AN ASSESSMENT OF SOME ALTERNATIVE CTOD DESIGN EQUATIONS

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The results of a recent ECSC collaborative programme of fracture toughness and wide plate tests on a BS 4360 Grade 50D steel are used for a preliminary statistical assessment of some alternative crack tip opening displacement (CTOD) design equations. The test data involved plain material and cleavage fractures at -65°C.

INTRODUCTION

The CTOD design curve approach (1-3) has been widely used over the past fifteen years, and presently forms the basis of the elastic-plastic fracture mechanics treatment in BSI PD 6493:1980 (4). The primary objective of the CTOD design curve approach has been the determination of tolerable (maximum allowable) crack sizes, i.e. crack sizes that are smaller than the critical ones for fracture or plastic collapse. With the impending revision of PD 6493, there has been increasing interest in using more accurate and generally less conservative CTOD analyses, and as a consequence The Welding Institute has made several proposals. These are:

1. A finite width corrected CTOD design curve approach (5)

2. A plastic collapse modified strip yield model (6)

3. A reference stress model (7)

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To provide more versatile fracture assessment procedures, Anderson et al (8) considered the above proposals and suggested an approach involving three tiers of increasing accuracy. The lowest tier was based on the CTOD design curve with modifications for finite size and stress gradient effects, the second tier on a modified strip yield model, and the third tier on the above reference stress model.

This paper provides a preliminary statistical assessment of the above CTOD design approaches, and examines their relative conservatism when applied to unwelded steel plates. The assessment uses the median of a large population of critical CTOD values, obtained from standard notched bend tests, to predict the fracture behaviour in pre-cracked wide plate tension tests. The safety factors on predicted crack size in the wide plates are compared for each CTOD design approach. Consideration is also given to the probability of making safe predictions of fracture behaviour using the critical CTOD values from small sets of data instead of the median of a large population.

NOMENCLATURE

a	Depth of surface crack or half depth of buried crack, and, in the case of equation [4] only, also equal to $\bar{a}$
a_{crit}	Critical value of a
a_m	a_{max} or $\bar{a}_{max}$
a_{max}	Maximum allowable value of a
$\bar{a}$	Length or equivalent length of through-thickness edge crack or half-length of centred crack
$\bar{a}_{crit}$	Critical value of $\bar{a}$
$\bar{a}_{max}$	Maximum allowable value of $\bar{a}$
$\bar{a}_e$	Value of $\bar{a}$ with plastic zone size correction
B	Section thickness containing crack
c	Half-length of surface or buried crack
CTOD	Crack tip opening displacement
ε_{ref}	Strain at σ_{ref} in a uniaxial tensile test
E	Young's modulus
K_I	Mode I stress intensity factor

W Half-width of centre or double-edge cracked plate

δ CTOD

δ_{crit} Critical value of CTOD, i.e. δ_c, δ_u, or δ_m, according to BS 5762

σ Nominal or gross-section stress at infinity

σ_n Effective net section stress (Ref. 7)

σ_N Net section stress (nominal)

σ_p Effective primary stress = $K_I^p/\sqrt{\pi a}$

σ_{pc} Stress at plastic collapse

σ_{ref} Reference stress = σ_n

σ_s Effective secondary stress = $K_I^s/\sqrt{\pi a}$

σ_{TS} Tensile strength at temperature of interest

σ_{YS} Yield or 0.2% offset proof strength at temperature of interest

THE ALTERNATIVE CTOD DESIGN EQUATIONS

These are briefly described below, together with the specific equations that are appropriate to the present assessment.

Modifications to the CTOD Design Curve

A finite width corrected CTOD design curve approach proposed by Dawes (5) constitutes an extension of the earlier CTOD design curve approach (2,3). It equates the results of full section thickness CTOD tests with a CTOD design curve, which has been modified to allow for finite plate width, and also to distinguish between applications involving plain and welded plates. The resulting equations give the maximum allowable half lengths of through thickness cracks ($\bar{a}_{max}$) in flat plates in tension. Additional equations are then used to convert the $\bar{a}_{max}$ values to the equivalent allowable dimensions of cracks of different shapes in different situations. This modified CTOD design curve approach (5) can be used without a separate analysis for failure by tearing instability or plastic collapse.

