_n)$ is a linear function, the framework is a linear damage cumulation law, which considers that each cycle induces for a same level of stress a constant damage, whatever the number of already applied cycles may be. From this point of view, the linear damage cumulation law is an arithmetical law and does not take into account the physical behaviour of the material. It is therefore necessary to search after cumulation laws better adapted to the physical damage phenomena.

III PROBLEMS UNDER CONSIDERATION

III.I Definitions adopted for the damage law and for the damage cumulation law

In the present study, the "damage law" is intended as the relationship between the factor $\mathcal{D}$ and the ratio n/N, with N being the total number of cycles with same stress variation under which the member will collapse.

As for the "damage cumulation law", it is intended as providing the relation between n_1/N_1, $n_2/N_2 \ldots n_n/N_n$. n_1, n_2, ... n_n are the numbers of cycles applied to different stress variations $\Delta\sigma_1, \Delta\sigma_2, \ldots \Delta\sigma_n$ liable to result in the structure collapse. N_1, $N_2, \ldots N_n$ represent the number of cycles for the same stress variations $\Delta\sigma_1, \Delta\sigma_2, .\Delta\sigma_n$ each of whom results in the structure collapse.

Let remark that the damage law deals with the null, partial or global damage of the structure whereas the damage cumulation law provides a relation between the ratios n_i/N_i at the instant of global damage, or collapse.

Each of both laws - damage or damage cumulation - may be formulated either in linear or in nonlinear form /3/ and we can therefore have at our disposal three types of laws for the damage evolution of a mechanical member, according to the expression given for the damage or for the damage cumulation.

III.2 Damage evolution laws

Type I - A linear damage law, Equation (1) is associated with a linear damage cumulation law $\Sigma n_i/N_i = 1$,
This association results from an evolution law of $\mathcal{D}$ to be written as follows: /4/

$$\frac{d\mathcal{D}}{dN} = A\ (\Delta\sigma)^b$$

with $\Delta\sigma$ - stress variation,
A and b - two constants.

Remark : The integration of this equation, $\Delta_0 \leq \mathcal{D} \leq \Delta_r$, $0 \leq N \leq N$ leads to a Wöhler law (S-N curve) /5/:

$$N = \frac{\Delta_r - \Delta_0}{A} \cdot (\Delta\sigma)^{-b} \qquad (3)$$

with N - number of cycles to failure.
This law is interpretative of a failure state (point W, figure 1-a).
As for $\Delta_0 \leq \mathcal{D} \leq \Delta \qquad 0 \leq N \leq n_1$

$$\Delta \leq \mathcal{D} \leq \Delta_r \qquad n_1 \leq N \leq n_1 + n_2$$

with $n_1 + n_2$ leading to the member collapse:

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \qquad (4)$$

This law is interpretative of a failure state (line M, figures 1-b) corresponding to the Miner cumulation rule.

Type II : A nonlinear damage law is associated with a linear damage cumulation law.

This type II allows to use both the Wöhler curve corresponding to the point W of the damage law and the Miner rule corresponding to the line M of the damage cumulation law. The essential difference between the two types I and II consists in the physical interpretation of the damage growth, but there is no difference when collapse occurs.

Type III : It is this Type III of damage evolution law that allows to take into account the behaviour of the member material in the damage cumulation. In this case, a nonlinear damage law is associated with a nonlinear damage cumulation law.

In relation to this type III, let mention the damage evolution law worked out by LEMAITRE and CHABOCHE /1/ which is written as:

$$\frac{d\mathcal{D}}{dN} = f\left(\frac{\Delta\sigma}{1-\mathcal{D}}, \bar{\sigma}, \mathcal{D}\right) \qquad (5)$$

with $\bar{\sigma}$ being the mean stress level.

