J ASTRAY AND BACK TO NORMALCY

David Broek*

Complicated J-estimation schemes are shown to be artificially elusive. They can be dramatically simplified, becoming almost trivial, by eliminating the artificialities. This results in a "new" and simple estimation scheme. New between quotation marks, because the essence of this and other schemes was published already in 1972 (Liebowitz and Eftis), but never given recognition.

INTRODUCTION

The J-integral is useful for fracture analysis of structures if it is possible to express J as a function of crack size and applied stress. Only in that case can the equation $J = J_R$ be solved for a certain crack size to obtain the fracture stress of a structure. Several such expressions have been proposed; since they are obtained indirectly, they are generally referred to as estimation schemes.

The oldest and best known estimation scheme is the one used in the ASTM specification for J_R testing. This scheme is useful because it permits generation of J_R curves from tests on simple specimens. However, it cannot be applied to structures using the reverse operation - obtaining the stress from J_R.

* FractuREsearch Inc. Colombus, Ohio, USA.

Paris and Tada (1) used the idea of expressing J in terms of the area under the load-displacement diagram. This can be applied to obtain the fracture stress if the load-displacement diagram for the structure is known, and can be expressed in mathematical form. Paris and Tada (1) developed an approximate procedure to estimate the shape of the diagram. The main difficulty with this scheme is that the displacement should be the one due to the crack only. This condition is almost automatically fulfilled in the case of a compact tension specimen because the load is applied close to the crack, so that practically all displacements are due to the crack. Estimations of the displacement due to the crack in a structure require rather coarse assumptions.

Kumar et al. (2), (3) developed a rational estimation scheme on the basis of a mathematical expression for the stress-strain curve, the Ramberg-Osgood equation. The resulting expression for J contains a geometry parameter that can be obtained for essentially any structure by means of finite element analysis in much the same way as the geometry parameter in the stress intensity factor. As Kumar et al. (2), (3) obtained geometry factors for a limited number of geometries, the procedure presently has applicability.

It is somewhat unfortunate that Kumar et al. (2), (3) elected to normalize with respect to an arbitrary flow stress and associated limit load, which has led to an elusive expression for J. It will be shown in this paper that (trivially) this expression can be much simplified, and more important, follows in the same form from Neuber's rule. An interesting consequence is that the geometry factors in J can be related to those in K, which leads to a new and simple estimation scheme.

THE ESTIMATION SCHEME BY KUMAR et al. (2), (3)
STRESS-STRAIN CURVE AND G

The Ramberg-Osgood equation consists of a linear part (Hooke's law) and an exponential part representing the plastic strain. In its simplest form the equation is written as:

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \frac{\sigma^n}{F} \qquad (1)$$

In a log-log plot of stress and strain (figure 1), the curve has two asymptotes, one representing the elastic strain, the other the plastic strain. The slope of the first is equal to 1 (elastic) and the intercept with the abscissa is log (1/E). The slope of the second is n, the intercept with the abscissa log (1/F). Note that three parameters E, F and n are needed and no more than three.

The plastic strain is given as:

$$\varepsilon_p = \frac{\sigma^n}{F} \qquad (2)$$

From a log-log plot of stress versus plastic strain the parameters F and n are obtained as slope and intercept of a straight line fitted through the data, as shown in figure 2. (In the original Ramberg-Osgood equation $F^{1/n}$ was called the "flow stress"; this definition of flow stress has some physical significance, because it is the stress for which the strain is unity).

Kumar et al. (2), (3) normalized the equation to:

$$\varepsilon_p = \varepsilon_o \, \alpha \left(\frac{\sigma}{\sigma_o}\right)^n \qquad (3)$$

by introducing a reference stress and a reference strain. Instead of two, the equation has now three parameters, two of which must necessarily be dependent. The reference stress must necessarily be dependent. The reference stress must be related to the reference strain by Hooke's law. A comparison of equations (2) and (3) shows that:

$$\alpha = \frac{\sigma_o^n}{\varepsilon_o F} \qquad (4)$$

Since, apart from n, the curve is governed exclusively by F, it is permissible to use any value for the reference stress (the reference strain follows from Hooke's law), and then determine α from F by means of equation (4). Calling the reference stress the "flow" stress is misleading, because it suggests that an arbitrary reference stress has a certain physical meaning.