For **plain** metals, the finite width modified CTOD design curve (5) can be re-arranged to give:

$$\bar{a}_{max} \leq \left[\frac{2\pi\,\sigma^2}{\sigma_{YS}\,\delta_{crit}E} + \frac{1}{W}\right]^{-1}, \qquad [1]$$

dependent on the restriction that $\sigma_N/\sigma_{YS} \leq 1.0$ and $\bar{a}/W \leq 0.5$

The appropriate equations for welded materials are given in Ref. (5).

Anderson et al (8) have proposed that the existing CTOD design curve (2) could be reformulated to include stress gradient effects. The reformulated design curve is given by the following equations.

$$\frac{\delta_{crit}E}{2\pi\,\sigma_{YS}\,a_m} = \left(\frac{\sigma_p + \sigma_s}{\sigma_{YS}}\right)^2, \text{ for } \frac{\sigma_p + \sigma_s}{\sigma_{YS}} \leq 0.5 \qquad [2a]$$

$$\frac{\delta_{crit}E}{2\pi\,\sigma_{YS}\,a_m} = \frac{\sigma_p + \sigma_s}{\sigma_{YS}} - 0.25, \text{ for } \frac{\sigma_p + \sigma_s}{\sigma_{YS}} > 0.5 \qquad [2b]$$

These equations are recommended for $\sigma_N/\sigma_{YS} \leq 0.85$ and constitute Level 1 in the three-tier approach (8).

Strip Yield Based CTOD Equations

A plastic collapse modification to Dugdale strip yield model proposed by Garwood (5) is intended to be used for estimating critical crack sizes. For the present application, the appropriate design equation is:

$$\delta_{crit} = \frac{8\,\sigma_{pc}^2\,\bar{a}_{crit}}{\pi\,E\,\sigma_{YS}}\ \ln\sec\left(\frac{\pi}{2}\frac{\sigma}{\sigma_{pc}}\right) \qquad [3]$$

where σ_{pc} is the assumed value of σ at plastic collapse and σ_{pc} is calculated by assuming that collapse occurs when the stress on the remaining section reaches the flow strength, i.e. $(\sigma_{YS} + \sigma_{TS})/2$.

Equation [3] applies only to the case of a uniformly loaded flat tension plate with no secondary stresses. For more general use, Eqn. [3] has been modified to incorporate both primary and secondary stresses, stress gradients and net section stresses (8). In this case the appropriate design equation is:

$$\delta_{crit} = \frac{\pi\,\sigma_{YS}\,a_{crit}}{E}\left[\frac{\sigma_p}{\sigma_n}\left(\frac{8}{\pi^2}\,\ell n\sec\left(\frac{\pi}{2}\frac{\sigma_n}{\sigma_{YS}}\right)\right)^{\frac{1}{2}} + \frac{\sigma_s}{\sigma_{YS}}\right]^2 \qquad [4]$$

This equation forms the basis for Level 2 in the Anderson et al three-tier approach (8).

The Reference Stress Method (7)

Equations [1], [2] and [4] are limited to net section yield and less, and equation [3] is limited to net section stresses less than the material flow strength. However, the reference stress method proposed by Anderson (7) can be used for stresses beyond the net section flow strength. For a uniformly loaded flat tension plate, and no secondary stresses, the basic design equation is:

$$\delta_{crit} = \frac{\pi \, \sigma_{YS} \, \bar{a}_{crit}}{E} \left(\frac{\sigma}{\sigma_{YS}}\right)^2 \left(\frac{\bar{a}_e}{\bar{a}_{crit}} + \frac{E\varepsilon_{ref}}{\sigma_{ref}} - 1\right), \qquad [5]$$

where $\bar{a}_e$ is the effective $\bar{a}_{crit}$ which includes a plastic zone size correction. The effective crack size is given by:

$$\bar{a}_e = \bar{a}_{crit} + \frac{1}{2\pi} \frac{(K_I)^2}{\sigma_{YS}^{\;2}} \frac{1}{(1 + \sigma_{ref}/\sigma_{YS})} \qquad [6]$$

In Ref. (8), Eqn. [5] has been modified to take stress gradients and secondary stresses into consideration. The "reference stress", σ_{ref}, in Eqns [5] and [6] is taken as the effective net section stress, σ_n. The strain at σ_{ref} in a uniaxial tensile test is termed the "reference strain", ε_{ref}, and the two are related to each other by the actual flow curve of the material.