Remark : It is theoretically impossible to associate a nonlinear damage cumulation law with a linear damage law since the latter one necessarily implies a linear cumulation law. In the technical literature, there are nevertheless several authors who propose linear cumulation laws depending on a parameter α, with $0 < \alpha < 1$ /6/. In this case, it is not a question of theoretical law based on an analytical approach but of a purely statistical approach that allows, for certain particular loading cases, to obtain satisfactory results. In fact, this is a nonlinear cumulation law interpreted by a parametric linear cumulation law.

IV DEVELOPMENT OF A PHYSICAL MODEL OF DAMAGE BY CRACKING

The crack propagation laws are a valuable tool for the adjustement of a fatigue damage law.

Let consider the following propagation law: /7/

$$\frac{da}{dN} = c\,(\Delta K - \Delta K_0)^m \qquad (6)$$ with

da/dN crack propagation rate,
c, m two parameters depending on the material quality in which

the crack extends,

ΔK variation of the stress intensity factor,

ΔK_0 threshold of ΔK under which no perceptible increase of the crack can be observed.
This factor would depend on the mechanical properties of the material in which the crack extends.

This law has been many a time confirmed by experience as concerns steel.

Let now adopt the assumption of the damage factor $\mathcal{D}$ being a function of the crack length a, according to the following relationship:

$$\mathcal{D} = \Delta_0 + (\Delta_r - \Delta_0)\left(\frac{a - a_0}{a_f - a_0}\right)^{1/1} \qquad (7)$$

a_0, a_f are the initial and final crack lengths.
1 is a factor depending in a general way on the material in which the crack extends.
Starting from equation (7), it may be written

$$\frac{da}{dN} = 1\left(\frac{\mathcal{D} - \Delta_0}{\Delta_r - \Delta_0}\right)^{1-1} \cdot \frac{1}{\Delta_r - \Delta_0} (a_f - a_0) \frac{d\mathcal{D}}{dN} \qquad (8)$$

Among the assumptions of the linear fracture mechanics, there is the relationship:

$$\Delta K = \Delta\sigma \sqrt{\pi a} \,.\, f(a) \qquad (9)$$

with f(a) being a factor of geometrical correction. If taking account of (7), $f(a) = g(\mathcal{D})$.
By replacing (7), (8) and (9) in equation (6), the following relationship is obtained :

$$\frac{d\mathcal{D}}{dN} = c\left(\frac{\mathcal{D} - \Delta_0}{\Delta_r - \Delta_0}\right)^{1-1} \cdot \frac{\Delta_r - \Delta_0}{1(a_f - a_0)} \cdot$$

$$\left[\Delta\sigma \sqrt{\pi\{a_0 + (a_f - a_0)\left(\frac{\mathcal{D} - \Delta_0}{\Delta_r - \Delta_0}\right)^{1}} \cdot g(\mathcal{D}) - \Delta K_0\right]^m \qquad (10)$$

The definitions for a_f, a_0, Δ_0 and Δ_r are based on the definition already given for the intact and collapse states of the structure. c, m and ΔK_0 are known by tests of fatigue induced cracking, carried out with test pieces of the same material as the structural member under consideration.
1 is a parameter to be determined by verifying the relationship in a and $\mathcal{D}$.

Equation (10) represents a damage evolution law. If integrating it for $\Delta_0 \leq \mathcal{D} \leq D$ and $0 \leq N \leq N$

a damage law is obtained: $D = NLF^*\left(\frac{n}{N}\right)$ (11)

If integrating it for $\Delta_0 \leq \mathcal{D} \leq D$ $0 \leq N \leq n_1$

$D \leq \mathcal{D} \leq \Delta_r$ $n_1 \leq N \leq n_1+n_2$

one obtains a damage cumulation law for two stress levels:

$$NLF^*(n_1/N_1, n_2/N_2) = 0 \quad (12)$$

Both integrations have been analytically carried out for $g(\mathcal{D})$ = constant $m \neq 1$ and 2 /3/.

Equation (7) necessarily imply a nonlinear damage cumulation law because of the ΔK_0 in the formulation, but this presence may not explain physically the effect of crack retardation.