For the case that n = 1, equation (2) reverts to the elastic equation. For that situation J is already known to be $J=G=K^2/E$. The expression for the stress intensity is:

$$K = \beta \, \sigma \sqrt{\pi a} \qquad (5)$$

so that J in the elastic case is given by:

$$J = \beta^2 \, \pi \, \sigma \, \varepsilon \, a = \pi \, \beta^2 \, \sigma^2 \, a/E \qquad (6)$$

THE "PLASTIC" J

For a material with an exponential stress-strain curve Kumar et al. (2), (3) obtained the following expression for J:

$$J = \alpha \, \sigma_o \, \varepsilon_o \, \frac{c}{a} \, a \, h_1 \left(\frac{P}{P_o}\right)^{n+1} \tag{7}$$

where h_1 is the function of geometry (and n) already referred to in the INTRODUCTION. Further, c represents the uncracked ligament, P the applied load and P_o the "limit" load based on the reference stress. Since the reference stress is arbitrary, P_o is not a "true" limit load.

Obviously:

$$\begin{aligned} P &= f(\text{geom}) \times \sigma \\ P_o &= g(\text{geom}) \times \sigma_o \\ c/a &= k(\text{geom}) \end{aligned} \tag{8}$$

where f, g and k are functions of geometry (and a).

Substitution of equations (4) and (8) in (7) provides:

$$J = \frac{\sigma_o^{\,n}}{\varepsilon_o \, F} \, \sigma_o \, \varepsilon_o \, k \, a \, h_1 \left(\frac{f \times \sigma}{g \times \sigma_o}\right)^{n+1} \tag{9}$$

which reduces to:

$$J = \frac{1}{F} \, \frac{k \, h_1 \, f^{n+1}}{g^{n+1}} \, a \, \sigma^{n+1} = \lambda \, \sigma \, \varepsilon \, a \tag{10}$$

with:

$$\lambda = \frac{k \, h_1 \, f^{n+1}}{g^{n+1}} \tag{11}$$

SIGNIFICANCE

The equation for J proposed by Kumar et al. (2), (3) gives the false impression that J depends upon the limit load. Naturally, J cannot depend upon an arbitrary limit load or on any limit load. Because the exponential equation for the stress-strain curve is unlimited (as is the linear equation), a limit load cannot enter the problem. Equation (10) shows that the limit load can be divided out indeed; it does not affect the equation.

Why then does λ in equation (10) depend upon the geometry function g for the limit load through equations (11)? Also this is artificial , because also h_1 depends upon this function since equation (9) was used to derive h_1 from the finite element results. Hence, that effect will be divided out as well. Had equation (10) been used to derive λ, the effect would not have appeared.

As a matter of fact it is easy to use equation (8) in the expression for G, which then becomes:

$$G = \beta^2 \pi \frac{g^2}{f^2} \sigma_o \varepsilon_o a \left(\frac{P}{P_o}\right)^2 = \sigma_o \varepsilon_o a \gamma \left(\frac{P}{P_o}\right)^2 \qquad (12)$$

with:

$$\gamma = \beta^2 \pi \frac{g^2}{f^2} \qquad (13)$$

Now G seemingly depends upon a limit load. But equation (13) is the same as equation (6). If equation (13) is used the limit load will be divided out automatically.

Obviously, the equation (10) for J would be much easier to use than the elusive equation (7), had λ been calculated. Only one geometry parameter would be necessary. Instead, four geometry functions must now be used, namely h_1, f, g and k. Of course any user can, once and for all, calculate λ from the given h_1, f, g and k, and from then on use the much simpler equation (10).

