CONTRIBUTION TO SOLUTION OF LIFE OF PRESSURE VESSELS SUBJECTED TO VARIABLE INTERNAL PRESSURE

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Classical solution of life of pressure vessels subjected to internal pressure (e.g. thick walled tubes) is based on calculation of critical wear. Increasing demands on the level of the cyclic internal pressure can be fulfilled only by using higher strength steels. This involves the danger of brittle fracture before the critical degree of wear is reached. The paper presents results of the application of fracture-mechanical concept to the calculation of the life of thick walled tubes.

INTRODUCTION

Pressure vessels may be used with different levels of internal pressure during service life. It is therefore of high importance to investigate the behaviour of the vessels under such conditions. The problem of multi-pressure level cyclic loading of thick walled tubes will be dealt with on the basis of the data for single--pressure level. The sequence of events during the life of pressure vessels subjected to cyclic pressure accompanied by heat-checking can be summarized on the basis of papers by Davidson et al. (1), Goldthrope (2) and Underwood (3) as follows:

(i) Nucleation of small cracks due to pressure and heat-checking during the first cycles,
(ii) fatigue crack propagation and
(iii) final failure caused either by brittle fracture or by leak-before-break.

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TYPES OF FINAL FAILURE

At the given pressure level, the fatigue crack growth leads to the increase of the stress intensity factor level. The stress intensity factor (K-factor) is generally given by

$$K = p \sqrt{a} \; f(a/T) \; , \qquad \ldots\ldots\ldots\ldots \quad (1)$$

where p is the internal pressure, a is crack length and T is wall thickness. The product $\sqrt{a} \; f(a/T)$ is an increasing function of the crack length. If the K-factor reaches its critical value K_{IC} (fracture toughness), the vessel fails in a brittle manner. This is illustrated in Fig. 1. This happens for a critical crack length a_c, which can be easily calculated from the above equation as follows

$$K_{IC} = p \sqrt{a_c} \; f(a_c/T) \; . \qquad \ldots\ldots\ldots\ldots \quad (2)$$

In this brittle case the critical crack size is lower than the wall thickness.

For low pressure levels and for materials with high fracture toughness, the fatigue crack can propagate through the full wall thickness without leading to a final brittle failure. Here the K-factor never reaches the value of fracture toughness. This is schematically depicted in Fig. 2. In this case, which is usually denoted as "leak-before-break" case, the critical value K_{IC} is not reached for any $a < T$.

For the given material and for the given geometry of the thick wall tubes there must be a transition between these two cases depending on the pressure level. We can define the transition pressure as a pressure at which the brittle fracture occurs just for the crack length equal to the wall thickness. In terms of equation 1 we get

$$K_{IC} = p_{trans} \sqrt{T} \; f(1) \; . \qquad \ldots\ldots\ldots\ldots \quad (3)$$

From the above discussion it is clear that for the working pressure above this transition value the final failure is of the brittle type and for the working pressure below this transition value the final failure is of the leak-before-break type. This point is schematically displayed in Fig. 3.

Another important quantity is the critical pressure. For each crack length there is always a critical pressure, at which the vessel would fail in the brittle manner. The value of the critical pressure is given by

$$K_{IC} = p_{crit} \sqrt{a} \; f(a/T) \; . \qquad \text{..........} \quad (4)$$

CYCLING BELOW TRANSITION PRESSURE

For the sake of simplicity, the case of two-level cycling as shown in Fig. 4 will be discussed. Moreover it is assumed that both pressure levels are below the transition pressure, i.e. $p_1 < p_{trans}$ and $p_2 < p_{trans}$. The crack starts to propagate at the depth a_o (crack caused by heat-checking) on the level p_1 . Altogether N_1 cycles are spent on this level and the resulting crack depth after the N_1 cycles is a_1. The crack propagation rate da/dN is given by the Paris law

$$da/dN = M \; \Delta K^m \; , \qquad \text{..........} \quad (5)$$

where M and m are material constants and ΔK is the stress intensity factor range. Integration of this equation gives

$$N_1 = \frac{1}{M \; \Delta p_1^m} \int_{a_o}^{a_1} \frac{da}{\left[\sqrt{a} \; f(a/T) \right]^m} \qquad \text{..........} \quad (6)$$