From now on, we consider in this paper, that the retardation effect is due to the occurrence of a plastic zone at the front of the crack; in which the cracking parameters relative to the material, i.e. c, m and ΔK_0, become c^*, m^* and ΔK_0^*.

Taking into consideration this modification at the moment at which the stress level changes would result in the retardation effect. The fact that this delay occurs at the moment of the loading changing from the high to the low stress level (figure 3-a) and that it "does not appear" in the reverse situation (figure 3-b) is due to the small plastic zone generated in figure 3-b on the one hand, and to the crack that extends in this zone under higher loading, on the other hand.

With m, c and ΔK_0 being known, equation (6) or even the damage growth (equation (10) may be used with $m = m^*$, $c = c^*$ and $\Delta K = \Delta K_0^*$. When the crack extends in the plastic zone, a preliminary calculation is necessary in order to determine the shape ot the plastic zone.

V NUMERICAL DETERMINATION OF THE NUMBER OF CYCLES AS A FUNCTION OF THE CRACK LENGTH AND VICE-VERSA

Let consider the crack propagation law given by equation (6) and suppose that the stress intensity factor varies linearly about the crack length a, between two values a_i and a_{i+1}, or $\Delta K = \Delta\sigma.\overline{\Delta k_i}$

$\overline{\Delta k_i} = \alpha_i a + \beta_i$

By replacing in (6) and by integrating for $a_j < a < a_{j+1}$ and $m \neq 1$ and 2, one obtains:

(*) NLF = nonlinear function

$$\left[N \right]_{N_j}^{N_{j+1}} = \frac{1}{c(1-m).\Delta\sigma.\alpha_i} \left[\{\Delta\sigma(\alpha_i a_{j+1}+\beta_i)-\Delta k_o\}^{1-m} - \{\Delta\sigma(\alpha_i a_j + \beta_i) - \Delta K_0\}^{1-m} \right] \quad (13)$$

The values of α_i and β_i change when changing the segment.
If integrating for $a_0 < a < a_f$, the life duration is determined as follows:

$$N = \Sigma_i \Sigma_j \frac{1}{c(m-1).\Delta\sigma.\alpha} \left[\{\Delta\sigma(\alpha_i a_{j+1}+\beta_i) - \Delta k_0\}^{1-m} - \{\Delta\sigma(\alpha_i a_j+\beta_i) - \Delta K_0\}^{1-m} \right]$$

For each given $\Delta\sigma$, the equation (13) allows to calculate one of the parameters N_{j+1}, N_j, a_{j+1} and a_j when the three of them are known and provided that the parameters α_i, β_i, c, m, and ΔK_0 are experimentally, numerically or analytically determined.

This equation (13) allows therefore to cumulate several stress variation levels. As for taking into account the plastic zone for the retardation effect, it will depend on the values assigned to m, c and ΔK_0.

VI NUMERICAL DETERMINATION OF THE CRACK PROPAGATION PARAMETERS IN THE PLASTIC ZONE

By considering the crack propagation law given by equation (6), it may be written :

$$S = X + Y. \log\ (t - Z) \quad (14)$$

with :

$$S = \log \frac{da}{dN}$$

$$t = \Delta k$$

$$X = \log c$$

$$Y = m$$

$$Z = \Delta K_0$$

By means of test results of crack propagation in mode I and for two stress levels, see figure 2, tables have been worked out of da/dN ratios (whose calculation is based on recordings of a_i –

crack lengths and N_i - numbers of cycles) versus ΔK (calculated by a finite elements program for each $\Delta\sigma_i, a_i$). As concerns the crack length a_f corresponding to the change of stress level, the shape of the plastic zone at the crack tip is determined before calculating $a_f + R_p$, with R_p being the distance of the plastic zone in the direction about which the crack extends.

The table (da/dN, ΔK) worked out for a values between $a = a_f$ and $a = a_f + R_p$ put at our disposal n_t equations with three unknowns X, Y, Z, that is :

$$S_i = X + Y \log (t_i - Z) \qquad i = 1 \text{ to } n_t$$

By using a program for the resolution of a nonlinear system of equations, for each group of 3 couples of values S_i, t_i a solution of X, Y, Z can be obtained, that means that for n_t couples of values (S_i, t_i), $n_t ! /(n_t - 3)!.3!$ solutions of X, Y, Z can be obtained.