A NEW ESTIMATION SCHEME

Equations (6) and (10) are repeated below:

$$J = \pi \beta^2 \sigma^2 a/E \qquad (14)$$

$$J = \lambda \sigma^{n+1} a/F \qquad (15)$$

Clearly, the two are identical. For the second equation to be valid, it must be valid for the case that n = 1 and E = F, which is obviously the case. It implies that:

$$\lambda(\text{geom},a,n) = (\beta\sqrt{\pi})^{n+1} \text{ or may be } \lambda = \pi(\beta)^{n+1} \qquad (16)$$

This equation opens a new perspective. It provides a means to estimate λ (or J) directly from the (already known) geometry functions for K. No non-linear finite element analysis is necessary. A new and very simple estimation scheme is available if its use can be demonstrated to provide accurate predictions of fracture stress. Although equation (16) may be questionable from a theoretical point of view, it may serve as a basis for an estimation scheme.

For cases with known β and h_1, λ can be obtained in two ways: with equation (11) and with equation (6). Good agreement was obtained for many cases, poor agreement in others. However, it is not the value of λ (i.e. J) that is important; the crucial question is how well the scheme would predict fracture stresses. Examples of predictions will be shown in the next section.

Confirmation of the connection between linear elastic (K) and non-linear (J) parameters can be obtained by using Neuber's rule. Neuber's rule essentially states that the product of local stress and local strain at a notch after local yielding is equal to the product of local elastic stress and local elastic strain obtained from purely elastic considerations (with the same applied or remote stress in the two cases).

Applying this to the crack tip, one obtains:

$$\sigma_1 \varepsilon_1 = \frac{q K^2}{2\pi E r} \qquad (17)$$

where q is a numerical factor depending upon Poisson's ratio. By using the exponential stress-strain relation this can be modified to:

$$\sigma_1 = B\left(\frac{K^2}{2\pi E r}\right)^{1/n+1} \qquad (18)$$

where B depends upon q and n.

Equation (18) is indeed the equation for the crack tip stress field as obtained on the basis of J for non-linear stress-strain curves. In that case J would appear in the numerator, but otherwise the equation would be the same. Then J and K are relatable through n, or rather λ and β (the geometry factors) are relatable through n, which is what equation (16) attempts.

Giving credit where credit is due, it should be mentioned that already in 1972, Liebowitz and Eftis (4) published a paper on the effects of non-linearity. Essentially, equation (15) as well as the Paris and Tada estimation scheme (1) follow from the work of Liebowitz and Eftis (4). A useful J-estimation scheme was available 15 years ago. Instead the paper (4) was ignored (not by the author of the present paper, but regretfully, he failed to see the implications).

APPLICATION

Fracture stress predictions (maximum load) were made using both the scheme by Kumar et al. (2), (3) and the new scheme. For comparison, test data reported by Kanninen et al (5) were employed. These are for SS304 center cracked panels. Predicted stress-crack size curves from both schemes are shown in figure 3. These curves show crack growth initiation and maximum stress (instability if load control) in comparison with actual test data (5). Figure 4 shows a comparison of the calculated J. Fracture stress predictions for pipes with circumferential cracks in bending are shown in figure 5. The β used for this case is the one obtained by Erdogan and Kibler (6); the test data shown in comparison, stem from Kanninen et al. (6). Figure 6 more clearly shows the predicted amount of stable crack growth up till maximum load (load control analysis); the amount of stable growth predicted at maximum load is larger than in the tests.

A more thorough evaluation of the new scheme is necessary. However, the scheme is a by-product. The main purpose of the above discussion was to demonstrate the stripped down expression for J. Also, the actual estimation scheme used is not of much practical importance, as will be demonstrated below. It is the value of n that is the biggest driver in the accuracy of predictions.

ACCURACY

There is often much concern about the very large variations occurring in J and (consequently) J_R. The reason for the large variability is obvious: in the calculation of J the stress is taken to the (n+1)th power. Hence, a slight error in stress of 5%, with e.g. n=9, leads to a difference of $(1.05)^{10} = 1.63$, i.e. a difference of 63% in J. (Note that the same occurs with other J-estimation schemes, that determine J from the load-displacement diagram: where the load-displacement diagram becomes almost horizontal, the value of J changes dramatically with a slight change in load or stress).

This may be bothersome for researchers, but it is of little practical importance. An engineer could care less about the value of J, as long as the procedures predict the stress or load a structure can carry. This is, of course, the saving grace in practical applications. If one calculates a fracture stress or load, the problem discussed in the previous paragraph works to our advantage: a difference of 63% in J_R with n = 9 will lead to only a difference of 5 % in the predicted fracture stress or load ($1.63^{1/10} = 1.05$). If the elastic part of J is small with respect to the plastic part, the stress for crack growth or fracture follows from $(a/J_R^{(n+1)})$.