In theory, this approach is more accurate than the strip yield based design equations, but it is also more complex. It requires more input data, and it is sensitive to the accuracy of the uniaxial tension stress/strain curve.

Calculation of Crack Sizes

Values of $\bar{a}_{max}$ may be obtained from Eqn. [1] directly, and if necessary, converted to a_{max} values using the diagrams and procedures in Reference (5). However, values of a_m may be calculated directly from Eqn. [2] only in the most simple cases of through thickness cracks in wide tension plates. For all other situations, an iterative procedure has to be used. Iterative procedures are also necessary to solve for the critical crack sizes in Eqn. [3], [4] and [5]. These require values of δ to be calculated for a range of assumed crack sizes. The allowable or critical crack sizes may then be inferred from the value of δ that is equal to δ_{crit}.

ASSESSMENT OF DESIGN EQUATIONS

The CTOD design approaches described above were compared with the experimental results of 102 full section thickness (52mm) BS 5762 (9) single edge notched bend (SENB), B x 2B section CTOD tests (Fig. 1), and 10 large scale wide plate tension tests (Fig. 2 and Table 1). These experimental data (10), from 14 co-operating laboratories, were obtained on a single BS 4360 Grade 50D steel plain plate material at -65°C. The uniaxial tensile test stress/strain data for the BS 4360 steel at -65°C gave a yield strength of $410 N/mm^2$ and a tensile strength of $635 N/mm^2$.

Comparison of Actual and Predicted Crack Sizes

The median critical CTOD from the SENB tests (0.29mm), and the fracture stresses for each wide plate test specimen (Table 1), were successively substituted into the different design equations to obtain predicted maximum allowable or critical crack sizes. These were then compared with the corresponding physical critical crack sizes in the wide plate test specimens. However, this was not a straightforward procedure for all the design equations, since all ten wide plates failed at net section stresses that were approximately equal to or greater than yield. This meant that:

a) the predicted maximum allowable crack sizes from Eqn. [1] were in all but one instance based on $\sigma_N = \sigma_{YS}$

b) it was not possible to obtain maximum allowable crack sizes from Eqn. [2] without exceeding the recommended limiting stress of $\sigma_N \leqslant 0.85\ \sigma_{YS}$

c) equation [4] could not be used at all, since the ln sec term goes to infinity as $\sigma \rightarrow \sigma_{YS}$

d) only Eqns [3] and [5] could be applied directly to the test data.

As a consequence of the above, Eqn. [4] was not assessed, and great care had to be taken in drawing conclusions from Eqns [1] and [2].

Figure 3(a) shows a comparison of the predicted maximum allowable crack sizes from Eqn. [1] and the physical crack sizes in the wide plate tests. Because nine out of the ten predicted crack sizes were for net section yield stresses, the physical crack sizes at these stresses were actually sub-critical, and the implied larger physical crack sizes for fracture at these stresses are indicated in Fig. 3(a) by vertical arrows. Thus, as illustrated in Fig. 3(a), the modified CTOD design curve approach, Eqn. [1], resulted in safe estimates of the **maximum allowable** through thickness and surface crack sizes in all 10 plain material

Table 1 Details and results of wide plate tension tests at −65°C.

Specimen type	Specimen No.	Plate width 2W, mm	Critical crack depth a_{crit}, mm	Critical crack length, $2c_{crit}$ or a_{crit}, mm.	Gross section fracture stress, σ, N/mm^2	Net section fracture stresses, σ_N N/mm^2
Surface notched	1	700	21	81	404	419 C
	2	700	26	186	387	432 C
	3	700	16	700	275	397 C
	4	700	21	179	385	418 U
	5	700	16	66	442	454 U
Through thickness notched	6	644	-	27	380	415 U
	7	695	-	15.3	399	415 C
	8	700	-	18.6	396	419 U
	9	698	-	51.6	367	430 U
	10	693	-	35.6	384	426 U

C = failure by cleavage fracture with no prior stable crack growth
U = failure by cleavage fracture with prior stable crack growth

wide plate tests, even though these fractured at stresses approximately equal to or greater than net section yield. This is an important observation, since the theoretical safety factors on crack size in this approach diminish with increasing stresses up to the maximum allowable values of net section yield (5).