The number of cycles reguired for the complete failure at this pressure level is given by

$$N_{f1} = \frac{1}{M \; \Delta p_1^m} \int_{a_o}^{T} \frac{da}{\left[\sqrt{a} \; f(a/T) \right]^m} \qquad \text{..........} \quad (7)$$

For the second level p_2 analogically we can write

$$N_2 = \frac{1}{M \; \Delta p_2^m} \int_{a_1}^{T} \frac{da}{\left[\sqrt{a} \; f(a/T) \right]^m} \qquad \text{..........} \quad (8)$$

If the tube was cycled at the level p_2 from the very beginning (crack length a_o) till complete failure, the corresponding number of cycles would be

$$N_{f2} = \frac{1}{M \; \Delta p_2^m} \int_{a_o}^{T} \frac{da}{\left[\sqrt{a} \; f(a/T) \right]^m} \qquad \text{..........} \quad (9)$$

Rearranging the above equations into ratio form, we get

$$N_1/N_{f1} + N_2/N_{f2} = 1 \quad \text{.......... (10)}$$

This is the well known empirical Miner's rule, which has been used in fatigue calculations for many years, but without the theoretical derivation. It is easy to show that the rule can be enlarged to the multi-level case as

$$\sum_{i=1}^{r} N_i/N_{fi} = 1 \, , \quad \text{.......... (11)}$$

where r is the number of pressure levels.

GENERAL PROCEDURE

The above derived rule is in our case of thick walled tubes only good for the case, in which all pressure levels are below the transition pressure. If one or more pressure levels in a multiple-level test are above the transition pressure, the above rule cannot be used. This is due to the fact that the expression for N_f would contain the critical crack depth depending on the pressure level. The calculation of the life in this case requires a computerized procedure based on the following steps (a two-level case is shown):

1. At the pressure level p_1 the crack grows from the inital depth a_o to the depth a_1. At this pressure level the critical crack depth for the brittle fracture is a_{c1} and can be calculated from eq. 2. For the whole period of cycling it must hold $a_1 < a_{c1}$.
2. The value of the crack depth a_1 is calculated from eq. 6.
3. At this crack depth the change of pressure level is performed. First, the critical crack depth, at which the brittle failure would occur, must be calculated from eq. 2.
4. The remaining number of cycles at the level p_2 can then be easily calculated from eq. 6 using the integral limits a_1 and a_{c2}.

This procedure, requiring computation of the critical crack-depths at each level and computation of the successive crack increments at each level, will be called: the method of successive calculations.

EXAMPLES OF LIFE ESTIMATIONS

Example 1: Residual life after a decrease of the pressure

It is required to decide whether it is possible to use a vessel for a prolonged period using a lower pressure level. Measurements on standard laboratory specimens yielded the data on the fracture toughness and on the fatigue crack propagation rate:

$$K_{IC} = 160 \text{ MPa} \sqrt{m}\,, \qquad da/dN = 4.37 \times 10^{-12}\ \Delta K^{3.06},$$

The expression for the K-factor according to a previous work (Rajab (4)) for 30 star cracks is

$$K = p\sqrt{a}\left(\frac{0.471}{a/T + 0.102} + 0.679\right) \quad .... \quad (12)$$

The transition pressure according to eq. 3 is 550 MPa. It is assumed that the working pressure is 588 MPa. The value of the critical crack depth from eq. 2 (for T = = 0.07 m) is 0.301 m. The integration according to eq.6 (from a_o = 0.001 m to a_f = 0.031 m) gives the number of cycles: N = 1650.

Now, using the vessel only for 700 cycles at the level p_1 and by decreasing the pressure level from 588 MPa to 450 MPa, the remaining life will be calculated by the method of successive calculations:

1. For the given value N_1 = 700 , the crack depth must be found from eq. 6; we get a_1 = 0.106 m.
2. As the second pressure level lies below the transition pressure, the value of a_2 is given simply by a_2 = T . For the brittle case, it would be necessary to solve eq. 2 to get the final crack depth.
3. Eq. 6 yields (for a_1 = 0.0106 m , a_2 = 0.7 m) the value N_2 = 5300 cycles.

This result means that by decreasing the pressure from 588 MPa to 450 MPa, the residual life is increased more than 5 times.

Example 2: Residual life after an increase of the pressure

By using the same pressure levels but in opposite sequence the following results were obtained:

Total life at level 1: 10 700 cycles
Residual life at level 2 (after 5300 cycles at level 1): 1300 cycles.