If the elastic J is not negligible, the stress is obtained by iterative solution, but the dependence on n is still strong.

For a difference in J_R by a factor of 2, and for n = 7, the fracture stresses would be different by $2^{1/8} = 1.09$; hence the error (difference) would be 9% only. This is clearly demonstrated in figures 7 and 8. These show two runs with exactly the same input, with n = 7. Two J_R-curves were used with differences in J_R of approximately a factor of 2 throughout. The scheme of Kumar et al. (2), (3) was used to obtain the predictions in figure 8. Clearly, the predicted fracture stresses differ only by a small amount. In general the stresses in a structure will not be known with better accuracy, so that any of the predictions in figure 8 would be satisfactory from an engineering point of view. Even the predicted amounts of stable crack growth at maximum load do not differ very much, as shown in figures 9 and 10.

Indeed, in particular for larger n, the value of J_R, and hence the calculated J, are of little influence on the predictions of fracture stress (which is the purpose of the analysis). Consequently, it does not matter very much which estimation scheme is used. Also, comparison of estimation schemes on the basis of how well they predict J, are rather meaningless, even if the schemes provide largely different J. The important question is how well the schemes predict fracture stresses. This depends much less on how J is calculated than on the value of n. For the same J (J_R) a difference of 10% in n can indeed have considerable effect on the calculated stress. However, n is not a parameter that can be adjusted or improved: its value is dictated by the material.

In view of the above, a final note is in place concerning the futility of sensitivity studies. Clearly, the results of the Kumar et al. (2), (3) scheme are insensitive to the choice of reference stress and reference strain, because both of these will be divided out. Remain the sensitivity to α and n.

Naturally, one can play analytical games varying α and n, but these are meaningless. After all, fracture prediction is not an analytical game: it deals with real materials. Both α and n are determined (fixed) by the material's stress-strain curve. Whether analysts like it or not, these parameters cannot be selected arbitrarily.

Then there remains only one sensitivity study of relevance. This is for the case that the material's stress-strain curve does not obey an exponential equation. Most alloys satisfy the equation fairly well, so that there can be no argument: α and n are what they are and cannot be played with. However, 304SS is an exception: its stress-strain curve cannot be fitted with an exponential equation. Yet, it must be fitted to an exponential equation, otherwise none of the schemes will work. In that case, and in such cases only, there is a choice: should the equation fit the lower or the upper part of the stress-strain curve? Note that in this case there is only one choice: either $F(\alpha)$ or n can be chosen freely, the other parameter then being fixed by the fit. Any arbitrary choices of both parameters are meaningless.

This paper is not the place to suggest whether the upper or lower part of the stress-strain curve should be fitted. However, common sense leads to the immediate conclusion that the decision will have to be made on a material-by-material basis and only for those materials not obeying an exponential stress-strain curve (for other materials there is no choice). When there is a choice, what was said about accuracy and the effect of n applies: it is not the value of J that matters, but the predicted stress.

CONCLUSION

The elusive complexity of existing J-estimation schemes was shown to be trivially artificial. A simple equation for J shows the way back to normalcy. It also shows the possibilities for a new scheme with geometry factors that are more easily obtainable. The new scheme requires much more evaluation. However, as was shown, J is not very influential in the calculation of fracture stresses, so that the simplest possible scheme is probably adequate. Large differences in J or J_R do not affect the predicted fracture stresses very much. The most influential parameter in the predictions is the strain hardening exponent.

SYMBOLS USED

a = crack size
c = uncracked ligament
E = Young's modulus
f = geometry function
F = "plastic" modulus
G = elastic energy release rate
g = geometry function
h = geometry function
J = J integral
k = geometry function
K = stress intensity
n = strain hardening exponent
P = load
r = distance from crack tip
α = stress-strain curve parameter
β = geometry factor in stress intensity
γ = geometry factor
ε = strain
λ = geometry factor in J
σ = stress
subscript e = elastic
subscript l = local
subscript o = reference value
subscript p = plastic

REFERENCES

1. Paris, P.C. and Tada, H. The application of fracture proof design methods using tearing instability theory. NUREG/CR-3464, 1983.