Figures 3(b) and (c) show that, on average, Eqns [3] and [5] predict slightly larger and therefore unconservative **critical** crack sizes compared to those in the wide plate tests.

TABLE 2 **Safety Factors on Predicted Crack Sizes** (Based on median critical CTOD value and wide plate fracture stresses, all approximately equal to net section yield)

Wide Plate Test Type	Specimen No.	$\frac{a_{crit}}{\text{predicted value}}$ or $\frac{\bar{a}_{crit}}{\text{predicted value}}$			
		Eqn. [1]*	Eqn. [2b]**	Eqn. [3]	Eqn. [5]
Surface Notched	1	>1.78	0.78	0.73	0.72
	2	>1.93	1.35	1.11	1.13
	3	1.00	1.06	0.81	0.77
	4	>1.53	1.22	0.93	0.94
	5	>1.24	0.64	0.54	0.70
Through Notched	6	>1.14	0.77	0.75	0.69
	7	>1.65	0.46	0.49	0.46
	8	>1.55	0.55	0.58	0.55
	9	>1.90	1.40	1.29	1.17
	10	>1.62	1.02	1.02	0.94
All Types	Average safety factor on crack size	>1.53	0.92	0.83	0.81

* Predicted a_{max} or $\bar{a}_{max}$; in all but one instance limited by $\sigma_N = \sigma_{YS}$

** Predicted a_m, but ignoring the recommendation for $\sigma_N/\sigma_{YS} < 0.85$

More quantitative comparisons between the predicted and physical critical crack sizes are summarized in Table 2. Here the comparisons are given as ratios of the physical crack sizes in the wide plate tests to the predicted crack sizes. Table 2 includes crack size ratios for Eqn. [2b]. Although this equation is not recommended (8) for values of $\sigma_N > 0.85\ \sigma_{YS}$, it was used in this assessment in order to check the recommendation. This is clearly justified by the fact that, for the stresses around net section yield, the average predicted **maximum allowable** crack sizes from Eqn. [2(b)] were greater than the physical crack sizes i.e. the average crack size ratios were less than unity, and therefore unsafe.

Comparing the crack size ratios in Table 2, it can be seen that only Eqn. [1] gives average factors of safety on predicted crack size that are greater than unity. However, a more appropriate comparison of the predictions of Eqns [1], [3] and [5] may be obtained by dividing the predicted critical crack sizes from Eqns [3] and [5] by a factor of 2.0, i.e. to define **maximum allowable** crack sizes for direct comparison with those obtained from Eqn. [1]. The factor of 2.0 on crack size was chosen since it was consistent with the background (5) to Eqn. [1]. The effect of introducing a factor of safety of 2 in Eqns [3] and [5] is to double the safety factors given in Table 2. Thus the average safety factors obtained from the two equations become 1.66 and 1.62 respectively. However it is not clear whether these factors are higher than the corresponding value of >1.53 obtained from Eqn. [1]. To shed more light on the application of the different equations, the wide plate and CTOD test data were re-analysed using a statistical approach similar to that used by Towers, et al (10).

Probability of Predicting the Failure Conditions Safely

For this analysis, the actual crack sizes and fracture stresses for each wide plate test specimen were substituted into Eqns [1], [3] and [5] in order to calculate the implied or predicted CTOD values at failure. These values were compared with the full distribution of CTOD test (BS 5762) results (Fig. 1). Calculations were then performed to give the probabilities of the implied critical CTOD values in the wide plate tests being greater than the minimum of three randomly selected CTOD values from the full distribution of CTOD test results. An example of this analysis procedure is illustrated in Fig. 4.

The calculated values represent the probabilities of predicting safe combinations of stresses and crack sizes when using the minimum of three CTOD test values randomly selected from a large population; which is a common basis for CTOD based approaches to design (11).

