DAMAGE TOLERANT STRATEGY IN GAS TURBINE DISKS

J. Drexler and J. StateČný*

A probabilistic method of estimating the allowable low-cycle fatigue life of a gas turbine disk involving cracks in blade fir-tree attachments is presented using the damage tolerant (DT) concept. Combination of possible fractographic marks stands herein for the fleet size, thus enabling the fatigue life of the worst disk in the fleet to be estimated.

INTRODUCTION

Gas turbine disks operating under low-cycle fatigue conditions represent typical machine parts which may often be apt for being designed as damage tolerant (DT) ones. Considering disks involving a system of identical critical zones such as the blade fir-tree attachments, the problems being met are as follows:

i) an adequate description of a short crack system in respect of the disk reliability parameters,

ii) detecting a short crack at its first occurrence,

iii) estimating the DT life, i.e. the life to safe crack occurrence in the worst one of the cracked blade attachments on the disk being considered to be the worst in the fleet,

iv) establishing an effective inspection programme in the operating conditions to be expected.

* Aeronautical Research and Test Institute, Praha, Czechoslovakia.

Problems ad i) have been dealt with by Nemec & Drexler (1), those ad ii) and partly ad iii) by Drexler & Statečný (2), (3). Based on some experience as gained with a considerable population of small gas turbine engines, the authors attempt to contribute to the last two mentioned problems. It seems useful to organize the problem solution in four succeeding sections taking into account the disk airworthiness criterion as well as that of a reliable crack detection.

THE DISK AIRWORTHINESS CRITERION. THE CRITERION OF A RELIABLE CRACK DETECTION. CORRESPONDING LIFE ESTIMATION

Let N_F be the test number of cycles after which the crack in the worst one of the blade attachments has grown to a critical length L_F resulting in hazardous or catastrophic consequences to the airplane. Analogously, define N_{DT} as the number of cycles corresponding to the safe crack length L_{DT} in the blade attachment considered as the worst one in the disk. Assuming N_F, N_{DT} for being adequate characteristics of both the hazardous and allowable damages, one can express the disk airworthiness criterion in the form:

$$\text{Prob}\left[N_F/N_{DT}\right] \leq q \tag{1}$$

bearing in mind the random character of the damage done to the disk. The conditional probability:

$$\text{Prob}\left[N_F/N_{DT}\right] = \text{Prob}\left[\{x \leq N_F(L_F)\}/N_{DT}(L_{DT})\right] \tag{2}$$

relating to the occurrence of catastrophical number of cycles, N_F, at the moment the disk has accomplished the last one of N_{DT} admissible cycles, reflects an a priori experience, q being about 0.001, see Meece & Spaeth (4).

The aim of estimating the admissible number of cycles, N_{DT}, according to equation (1) corresponds with finding the joint occurrence probability for pairs of N_F, N_{DT} number of cycles as follows:

$$\text{Prob}\left[N_{DT} \cdot N_F\right] = \text{Prob}\left[N_{DT}/N_F\right] \cdot \text{Prob}\left[N_F\right] = $$
$$= \text{Prob}\left[N_F/N_{DT}\right] \cdot \text{Prob}\left[N_{DT}\right] \tag{3}$$

From equation (3) one gets:

$$\text{Prob}\left[N_{DT}/N_F\right] = \text{Prob}\left[N_F/N_{DT}\right] \cdot \frac{\text{Prob}\left[N_{DT}\right]}{\text{Prob}\left[N_F\right]} \qquad (4)$$

Using the theory of geometrical probabilities, one finds (cf. figure 1 for reference):

$$\text{Prob}\left[N_{DT}/N_F\right] \doteq \frac{N_{DT}}{N_F} \qquad (5)$$

Substituting equation (5) in equation (4) one obtains:

$$N_{DT} \doteq N_F \cdot q \cdot \frac{\text{Prob}\left[N_{DT}\right]}{\text{Prob}\left[N_F\right]} \qquad (6)$$

Equation (6) is of use in the case that the number of cycles N_F and the probability ratio (Prob $[N_{DT}]$ / Prob $[N_F]$) are obtained experimentally, as a result of cyclic fatigue tests on real disks.