2. Kumar, V. et al., An engineering approach for elastic-plastic fracture analysis. Electric Power Research Institute, Report EPRI NP-1931, 1981.

3. Kumar, V. et al., Advances in elastic-plastic fracture analysis. Electric Power Research Institute, Report EPRI NP-3607, 1984.

4. Liebowitz, H. and Eftis, J., Eng. Fract. Mech.3, pp 267-281, 1971.

5. Kanninen, M.F. et al., Nucl. Eng. and Design 48, pp 117-134, 1978.

6. Erdogan, F. and Kibler, J.J., Int. J. Fract. Mech. 9, pp 229, 1969.

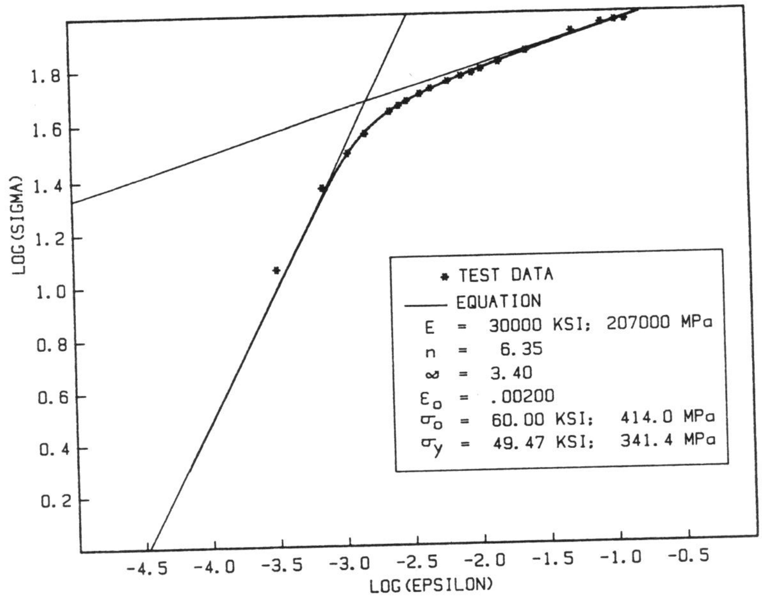

FIGURE 1. STRESS - STRAIN CURVE
ASYMPTOTES DETERMINED BY E; F; AND n