At each solution X_j, Y_j, Z_j, there is a calculation of:

$$V_j = \Sigma_i v_i^2$$

with

$$v_i = S_i - \left[X_j + Y_j \log (t_i - Z_j) \right], \; i = 1 \text{ to } n_t$$

The values of X_j, Y_j, Z_j corresponding to minimum V_j are taken into account from which c, m and ΔK_o are calculated in the plastic zone.

VII APPLICATION

The method of calculation of c, m and ΔK_o as well as of the number of cycles due to the crack retardation has been applied by means of a computer program (CUMUL).
CUMUL program requests 5 types of data:

a) DATA I : data relative to a fatigue test with two stress levels $\Delta\sigma_1$, $\Delta\sigma_2$, carried out on a test piece with an initial crack of length a_o. The fracture occurs in mode I, after application of n_1 and n_2 for $a = a_R$. See figure 2.

b) DATA II : consists of a table of K as a function of a. The calculation proceeds from a given geometry of a plane specimen in which a crack extends in mode I.

c) DATA III : data resulting from propagation tests on standard CT specimens /8/.

d) DATA IV : a, as a function of N, obtained by recordings on the same specimen with the crack propagating in mode I, under fatigue at two stress levels with n_1 and n_2 cycles respectively. $n_1 + n_2$ leads to collapse.

e) DATA V : da/dN as a function of ΔK , based on DATA II and IV.

For the algorithm of the CUMUL program, see figure 4.
The successive steps of the program are :

1) SUBROUTINE CRACKT (I) - starting from DATA I, DATA II and DATA III, the equation (13) is used to cumulate the loading of two stress levels.
The calculation results provide the values a_f = crack length for n_1 cycles, and $N_R = n_1 + n_2$ = number of cycles to $a = a_R$.

2) SUBROUTINE ZPLAST - starting from a_f, σ_2 and DATA II, the shape of the plastic zone is determined and then its depth R_p in the direction of crack propagation; see figure 3.
3) Starting from ZPLAST and DATA V, DATA V' is composed, which corresponds to the table da/dN, ΔK in the plastic zone, i.e. between $a = a_f$ and $a = a_f + R_p$
4) By means of OPTCM , c^*, m^* and ΔK_o^* are calculated according to the analysis developed in paragraph VI and by using NLEQ that solves the nonlinear system of equations (14).
5) CRACKT (III) - allows to calculate the number of cycles applied in the plastic zone for :
$\Delta\sigma = \Delta\sigma_2$ and $a = a_f$ to $a_f + R_p$
with taking into account the influence of the plastic zone ($c = c^*$, $m = m^*$ and $\Delta K_o = \Delta K_o^*$)
6) CRACKT (II) - allows to proceed to the same calculation as CRACKT (III) but with neglecting the influence of the plastic zone, that means by considering the c, m and ΔK_o of the not plastified material.

The application of the CUMUL program by using the results of tests carried out by /9/ allowed to calculate c^*, m^* and ΔK_o^* in view of taking into account the retardation effect. But many other test results in form of DATA I, II, III, IV and V are still necessary to substantiate the proposed model.

CONCLUSIONS

The damage evolution under variable stress amplitude depends on the physical interpretation of both the damage and the damage cumulation laws.
A physical interpretation through the propagation into the material of a macroscopic crack allows a more rational development of the laws in view of calculating the life duration of a mechanical piece under loading of variable amplitude and determining the damage rate at any instant. This interpretation also allows to explain physically the effect of crack retardation due to an existent plastic zone at the crack front.
The crack propagation law constants in this zone may be determined by using test results of crack propagation under variable amplitude.

The crack retardation therefore depends on the size of the plastic zone at the moment corresponding to the change of stress amplitude, on the mechanical properties of the material relative to the crack propagation and on the eventual loading levels that enables the crack to extend through the plastic zone.