There is an other condition to be satisfied for estimating the number of cycles N_{DT}, i.e. that of a reliable crack detection as given by the ratio:

$$\frac{N_F - N_{DT}}{N_{DT} - N_d} \geq 2 \qquad (7)$$

Substituting N_F from equation (7) in equation (6) one finds another useful expression for N_{DT} estimation, viz.:

$$N_{DT} \doteq \frac{2q \cdot N_d}{3q - \left(\text{Prob}\left[N_F\right] / \text{Prob}\left[N_{DT}\right]\right)} \qquad (8)$$

This holds in the case the value of the probability ratio (Prob $[N_{DT}]$ / Prob $[N_F]$) can be estimated for the operating fleet of disks on the basis of the field data. In this sense, the number of cycles N_d corresponds to the first crack detection in the worst one of the disks in the fleet.

Neither the N_{DT} estimate of equation (6) nor that of equation (8) accounts for possible intervening of other adverse damage mechanisms under disk operating conditions. Profiting of the wise B.C.A.R. recommendation, such an intervening - when lacking more information - will be covered by a factor k = 2, thus having:

$$N^*_{DT} = \frac{1}{k} \cdot N_{DT} \tag{9}$$

as a final result.

ESTIMATING THE (Prob $[N_F]$ / Prob $[N_{DT}]$) RATIO IN EQUATIONS (6) AND (8)

Let us now consider this ratio estimate, based on the disk cyclic fatigue test data only, referring to the whole fleet of M disks featuring the same material, technology as well as operating conditions.

The low-cycle fatigue test data obtained when investigating a single disk specimen are as follows: N_d, N_F, L_F as specified before, m the number of the fir-tree attachments on the disk, n_F standing for the attachment number being damaged by cracks at the moment the disk has achieved the last of N_F cycles. The fractographic mark referring to the n-th cracked attachment is denoted by Z_n.

Hence, the complex information obtained from low-cycle fatigue test data of the disk may be written as a compound statement:

$$N_d \cdot N_F \cdot L_F \cdot n_F \cdot (Z_1 + Z_2 + \ldots + Z_n + \ldots Z_\xi)$$

where a point (•) is denoting a logic product, the mark + a logic sum assuming either one or several or all possible fractographic marks Z_n may occur on the crack damaged attachments, cf. Figure 2.

The occurrence probability of such a complex information at the moment N_F, when the low-cycle fatigue test of the disk has been terminated, may be written in the form:

$$\text{Prob}\,[N_d \cdot N_F \cdot L_F \cdot n_F \cdot (\ldots Z_n \ldots)] =$$

$$\doteq \text{Prob}\,[(x \le N_d)\cdot(y \le N_F)\cdot(1 < L_F)\cdot(n \le n_F)\cdot(\mu \le Z)] \tag{10}$$

The probability of equation (10) involves desired information about Prob $[N_F]$ as well. To obtain it we can write:

$$\text{Prob}\left[N_d \cdot N_F \cdot L_F \cdot n_F \cdot (\ldots Z_n \ldots)\right] = \text{Prob}\left[(\ldots Z_n \ldots)/N_d \cdot N_F \cdot L_F \cdot n_F\right] \cdot$$

$$\cdot \text{Prob}\left[n_F/N_d \cdot N_F \cdot L_F\right] \cdot \text{Prob}\left[L_F/N_d \cdot N_F\right] \cdot \text{Prob}\left[N_d/N_F\right] \cdot \text{Prob}\left[N_F\right] \quad (11)$$

using the probability product rule.
Formally, the left hand side of equation (11), presenting the experimental distribution function as obtained from low-cycle fatigue test data processing, may be transcribed in a simple form:

$$\text{Prob}\left[N_d \cdot N_F \cdot L_F \cdot n_F \cdot (\ldots Z_n \ldots)\right] = 1 - \exp\left\{-\bar{\lambda} \cdot (N_F - N_o)\right\} =$$

$$\doteq \bar{\lambda} \cdot (N_F - N_o) \quad \text{for} \quad \bar{\lambda} \cdot (N_F - N_o) \ll 1 \quad (12)$$

where $\bar{\lambda}$ is standing for the mean crack occurrence rate in the disk attachments under the testing conditions. N_o is the threshold value of the number of cycles below which no crack in disk blade attachments appears, cf. both figures 1 and 2.

The conditional occurrence probability of Z_n marks is approximated by the relative frequency:

$$\text{Prob}\left[(\ldots Z_n \ldots)/N_d \cdot N_F \cdot L_F \cdot n_F\right] \doteq \frac{1}{m+1} \cdot \sum_{z=1}^{n_F} n_z \quad (13)$$

in which every one of Z_n marks refers to one crack damaged attachment only.