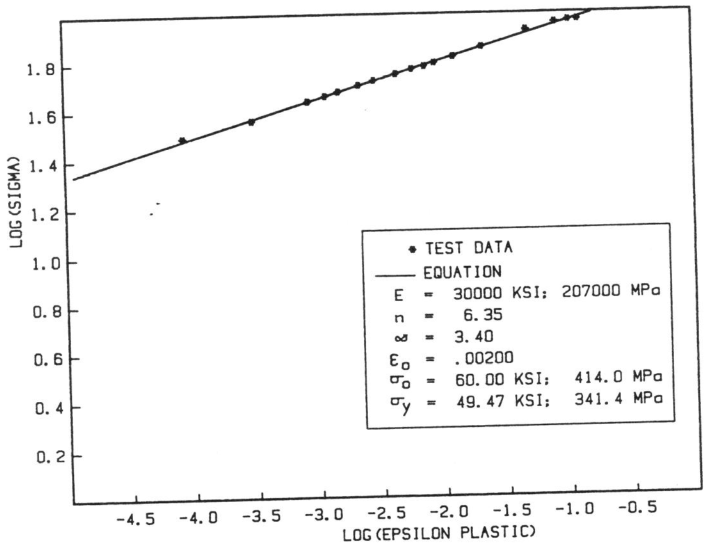

FIGURE 2. STRESS - STRAIN CURVE
DETERMINING F AND n

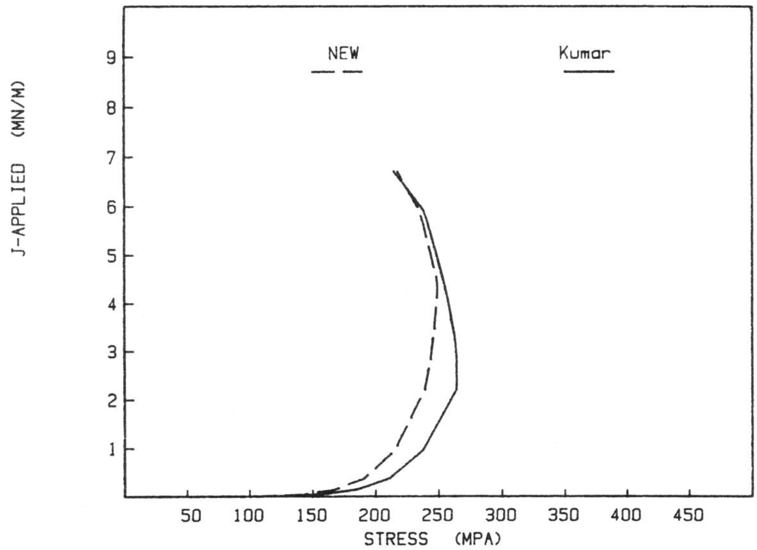

FIGURE 3. STRESS - JAPPLIED ; RUN ID # 22288
COMPARISON OF KUMAR SCHEME AND NEW SCHEME
127 MM CENTER CRACK IN 300 MM SS304 PANEL

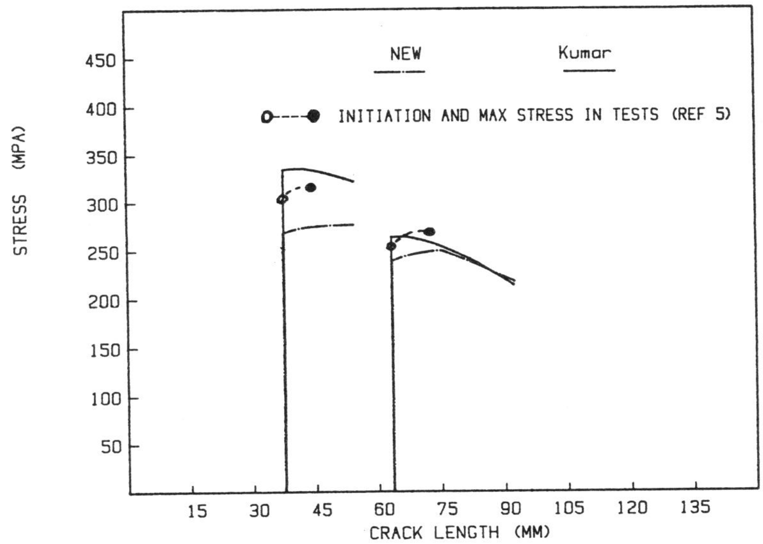

FIGURE 4. STRESS - CRACK SIZE ; RUN ID # 13715
CENTER CRACKS IN 300 MM 304SS PANELS
PREDICTIONS; TEST DATA FROM REF (5)

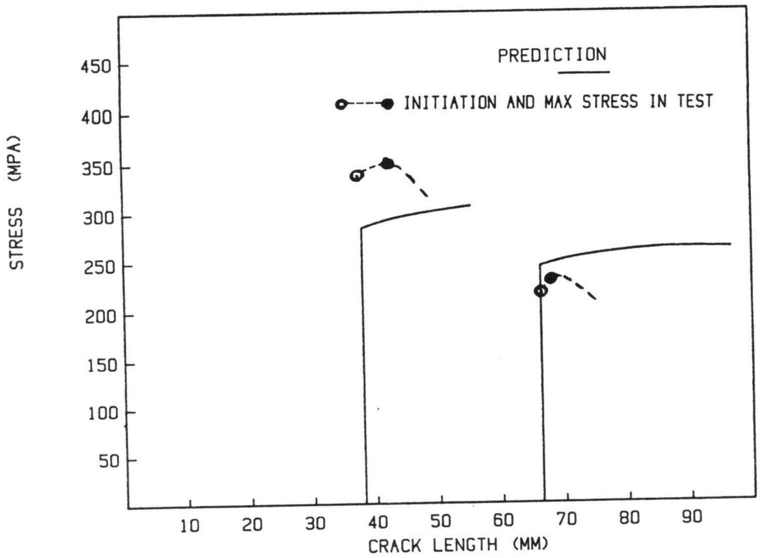

FIGURE 5. STRESS - CRACK SIZE ; RUN ID # 13715
100 MM DIAMETER SS304 PIPE IN BENDING
BETA FROM REF (6); TEST DATA FROM REF (5)