It means that by increasing the pressure level from 450 MPa to 588 MPa the residual life is decreased more than 7 times.

CONCLUSION

The method presented in the paper makes it possible to compute the life of cyclically loaded thick walled tubes under condition of multiple-level loading. The practical importance of the procedure lies in the simple possibility to compute the residual life after each change of the pressure level.

SYMBOLS USED

a = crack length (m)

K = stress intensity factor (MPa $\sqrt{m}$)

K_{IC} = fracture toughness (MPa $\sqrt{m}$)

N = number of cycles

p = internal pressure (MPa)

T = wall thickness (m)

REFERENCES

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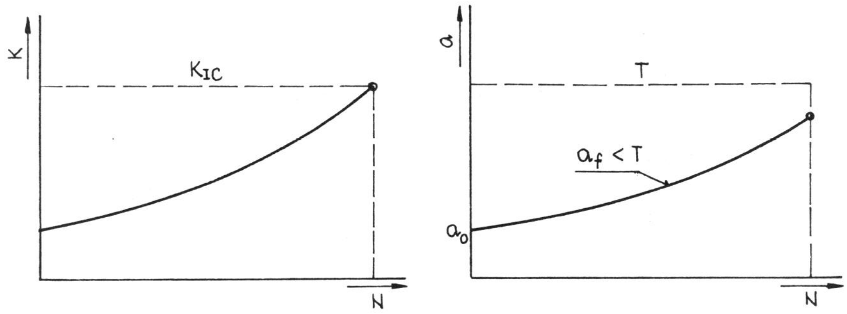

Fig. 1 Representation of brittle fracture

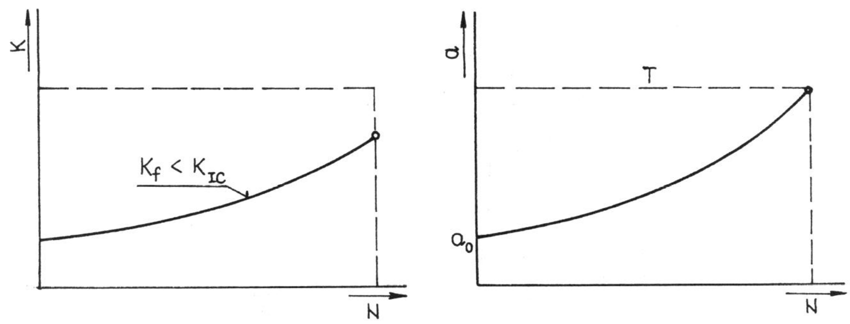

Fig. 2 Representation of leak-before-break

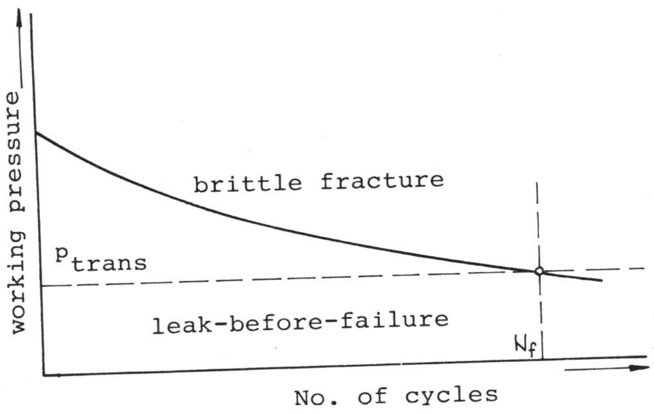

Fig. 3 Regions of the failure

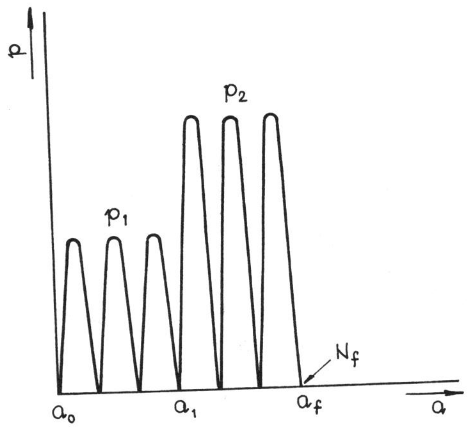

Fig. 4 Two level cycling