

**THE CTOD DESIGN CURVE APPROACH: LIMITATIONS AND CHANGES**

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This paper reviews the existing crack tip opening displacement design curve approach and draws attention to the correct application of the approach to small cracks in elastically contained yielding situations. A simple modification to the CTOD design curve approach is proposed. Recommendations are given concerning the various assumptions that may be necessary when applying the CTOD design curve approach to plain materials and welded structures.

**INTRODUCTION**

The primary objective of the crack tip opening displacement (CTOD) design curve approach (1-3) is to determine tolerable or maximum allowable crack sizes, i.e. the crack sizes that are significantly smaller than the critical values for brittle fracture or plastic collapse.

The CTOD design curve **approach** is based on the results of full section thickness BS 5762 CTOD tests (4), a design curve, and a number of simplifying assumptions. These provide an easy and rapid basis for determining the tolerable sizes of cracks in welded structures.

Experience over the last 15 years has shown many instances where the CTOD design curve approach has saved time and money by avoiding such alternatives as more sophisticated analyses, unnecessary weld repairs, and even some unnecessary post weld heat treatments (5). It is mainly in this context, that the CTOD design

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curve approach continues to have a valuable rôle. The approach may be regarded as a first coarse filter in fitness-for-purpose assessments, whilst retaining the option of using alternative more accurate, but less conservative fracture mechanics methods. Two such alternatives have been proposed in complimentary studies at The Welding Institute (6,7). These have led to a further proposal for a three tier fitness-for-purpose assessment procedure (8), which includes the CTOD design curve approach as the lowest tier of sophistication.

Although the CTOD design curve approach has been widely publicised by its inclusion in PD 6493 (9), this document does not draw attention to some of the limitations of the approach. For example, the present CTOD design curve approach should be limited to crack length to plate width ratios less than approximately 0.1 and uniform gross-section stresses less than yield. Fortunately, most industrial applications of PD 6493 fall within these limitations. On the other hand, guided only by PD 6493, some experimentalists and analysts have worked outside of the limitations and have concluded that the CTOD design curve approach may be unsafe in some instances.

This paper summarises a more detailed report (10) of the development, limitations and general application of the CTOD design curve approach, and also some simple modifications.

#### EARLY DEVELOPMENT

Wells (11) and Cottrell (12) independently proposed CTOD as a parameter for characterising fracture toughness under elastic-plastic conditions. These proposals stimulated the development and application of associated elastic-plastic fracture mechanics relationships, some based on a consideration of dislocation movements (13) and others on the macroscopic displacements (14-16) resulting from what is generally referred to as the Dugdale plastic zone (17). The CTODs perpendicular to the plane of a through-thickness rectilinear crack (Mode I displacements) in an infinite flat plate in tension were generally found to reduce to (see the nomenclature at the end of the paper),

$$\delta = \frac{8\sigma_{YS} \bar{a}}{\pi E} \ln \sec \left( \frac{\pi \sigma}{2\sigma_{YS}} \right) \quad (1)$$

Burdekin and Stone (16) re-expressed Eq.(1) in the following non-dimensional form:

$$\phi = \frac{\delta E}{2\pi\sigma_{YS} \bar{a}} = \frac{4}{\pi} \ln \sec \left( \frac{\pi}{2} \frac{\sigma}{\sigma_{YS}} \right) \quad (2)$$

APPLICATION TO DESIGN

Flat Plates Containing Through-Thickness Cracks

The above considerations led to the idea of using a design approach based on the combination of a single CTOD design curve (1,2) and full section thickness CTOD tests (4). Thus, by rearranging the expression for non-dimensional CTOD, Eq.(2), and replacing  $\phi$  by a larger design curve value,  $\phi_D$ , we obtain

$$\bar{a}_{\max} = \frac{\delta_{\text{crit}} E}{2 \pi \phi_D \sigma_{YS}} \quad (3)$$

The CTOD design curve approach, as represented by Eq.(3), is intended to give simple but safe estimates of maximum allowable (and, therefore, subcritical) crack sizes in relatively large welded structures subjected to normal gross-section design stresses, i.e.  $\sigma < \sigma_{YS}$ . However, within these restrictions, the design curve approach allows a consideration of effective **local stresses**,  $\sigma_1$ , and **local strains**,  $e_1$ , greater than yield. Referring to Fig.1, the effective local stresses or strains may be estimated from:

$$\frac{\sigma_1}{E} = e_1 = \frac{1}{E} [(\sigma \times \text{SCF}) + \sigma_S], \text{ for } \frac{\sigma}{\sigma_{YS}} < 1.0 \quad (4)$$

Note that some authorities (9) recommend that the product  $\sigma \times \text{SCF}$  should not exceed  $2\sigma_{YS}$ . Beyond this value, it is suggested that a full elastic-plastic stress analysis is carried out to determine the maximum plastic strain levels that would occur in the region containing the crack if it were not present.