The estimate of the conditional probability $\text{Prob}[n_F/N_d \cdot N_F \cdot L_F]$ for the given attachment number m was already determined in ref. (3), i.e.:

$$\text{Prob}\left[n_F/N_d \cdot N_F \cdot L_F\right] = \sum_{n=0}^{n_F - 1} C_m^n \cdot p_a^n \cdot (1-p_a)^{m-n} \quad (14)$$

for one disk in the fleet; the probability of an arbitrary blade attachment to the disk being damaged by a crack is estimated therein by:

$$p_a = \frac{n_F}{m+1} \quad (15)$$

The conditional probability Prob $[L_F/N_d \cdot N_F]$ of the critical crack length occurrence in the worst one of the blade to disk attachments is to be obtained as follows:

$$\text{Prob}\ [L_F/N_d \cdot N_F] = \frac{n_F}{n_F+1} \tag{16}$$

because of the correlation between the numbers of cycles N_d, N_F with the number of cycles n_d, n_F respectively, (3). As mentioned for Prob $[N_{DT}/N_F]$ we can take analogously:

$$\text{Prob}\ [N_d/N_F] \doteq \frac{N_d}{N_F} \tag{17}$$

Finally, substituting equations (12) to (17) in (11), we find after some rearrangements:

$$\text{Prob}\ [N_F] \doteq \frac{\bar{\lambda} \cdot (N_F - N_o) \cdot N_F \cdot (n_F+1) \cdot (m+1)}{n_F^2 \cdot N_d \cdot \sum_{n=o}^{n_F-1} C_m^n \cdot p_a^n \cdot (1-p_a)^{m-n}} \tag{18}$$

as the result being aimed at.

Let equation (18) be interpreted from a physical point of view: it presents the estimate of an unconditioned occurrence probability of the number of cycles to failure, N_F, of the disk as a whole, i.e. for any arbitrary data $(N_d \cdot N_F \cdot L_F \cdot n_F \cdot (Z_1 + \ldots Z_\xi))$ combination, on the basis of a single fatigue test. In this estimate, the extent M of a disk fleet is substituted by the possible data $(N_d \cdot N_F \cdot L_F \cdot n_F \cdot (Z_1 + \ldots Z_\xi))$ combination number reflecting variance in material, technology and in operating conditions of the investigated disk type: practically, this combination number exceeds any real M number.

Now, let us come back to the problem of estimating the unconditioned probability Prob $[N_{DT}]$ of the occurrence of N_{DT} cycles. Proceeding analogously as in the preceding Prob $[N_F]$ case, we get:

$$\text{Prob}\ [N_{DT}] \doteq \frac{\bar{\lambda} \cdot (N_{DT} - N_o) \cdot N_{DT} \cdot (n_F+1) \cdot (m+1)}{n_{DT}^2 \cdot N_d \cdot \sum_{n=o}^{n_{DT}-1} C_m^n \cdot p_a^n \cdot (1-p_a)^{m-n}} \tag{19}$$

where n_{DT} is the allowable number of cracked attachments and the parameters $\bar{\lambda}$, N_d, m, n_F are identical to those in equation (18) because of investigating the same disk. It is obvious that the N_{DT} maximum is obtained in case of Prob $[N_{DT}] \to 1$. Equations (18) and (19) yield the probability ratio to be found, i.e.:

$$\frac{\text{Prob}\,[N_{DT}]}{\text{Prob}\,[N_F]} = \left(\frac{n_F}{n_{DT}}\right)^2 \cdot \frac{N_{DT}}{N_F} \cdot \frac{(N_{DT}-N_o)}{(N_F-N_o)} \cdot \text{Prob}\,[n_{DT}/n_F;m;M=1] \quad (20)$$

where:

$$\text{Prob}\,[n_{DT}/n_F;m;M=1] = \frac{\sum_{n=o}^{n_{DT}-1} C_m^n \cdot p_a^n \cdot (1-p_a)^{m-n}}{\sum_{n=o}^{n_F-1} C_m^n \cdot p_a^n \cdot (1-p_a)^{m-n}} \quad (21)$$

represents the probabilistic damage rate of the disk involving n_F cracked attachments from a total number of m attachments.

RESULTING ALGORITHM FOR ESTIMATING THE DT LOW-CYCLE FATIGUE LIFE OF THE DISK

Substituting equation (20) in equation (16) one gets after some manipulation the disk DT fatigue life in the form:

$$N_{DT} = N_o + \left\{\left(\frac{n_{DT}}{n_F}\right)^2 \cdot (N_F-N_o) \cdot \frac{\text{Prob}\,[n_{DT}/n_F;\ m;\ M=1]}{q}\right\} \quad (22)$$

where the conditions stated in equations (1), (7) and (9) should be satified.