REFERENCES

1 LEMAITRE J. and CHABOCHE J.L.
"Rupture par endommagement". Journal de Mécanique Appliquée Volume 2, n° 3, 1978.

2 MINER M.A.
"Cumulative Damage in Fatigue". Trans. ASME Journal of Applied Mechanics, 12 n° 3, 1945.

3 RECHO N. and BROZZETTI J.
"Endommagement par fatigue". Construction Métallique n° 3, 1984.

4 LEMAITRE J.
"Cours de mécanique appliquée à la construction". University PARIS VI, Cachan 1977.

5 WOEHLER A.
"Versuche zur Ermittlung der auf die Eisenbahnwagen-Achsen einwirkenden Kräfte und der Widerstandsfähigkeit der Achsen". Zeitschrift für Bauwesen, 1860.

6 SCHUTZ W.
"Procedures for the Prediction of Fatigue Life of Tubular Joints". International Conference "Steel in Marine Structures", Paris 5-8, October 1981.

7 PARIS P.C., GOMEZ M.P. and ANDERSON W.E.
"A Rational Analytical Theory of Fatigue". University of Washington, Wa. volume 13 n° 1, January 1961.

8 BROWN W.F. and STRAWLEY J.E.
"Plane Strain Crack Toughness testing of High Strength Metallic Materials". ASTM - STP 410, March 1969.

9 Mémoires de fin d'études à l'ENSAM (Ecole Nationale Supérieure des Arts et Métiers) - France - Projets Fatigue 1983 - 1984 - 1985