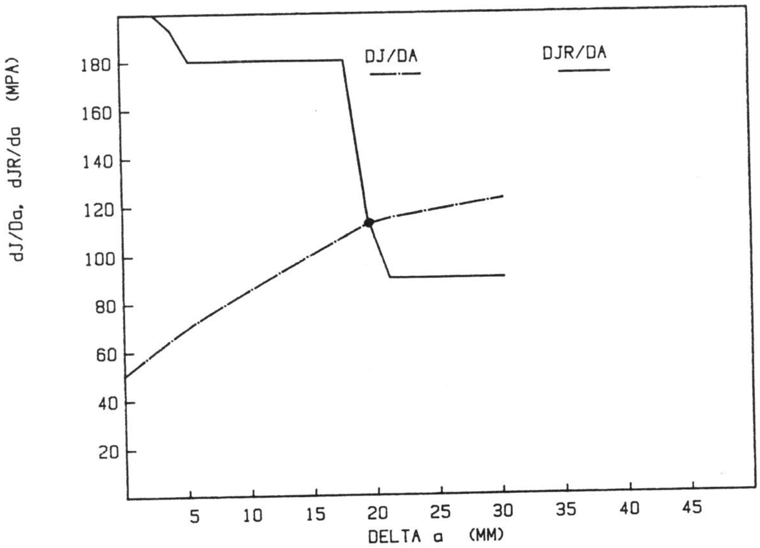

FIGURE 6. DELTA A-DJ/DA, DJR/DA ; RUN ID # 9429
CRACK SIZE AT INSTABILITY IN LOAD CONTROL

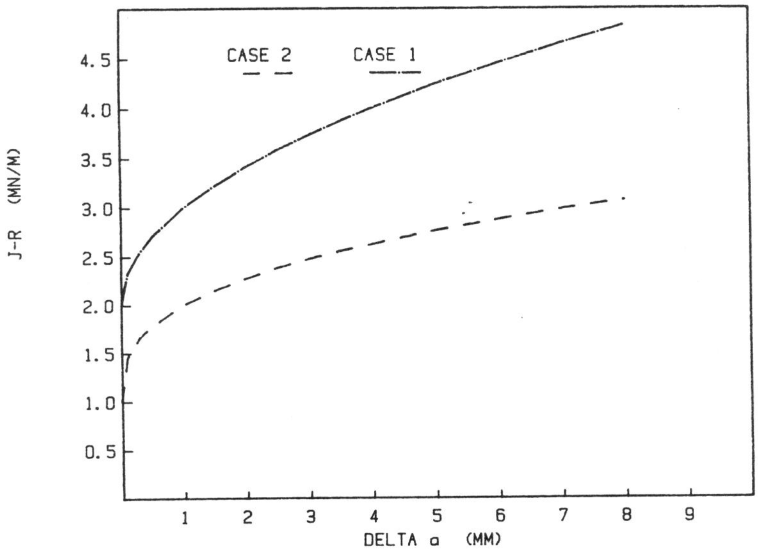

FIGURE 7. R-CURVE ; RUN ID # 5861
TWO DIFFERENT JR-CURVES USED IN ANALYSIS
IN NEXT FIGURE

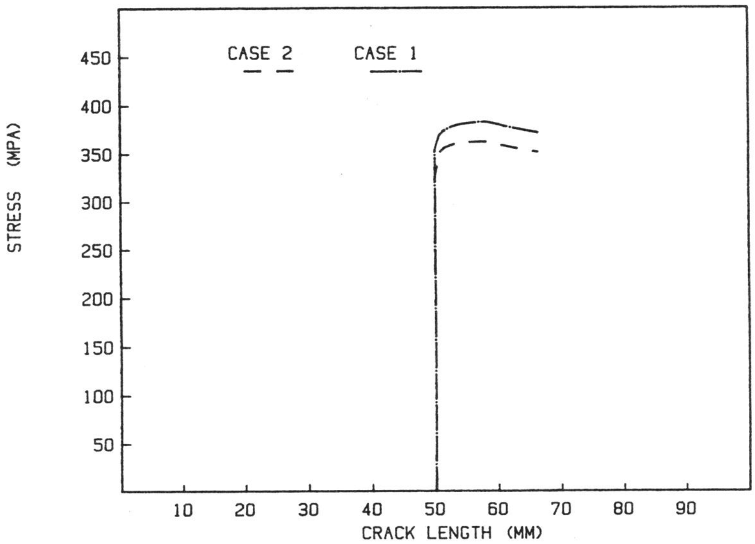

FIGURE 8. STRESS - CRACK SIZE ; RUN ID # 18002