When estimating N_{DT} from equation (22) one more condition as resulting from equations (1) and (21) is to be kept in mind:

$$\text{Prob}\,[n_{DT}/n_F;\ m;\ M=1] \le q \quad (23)$$

Taking the equality in equation (23), the maximum admissible value of N_{DT} is obtained, whereby a useful relation can be derived from equation (22), i.e.:

$$\left(\frac{n_c}{n_F}\right)_q = \sqrt{\frac{N_q - N_o}{N_F - N_o}} \tag{24}$$

expressing the relative number of cracked attachments $(n_c/n_F)_q$ as a function of the number of cycles N_q as a quantile curve with probability q.

CASE STUDY: ESTIMATION OF A DISK DT FATIGUE LIFE

A randomly chosen turbine disk of a small gas turbine engine (cf. figure 2) has been tested for low-cycle fatigue resistance using load programme cycle units. The test has been stopped at N = 18300 cycles, without failure of the disk as a whole. Fractographic marks of the cracked blade to disk attachments were the same as shown in Fig. 2. The sample of corresponding crack growth curves as well as the number of cracked attachments as a function of (N_q-N_o) are given in figure 3. Let us summarize the data resulting from this test as follows:

$N_o(L= 0)$ = 11500 cycles (obtained by extrapolation)

$N_d(L=L_d)$ = 12800 cycles; L_d = 0.41 mm, cf. (2) and (3)

$N_F(L=L_F)$ = 21000 cycles; $L_F \doteq 7.50$ mm (extrapolation)

n_F = 25 (by extrapolation)

m = 28

First, the allowable number of cracked attachments n_{DT} was estimated by using equation (23), cf. table 1. Next, the corresponding fatigue life was calculated from equation (24). The obtained N_{DT} estimate N_{DT} = 15892 does not satisfy in some extent the condition of equation (7) yielding 1.654 instead of 2.0 when taking n_{DT} = 17. Keeping in mind that the B.C.A.R. k-factor is rather conservative, we decide to take N^*_{DT} = 8000 test programme cycle units as a good result.

TABLE 1 - N_{DT} estimation for the disk from figure 2 on the basis of equation (23) testing $n_F = 25$.

n	Prob $[n_{DT}/n_F;\ m;\ M=1]$	q	$N_{DT}-N_o$	N_{DT}
16	5.748×10^{-5}			
17	3.005×10^{-4}	1.10^{-3}	4392	15892
18	1.137×10^{-3}			

$17 \leq n_{DT} \leq 18$

SYMBOLS USED

k	factor respecting the intervening of other adverse damage mechanisms than fatigue	[1]
L_{DT}	allowable (safe crack) length	[mm]
M	size of disk fleet	[1]
m	total number of blade attachments on the disk under investigation	[1]
N_o	threshold number of cycles below which no crack appears	[1]
N_d	number of cycles to first crack detection in the worst blade attachment to the disk	[1]
N_{DT}	damage tolerant life number of cycles estimate for the blade attachment in the fleet	[1]
N_F	number of cycles to critical crack length in the worst blade attachment to the disk	[1]
Prob [ ]	cumulative probability operator	
p_a	probability of an arbitrary blade attachment to be cracked	[1]
n_c	number of cracked blade attachments	[1]
n_{DT}	allowable number of cracked blade attachments	[1]
n_F	number of cracked blade attachments corresponding to the critical crack length occurrence	[1]
Z_n	symbol of a specific fractographic mark being found on the n-th cracked blade attachment	

REFERENCES

(1) Nemec, J. and Drexler, J., Advances in Fract. Res., Proc. ICF6, Vol. 3, 1984, pp. 1919-1926.

(2) Drexler, J. and Statečný, J., Advances in Fract. Res. Proc. ICF6, Vol. 5, 1984, pp. 3541-3548.

(3) Drexler, J. and Statečný J., Proc. ISABE, paper 83-7071, 1983.

(4) Meece, C.E. and Spaeth, C.E., Proc. AIAA, paper 70-1189, 1979.

(5) B.C.A.R., Section C, Issue 12, 29 August 1980, C.A.A. (explanatory note).