The results of the probability calculations are summarised in Table 3. The last two columns in Table 3 were obtained after modifying Eqns [3] and [5] to give allowable instead of critical conditions, thereby making the equations directly comparable with Eqn. [1]. As in the previous section (concerning predicted crack sizes), the modification was to put a factor of safety of 2.0 on the initial crack size. Therefore it was assumed that the crack sizes in the wide plate test specimens were double the actual sizes. This had the effect of increasing the implied critical CTOD, and increasing the probability of the minimum of any three measured CTOD values being less than the implied values. As shown in Table 3, this factor of safety of 2.0 on initial crack size resulted in the collapse modified strip yield model, Eqn. [3], giving possibly the highest average probability of safe predictions at net section stresses approximately equal to yield. The fact that the reference stress model gave a slightly lower probability of safe predictions at this stress level was due to the particular method of considering a continuous surface crack (specimen 3 in Table 3), the particular small scale uniaxial tensile test stress/strain data that was used, and the fact that this model gives a more accurate estimate of the critical conditions. Thus, it would seem that, in order to use the references stress model (Eqn. [5]) to determine a higher probability of safe predictions than the plastic collapse modified model (Eqn. [3]), it would be appropriate to assume a correspondingly higher factor of safety on crack size.

TABLE 3 Probabilities of Predicting Fracture Conditions Safely at Net Section Stresses Approximately Equal to Yield.

	$Pr(\delta_{3min} < \delta_{calculated})$				
Spec. No.	Eqn. [1]	Eqn. [3]	Eqn. [5]	Eqn. [3]*	Eqn. [5]*
1	> 0.82	0.66	0.61	0.931	0.92
2	> 0.986	0.931	0.91	0.997	0.994
3	0.83	0.58	0.25	0.90	0.69
4	> 0.941	0.82	0.81	0.978	0.978
5	> 0.76	0.71	0.66	0.955	0.931
6	> 0.89	0.74	0.69	0.96	0.96
7	> 0.69	0.45	0.37	0.83	0.81
8	> 0.76	0.58	0.53	0.90	0.87
9	> 0.992	0.953	0.926	0.998	0.996
10	> 0.967	0.86	0.84	0.991	0.986
Average	= >0.86	= 0.73	= 0.66	= 0.94	= 0.91

* Modified by substituting $2\bar{a}_{crit}$ for $\bar{a}_{crit}$

As implied by the previous section (concerning predicted crack sizes) the probabilities of safe predictions from the modified CTOD design curve approach, Eqn. [1], are likely to be even greater than the tabulated values (Table 3) at stresses significantly below net section yield, especially when the crack sizes are relatively large compared to those in the present wide plate test specimens (5).

DISCUSSION AND CONCLUSIONS

Because the wide plate test data in the present analysis involved net section cleavage fracture stresses that were approximately equal to or greater than yield, it was not possible to investigate the relative conservatism of the different assessment levels in the Anderson et al (8) three tier approach. This was due to the fact that the Level 1 approach, Eqn. [2b], is only recommended for net section stresses less than 85% of the material yield strength, and the Level 2 approach, Eqn. [4], predicts infinitely small crack sizes as the effective net section stresses approach yield. Nevertheless, some calculations were carried out using Eqn. [2b], and in five out of ten wide plate tests, these gave unsafe estimates of maximum allowable crack sizes, thereby justifying the recommended limits on net section stresses for this equation.

For the present test data and analyses, it was shown that Eqn. [1], from the finite width modified CTOD design curve approach (5), is more conservative than both Eqn. [3], the plastic collapse modified strip yield model (6), and Eqn. [5] the reference stress model (7); which also constitutes Level 3 in the Anderson et al three tier approach (8).