DIFFERENCE IN PREDICTIONS WITH JR-CURVES FROM
PREVIOUS FIGURE; KUMAR SCHEME

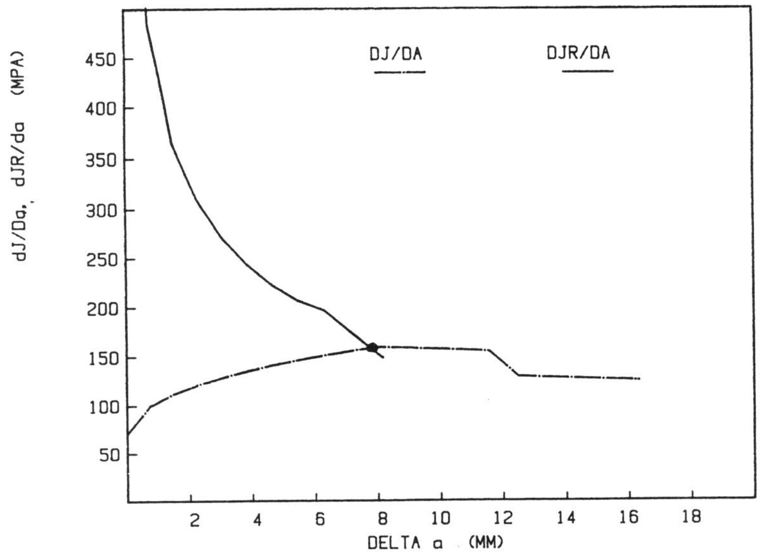

FIGURE 9. DELTA A-DJ/DA, DJR/DA ; RUN ID # 18002
CASE 1 OF PREVIOUS FIGURES

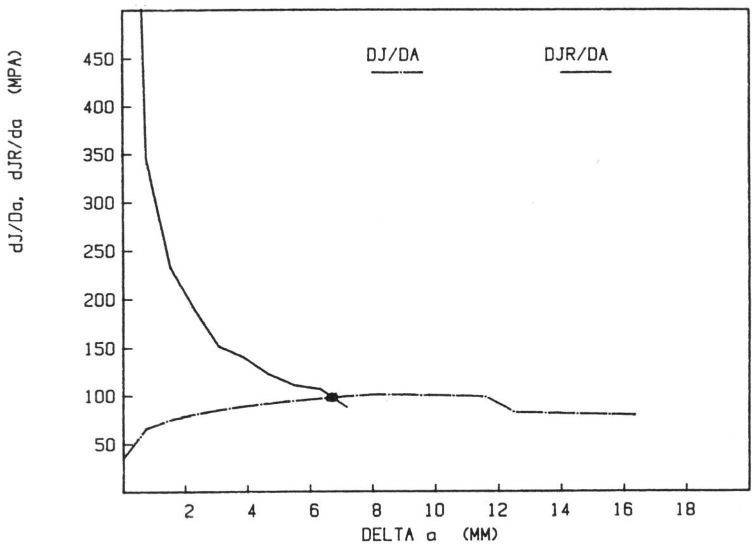

FIGURE 10. DELTA A-DJ/DA, DJR/DA ; RUN ID # 5861
CASE 2 OF PREVIOUS FIGURES