What is widely known as the Burdekin and Dawes design curve was published in a graphical form in 1971 (1), and in the following modified form by Dawes in 1974 (2):

$$\phi_D = \left(\frac{e_1}{e_{YS}}\right)^2, \text{ for } \frac{e_1}{e_{YS}} < 0.5 \text{ and } \frac{\bar{a}}{W} < 0.1 \quad (5a)$$

$$\phi_D = \frac{e_1}{e_{YS}} - 0.25, \text{ for } \frac{e_1}{e_{YS}} > 0.5, \frac{\sigma}{\sigma_{YS}} < 1.0 \text{ and } \frac{\bar{a}}{W} < 0.1 \quad (5b)$$

The full background to Eq.(5) is given in Ref.(3). Unfortunately, in the past (1-3), the limitations  $e/e_{YS} < 1.0$  and  $\bar{a}/W < 0.1$  have not been included in the statement of Eq.(5), and also a clear distinction has not always been made between  $e$  and  $e_1$ . Although this does not seem to have been a problem to designers, it has led

to some confusion amongst experimentalists, especially in relation to the differences between gross-section and local strains in the context of Eq.(5b). Part of the confusion arises from the **derivation** as compared to the **application** of the design curve.

As illustrated in Fig.1, the derivation of Eq.(5b) was based on the upper bound of experimental plain plate data of  $\Phi$  in relation to strain, extending to final gross-section yielding strains,  $e_{gsy}$ , rather than final net-section yielding strains; the former subsequently being associated with  $a/W < 0.1$ , and approximately gauge length independent measurements of  $e_{gsy}$  across the notched section (3). However, as Fig.1 indicates, for safe application to design situations ( $e \leq e_{ys}$ ), the simple, and already conservative definition of local strain,  $e_l$  (Eq.(4)), was substituted for the gross-section yielding strains that formed the original basis of Eq.(5b). Similarly, the local strains were also substituted for the original gross-section strains in Eq.(5a).

As far as the author is aware, the correct application of Eq.(5) to welded joints has not caused problems. However, difficulties may be experienced for some plain materials or weldments when Eq.(5) is applied to finite width plates, which are here defined as those having  $a/W > \approx 0.1$ . These situations are considered below.

#### Finite Width Flat Plates Containing Through-Thickness Cracks

Although it is quite difficult to imagine an initial design situation where it would be acceptable to have  $a/W > 0.1$ , such large cracks could occur in service by fatigue cracking and various other crack growth mechanisms. In these cases it would be important to determine the maximum allowable crack sizes to avoid brittle or ductile fracture within the time available for replacement or repair.

Several attempts have been made to produce Dugdale (17) strip yield based solutions for CTODs in finite width plates. Notable amongst these are the finite element analysis solutions by Hayes and Williams (18) and the closed form analytical expression by Schwalbe (19), which may be expressed as:

$$\Phi = \frac{4}{\pi^2} \ln \left[ \frac{2W}{\pi a} \arcsin \left( \sin \frac{\pi a}{2W} \sec \frac{\pi \sigma}{2\sigma_{YS}} \right) \right], \quad (6)$$

$$\text{for } \sigma \leq \sigma_{YS} \left( 1 - \frac{\pi a}{2W} \right)$$

The restriction on Eq.(6) ensures that it is not applied to net section stresses ( $\sigma_N$ ) greater than yield. However, when  $\sigma_N = \sigma_{YS}$ , Eq.(6) gives a finite value of CTOD, and the equation reduces to

$$\Phi = \frac{4}{\pi^2} \ln \left( \frac{2W}{\pi a} \arcsin 1 \right) = \frac{4}{\pi^2} \ln \left( \frac{2W}{2a} \right) \quad (7)$$

Based on earlier analytical and experimental studies (3), the writer has used the following alternative expression for finite-width,

$$\Phi = \left( 1 - \frac{\bar{a}}{W} \right)^2 \frac{4}{\pi^2} \ln \sec \left[ \frac{\pi \sigma}{2\sigma_{YS} \left( 1 - \frac{\bar{a}}{W} \right)} \right] \quad (8)$$

Note that Eq.(8) reduces to Eq.(2) when  $\bar{a}/W = 0$ . Also, as  $\sigma_N \rightarrow \sigma_{YS}$ ,  $\Phi \rightarrow \infty$ .