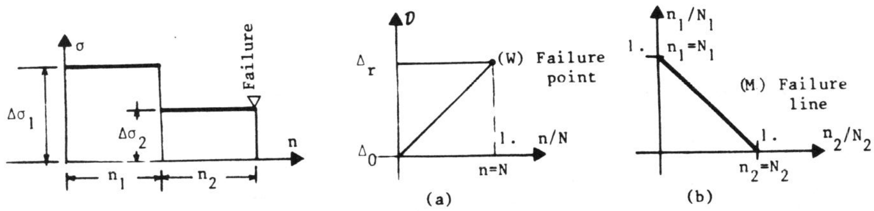

Figure-1 - LINEAR DAMAGE AND NONLINEAR CUMULATIVE DAMAGE LAWS
(W) - WHOLER CURVE
(M) - MINER RULE

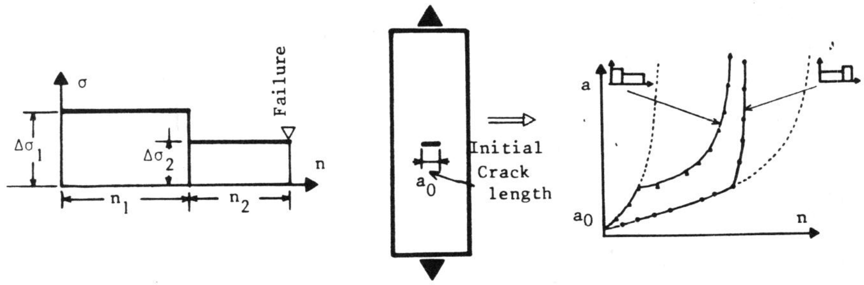

Figure-2 - MODE I CRACK PROPAGATION UNDER TWO VARIABLE STRESS LEVELS

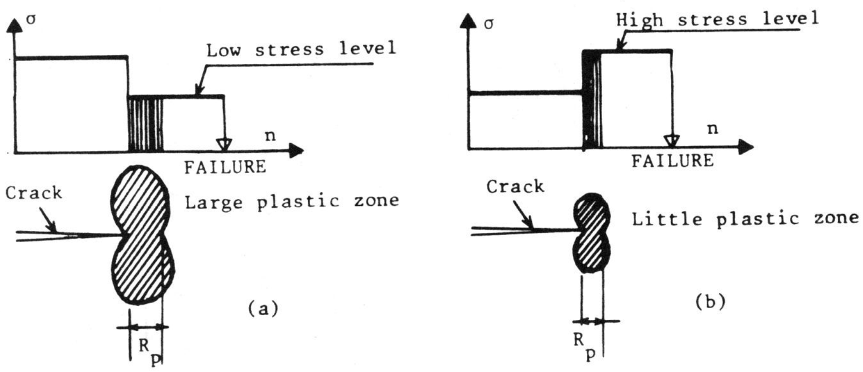

Figure-3 - RETARDATION DUE TO THE SIZE OF THE PLASTIC ZONE

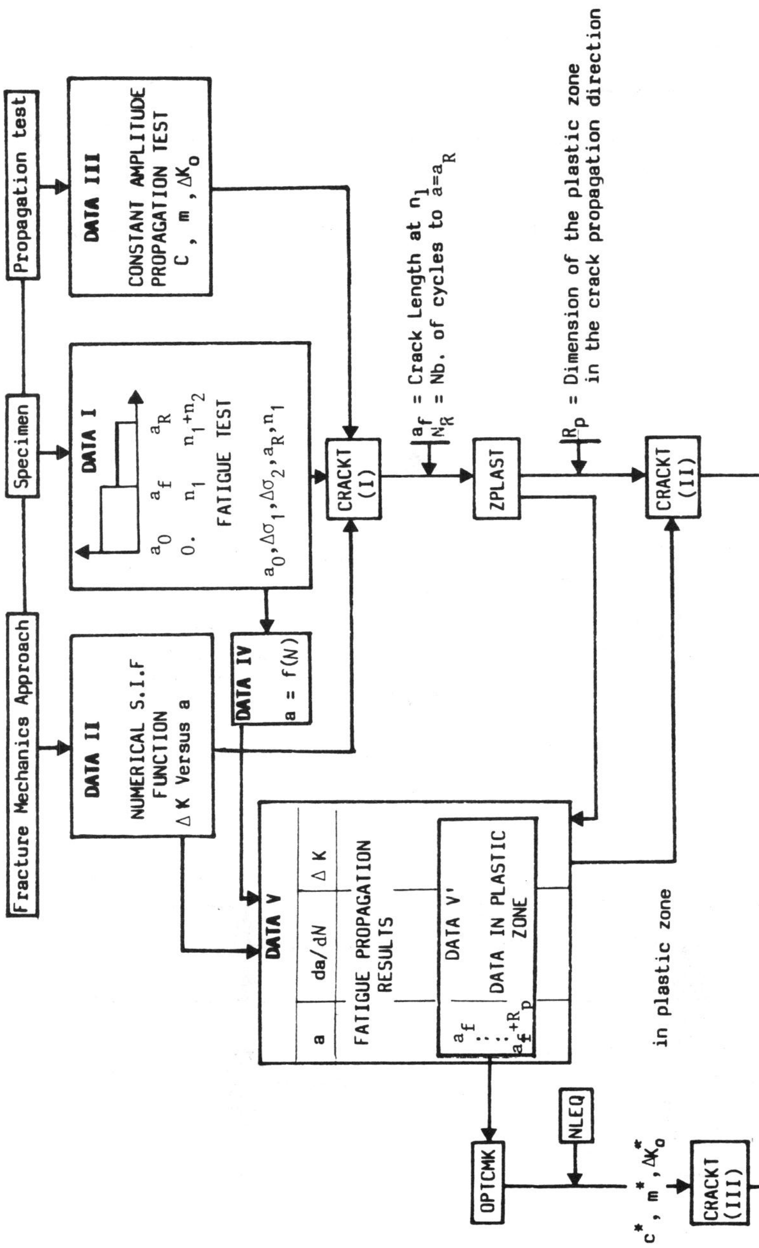

Figure-4 - PROGRAMME TO ADJUST A FRACTURE MECHANICS MODEL IN VARIABLE AMPLITUDE LOADING USING FATIGUE PROPAGATION RESULTS