It is important to note that the above observations are in general agreement with mathematical comparisons of the equations, which, for primary design stresses below approximately 85% of yield, indicate that the equations become less and less conservative in the order Eqn. [1], Eqn. [2], Eqn. [4], Eqn. [3] and Eqn. [5]. Indeed, it will be recalled that Eqns [1] and [2] are concerned with 'allowable' conditions, and Eqns [3] - [5] with 'critical' conditions. This was the reason why the critical equations were factored before using them to predict allowable conditions. For example, when a theoretical factor of safety of two on crack size was chosen for the present analyses, and the minimum of 3 randomly selected CTOD values was used, the probabilities of predicting safe conditions to avoid fracture increased to 0.94 and 0.91, for Eqns [3] and [5], respectively. Unfortunately, these can not be directly compared with the corresponding probability of >0.86 from Eqn. [1]. Nevertheless, the present probabilities were determined for failure stresses approximately equal to net section yield, and higher probabilities of safe predictions would be expected from Eqn. [1] at lower net section stresses. Indeed, a corresponding analysis by Kamath (11), using the earlier and slightly less conservative CTOD design curve (2,3) and data from 28 plain material wide plate tests covering a wide range of failure stresses, indicated a 0.96 probability of safe predictions of failure conditions.

It is concluded that, for stresses up to net section yield, there is a high probability of avoiding fracture when using Eqn. [1] and the minimum of three randomly selected critical CTOD values. The same conclusion applies to Eqns. [3] and [5], except that in these instances the equations may also be applied to service applications involving net section stresses greater than yield.

The use of a minimum value from a set of data containing three or more CTOD values could be very misleading. This is because as more results become available, the overall range of the values obtained will increase, and in particular it is possible that the lowest value will drop. This can lead to increasing conservatism which may not be required. An alternative approach, proposed by Williams and Jutla (12), is to use an intermediate value of CTOD from a larger data set; which is at least equivalent to the minimum of three random tests. In this way it may be possible to avoid using an exceptionally low minimum value of CTOD, while maintaining a high probability of safe predictions. Therefore, when an extremely low CTOD value is obtained in a set of three results, there would be an advantage in carrying out re-tests to increase the sample size, as suggested in Ref. 12.

Because the present comparison of alternative CTOD design equations was limited to a consideration of plain materials, further comparisons are in hand for the more complex situations in weldments. These are expected to pose a particularly difficult problem for the application of Eqn. [5], i.e. since this is sensitive to the shape of the assumed tensile stress/strain behaviour, which normally varies significantly with position within a welded joint.

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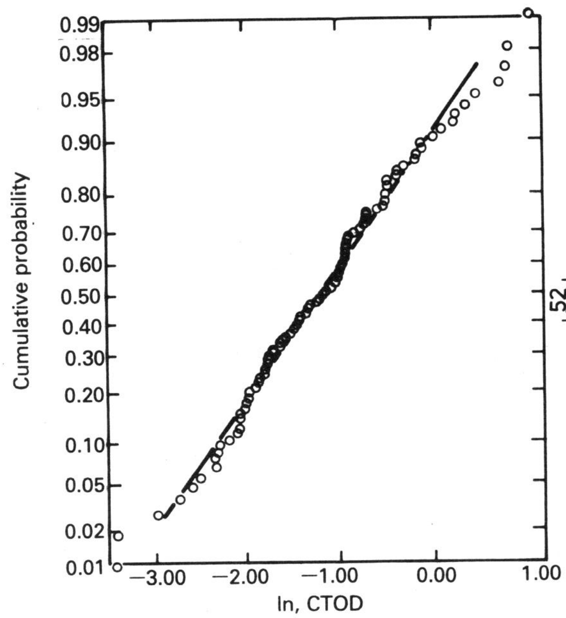

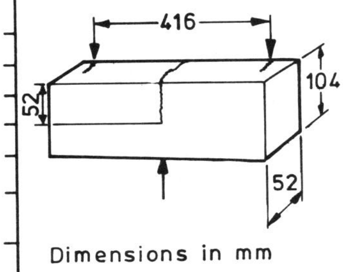

Fig.1. Critical CTOD values for the BS 4360 Grade 50D steel at −65°C obtained from 14 laboratories (Ref.10.).