The Hayes and Williams (18) finite element solutions, and those of Eqs (6) to (8) are compared in Figs 2 and 3 for values of  $\bar{a}/W = 0.1$  and  $0.5$ , respectively. It can be seen that there is a close correspondence between the three finite width solutions up to approximately  $\sigma_N/\sigma_{YS} = 0.8$ . Beyond this value, Eq.(8) generally predicts significantly higher values of  $\Phi$  and increasing differences compared to the other solutions.

Unfortunately, compared to the existing design curve, Eq.(5), the above finite width relationships require relatively difficult iterative calculations in order to determine crack sizes.

**Modified CTOD Design Curves for Flat Plates Containing Through Thickness Cracks.** In the spirit of the existing CTOD design curve (Eq.(5)), it was proposed (10) that the following modified but simple stress-based finite width design curves should be adopted in all instances, cf Eqns (3) - (5).

**For all plain materials and welded non-ferritic metals,**

$$\Phi_{FWD} = \left( \frac{\sigma_1}{\sigma_{YS}} \right)^2 \left[ 1 - \frac{\bar{a}_{max}}{W} \right]^{-1}, \quad (9a)$$

$$\text{provided that } \sigma_N/\sigma_{YS} \leq 1.0 \text{ and } \bar{a}_{max}/W \leq 0.5 \quad (9a)$$

**Also, for all welded ferritic steels,**

$$\Phi_{FWD} = \left( \frac{\sigma_1}{\sigma_{YS}} \right)^2 \left[ 1 - \frac{\bar{a}_{max}}{W} \right]^{-1}, \text{ for } \frac{\sigma_1}{\sigma_{YS}} \leq 0.5 \quad (9b)$$

$$\Phi_{FWD} = \left( \frac{\sigma_1}{\sigma_{YS}} - 0.25 \right) \left[ 1 - \frac{\bar{a}_{max}}{W} \right]^{-1}, \text{ for } \frac{\sigma_1}{\sigma_{YS}} \leq 0.5, \quad (9c)$$

provided that  $\sigma_N/\sigma_{YS} \leq 1.0$  and  $\bar{a}_{max}/W \leq 0.5$  (9c)

It is important to note that, for most practical applications, where  $\bar{a}/W \rightarrow 0$ , Eqs (9b) and (9c) will reduce to the stress equivalent of Eq.(5), i.e. the existing near-infinite width CTOD design curve which is used in PD 6493 (9).

Equation (9) is shown in diagrammatic form in Fig.4. Here, the distinction between plain materials and welded ferritic steels allows the finite width design curve to be applied more confidently to cracks in plain materials remote from regions of geometric stress concentration, including cracks subjected to uniaxial loading.

**Modified Maximum Allowable Sizes for Through Thickness Cracks.** The maximum allowable or tolerable crack sizes may be obtained from Eqns (3) and (9) as given below.

**For all plain materials and welded non-ferritic metals,**

$$\bar{a}_{max} = \left[ \frac{2\pi \sigma_1^2}{\sigma_{YS} \delta_{crit} E} + \frac{1}{W} \right]^{-1} \tag{10a}$$

provided that  $\sigma_N/\sigma_{YS} \leq 1.0$  and  $\bar{a}_{max}/W \leq 0.5$

**Also, for all welded ferritic steels,**

$$\bar{a}_{max} = \left[ \frac{2\pi \sigma_1^2}{\sigma_{YS} \delta_{crit} E} + \frac{1}{W} \right]^{-1}, \text{ for } \frac{\sigma_1}{\sigma_{YS}} \geq 0.5 \tag{10b}$$

$$\bar{a}_{max} = \left[ \frac{2\pi (\sigma_1 - 0.25 \sigma_{YS})}{\delta_{crit} E} + \frac{1}{W} \right]^{-1}, \text{ for } \frac{\sigma_1}{\sigma_{YS}} \leq 0.5, \tag{10c}$$

provided that  $\sigma_N/\sigma_{YS} \leq 1.0$  and  $\bar{a}_{max}/W \leq 0.5$

Table 1 summarises the recommended values of  $\sigma_1$  for use with Eq.(10). The post weld heat treated condition is not considered to be equivalent to a plain material condition, and cracks in thermally stress relieved welds remote from stress concentration are considered in relation to  $\sigma_1 \geq \sigma + 0.25\sigma_{YS}$ .