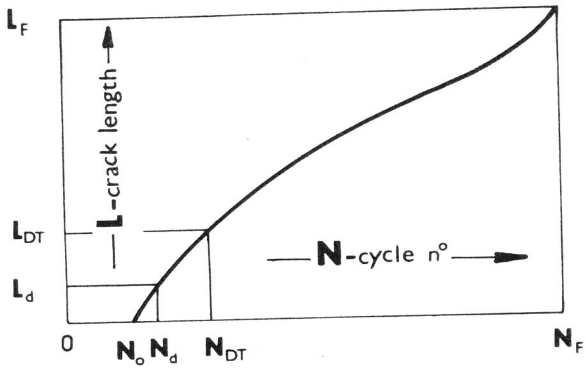

Figure 1. Basic parameters of the crack growth curve in the blade to disk attachment

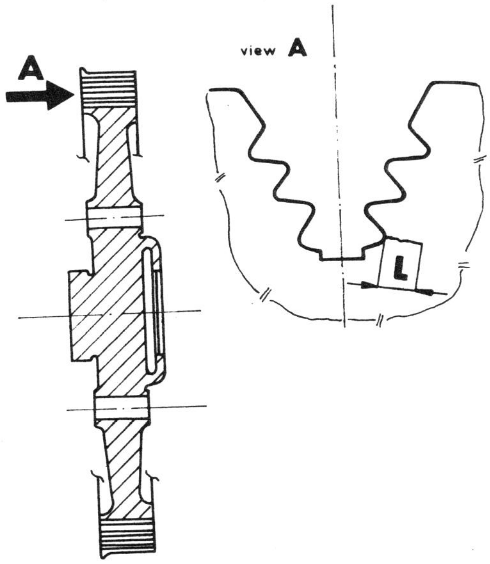

Figure 2. A simplified scheme of the critical area on the disk - the blade to disk fir-tree attachment

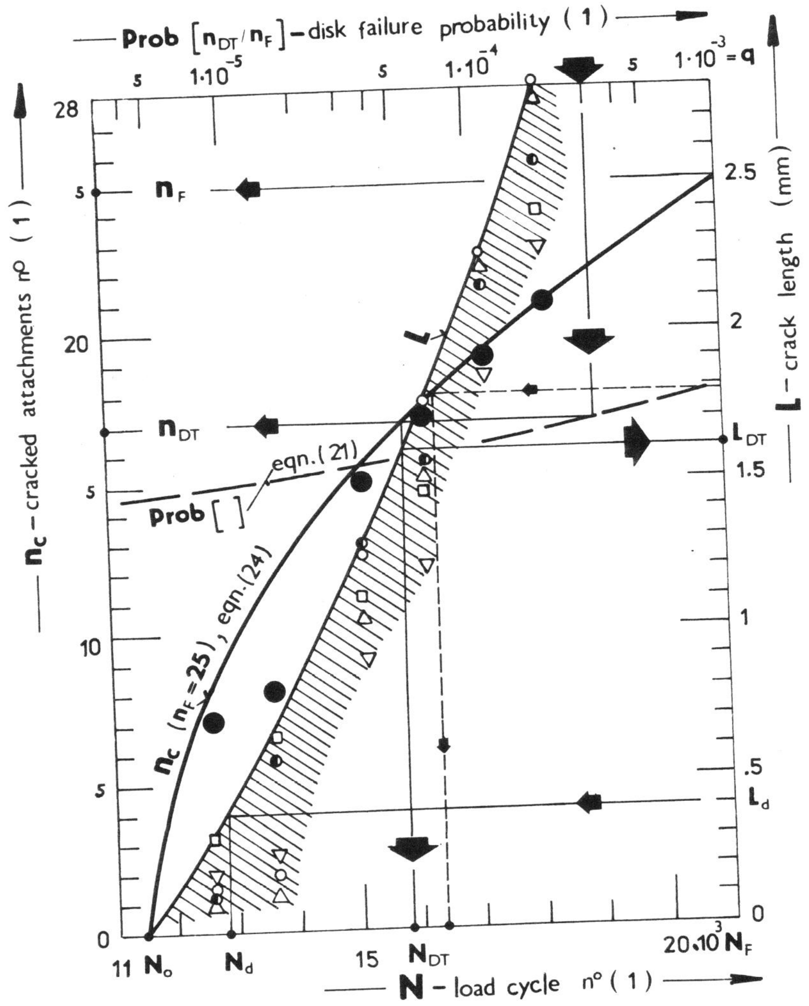

Figure 3. Cracking map used for estimating the DT life of a free turbine disk on the basis of a fatigue test