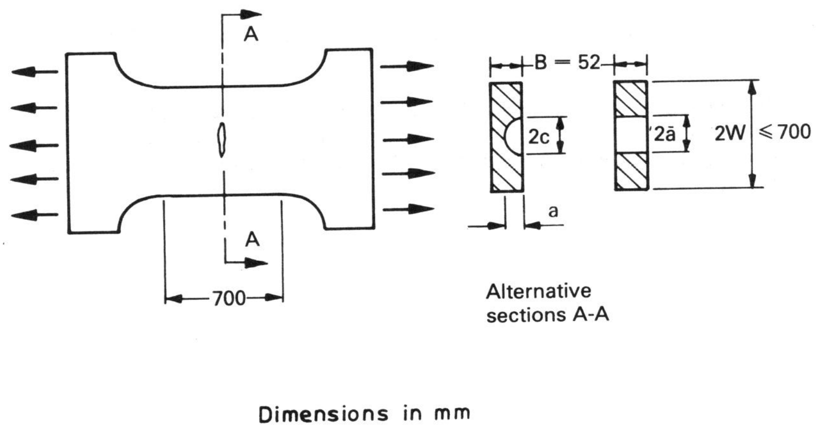

Fig.2. Overall dimensions of wide plate tension test specimens (cf. Table 1).

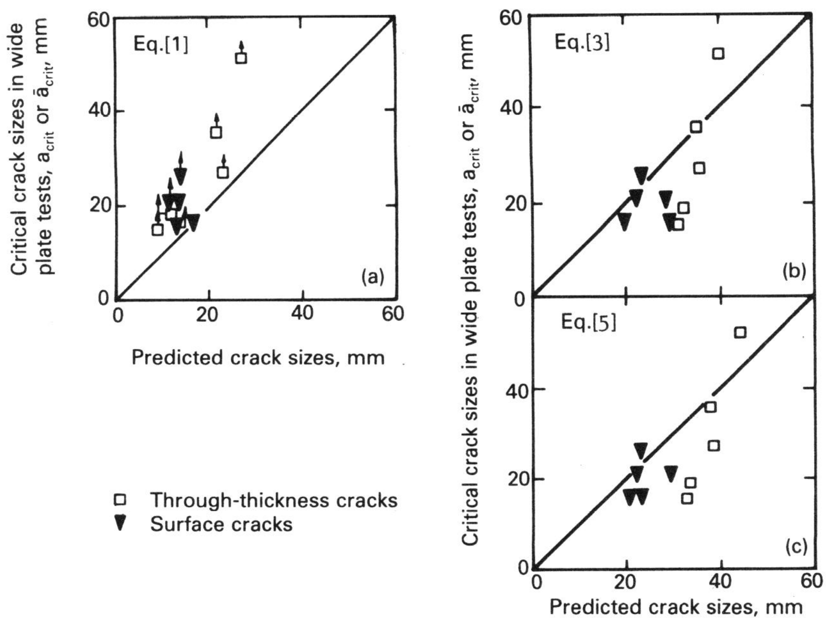

Fig.3. Comparison of critical crack sizes in wide plate tests and predicted crack sizes.

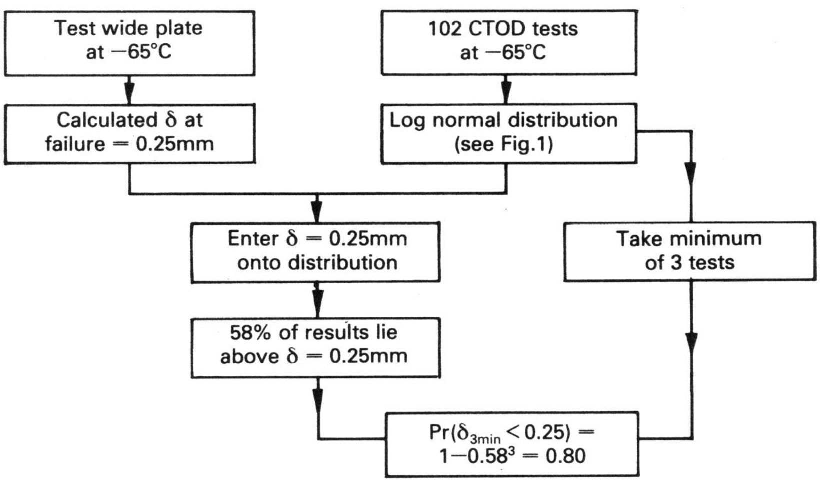

Fig.4. Example of the procedure for calculating the probability of safe predictions of fracture conditions.