It is also important to note that the limitation  $\sigma_N/\sigma_{YS} \leq 1.0$ , on Eq.(10), places an upper bound on the maximum allowable through-thickness crack sizes for very ductile tear resistant materials, given by:

$$\bar{a}_{max} \leq W \left( 1 - \frac{\sigma}{\sigma_{YS}} \right) \tag{11}$$

This is a very conservative limitation, which, when combined with Eq.(10), will make it unnecessary to consider failure by tearing instability or plastic collapse.

**TABLE 1** Recommended effective local stresses,  $\sigma_1$ .

Crack location	Condition	$\sigma_1$ (See Notes 1 & 2)
Unwelded parent material (PM) remote from stress concentration (SC)	Normalised As-rolled	$\sigma + ?$
Unwelded PM near SC	Normalised As-rolled	$(\sigma \times SCF) + ?$
PM remote from non-PWHT welds	Normalised As-rolled As-fabricated	$\sigma + ?$
Weld metals and HAZs remote from SC	After PWHT As-welded	$(\sigma + 0.25\sigma_{YS} + ?)$ $\sigma + \sigma_{YS} + ?$
Weld metals and HAZs adjacent to SC	After PWHT As-welded	$(\sigma \times SCF) + 0.25\sigma_{YS} + ?$ $(\sigma \times SCF) + \sigma_{YS} + ?$

NOTES:

1. It may be necessary to consider the following: uniform or localised thermal stresses; long range or reaction residual welding stresses and residual stresses due to cold working during fabrication. All uniform stresses should be considered as part of the gross-section stress,  $\sigma$ , when dealing with cracks adjacent to SCs.
2. The residual stresses remaining after postweld heat treatments (PWHTs) should be investigated. The value of  $0.25\sigma_{YS}$  should be considered as the minimum value remaining after PWHT.

The implications of finite plate width effects to part-wall cracks are considered below.

**Finite Width Flat Plates Containing Part-Wall Cracks**

To be consistent with the derivations used in the past (2,3), the maximum allowable dimensions of a semi-elliptic surface crack in a finite width plate may now be obtained from:

$$a_{\max} = \bar{a}_{\max} \times f' \left( \frac{a}{B}, \frac{a}{2c} \right), \text{ for}$$

$$a_{\max} \leq B \left( 1 - \frac{2\sigma}{\sigma_{YS} + \sigma_{TS}} \right), \text{ and } c_{\max} \leq W \left( 1 - \frac{\sigma}{\sigma_{YS}} \right) \text{ when}$$

$$a_{\max} > B \left( 1 - \frac{\sigma}{\sigma_{YS}} \right) \tag{12}$$

Similarly, for buried elliptical cracks,

$$a_{\max} = \bar{a}_{\max} \times f'' \left( \frac{a}{B}, \frac{a}{2c} \right), \text{ for}$$

$$2a_{\max} \leq B \left( 1 - \frac{2\sigma}{\sigma_{YS} + \sigma_{TS}} \right), \text{ and } c_{\max} \leq W \left( 1 - \frac{\sigma}{\sigma_{YS}} \right), \text{ when}$$

$$2a_{\max} > B \left( 1 - \frac{\sigma}{\sigma_{YS}} \right) \tag{13}$$

The functions of  $(a/B, a/2c)$  are obtained (2,3) by equating  $K_I$  expressions (20,21) for through-thickness and surface or buried cracks in finite thickness plates for equal values of  $\sigma$  and  $K_I$  applied to each geometry. Non-dimensional graphs of Eqs (12) and (13) are given in Figs 11 and 12. Hence Eqs (12) and (13) may be solved by entering the 'x' axes of Figs 5 and 6 with a value of  $\bar{a}_{\max}$  from Eq.(10), and then reading values of  $a_{\max}$  from the 'y' axes via the curves for the crack aspect ratios  $(a/2c$  or  $2a/2c)$  of interest. Known or postulated part-wall cracks are only of acceptable size when they have dimensions that are equal to or smaller than the corresponding values of  $a_{\max}$  and  $c_{\max}$ .

Except for the finite width limitations on crack length (2c), Eqs (12) and (13) are unaltered compared to earlier proposals (2,3). For ligament yielding ahead of the part-wall crack, the finite width limitations are treated in the same manner as a through-thickness crack, cf Eqs (10) to (13).

**Other Geometries**

In some applications it will be necessary to reduce the "flat plate  $a_{\max}$  and  $a_{\max}$  values" to the smaller values that will be relevant to some asymmetric or adjacent cracks and cracks in



curved, distorted and/or misaligned plates and welds. Further guidance on the adjustments to the flat plate maximum allowable crack sizes may be obtained from PD 6493 (9).

**Application to Welded Structures**

To apply Eqs (10) to (13) to welded structures, it is necessary to know, or make realistic assumptions, for the crack shapes,  $\delta_{crit}$ ,  $\sigma_{YS}$ ,  $\sigma_{TS}$ ,  $\sigma$ ,  $\sigma_s$ , E and W in representative welded joints at the temperatures and strain rates of interest. It is also necessary to consider the inherently heterogeneous nature of welded joints. In this context, the welded joint should be regarded as comprising at least three regions, namely the weld metal, the transformed weld heat affected zone (HAZ) and the parent material close to the weld, which is sometimes referred to as the non-transformed or sub-critical HAZ. Each region contains material of varying strength and toughness, and appropriate tests and assumptions are necessary in order to obtain the relevant data for fracture mechanics analyses. Appropriate assumptions may also be necessary concerning the residual stresses in the different weld regions. All of these requirements are reviewed in more detail in Ref.(10).

**CONCLUSIONS**

The CTOD design curve approach should be regarded as providing a first coarse filter in fitness-for-purpose assessments, whilst retaining the option of using more accurate but less conservative fracture mechanics methods (6,7). In this rôle, the CTOD design curve approach continues, after approximately 15 years of industrial use, to provide significant savings in terms of time and money. Nevertheless, there are some doubts about applying the existing design curve approach to plain materials, welded non-ferritic materials, and crack length to plate width ratios greater than approximately 0.1.

Thus, for all **plain materials and non-ferritic weldments**, a modified CTOD design curve is recommended (see Fig.4). This leads to the following simple relationship for maximum allowable half lengths of through thickness cracks in flat tension plates.

$$\bar{a}_{max} \leq \left[ \frac{2\pi \sigma_1^2}{\sigma_{YS} \delta_{crit} E} + \frac{1}{W} \right] \quad (10a)$$

Similarly, for **welded ferritic steels**, a modified CTOD design curve is recommended, which leads to:

$$\bar{a}_{max} \leq \left[ \frac{2\pi \sigma_1^2}{\sigma_{YS} \delta_{crit} E} + \frac{1}{W} \right]^{-1}, \text{ for } \frac{\sigma_1}{\sigma_{YS}} < 0.5 \quad (10b)$$

$$\bar{a}_{\max} \leq \left[ \frac{2\pi (\sigma_1 - 0.25\sigma_{YS})}{\delta_{\text{crit}} E} + \frac{1}{W} \right]^{-1}, \text{ for } \frac{\sigma_1}{\sigma_{YS}} \geq 0.5 \quad (10c)$$

As  $\bar{a}_{\max}/W \rightarrow 0$ , Eq.(10) reduces to the expressions given by the existing CTOD design curve (3), a version of which is currently used in PD 6493 (9). However, it is very important to note that **Eq.(10) is limited to values of  $\sigma_1/\sigma_{YS} \leq 1.0$ , and values of  $\bar{a}_{\max}/W \leq 0.5$ .** Because these are very conservative limitations, the modified CTOD design curve approach has an advantage over the PD 6493 version, since it can be used without a separate analysis for failure by tearing instability or plastic collapse.

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#### NOMENCLATURE

a	Depth of surface crack or half-depth of buried crack
$\bar{a}$	Length of through-thickness edge crack or half-length of centred crack in a flat tension plate.
$a_{\max}$	Maximum allowable value of a
$\bar{a}_{\text{crit}}$	Critical value of $\bar{a}$
$\bar{a}_{\max}$	Maximum allowable value of $\bar{a}$
B	Section thickness containing crack
c	Half-length of surface or buried crack
$c_{\max}$	Maximum allowable value of c
e	Nominal strain on gross-section
$e_{\text{gsy}}$	Strains accompanying gross-section yielding behaviour.
$e_{\text{YS}}$	Yield strain = $\sigma_{\text{YS}}/E$
$e_l$	Local strain
E	Young's modulus
SCF	Stress concentration factor
W	Half-width of centre or double-edge cracked plate
$\delta$	CTOD

$\delta_{crit}$	Any of $\delta_c$ , $\delta_m$ , $\delta_u$ , according to BS 5762(4)
$\sigma$	Nominal or gross-section stress at infinity
$\sigma_N$	Net section stress
$\sigma_S$	Secondary stress, eg. residual stress
$\sigma_{TS}$	Tensile strength at temperature of interest
$\sigma_{YS}$	Yield or 0.2% offset strength at temperature of interest
$\sigma_l$	Effective local stress
$\phi$	Non-dimensional $\delta$
$\phi_D$	$\phi$ in near-infinite-width design curve
$\phi_{FWD}$	$\phi$ in finite-width design curve

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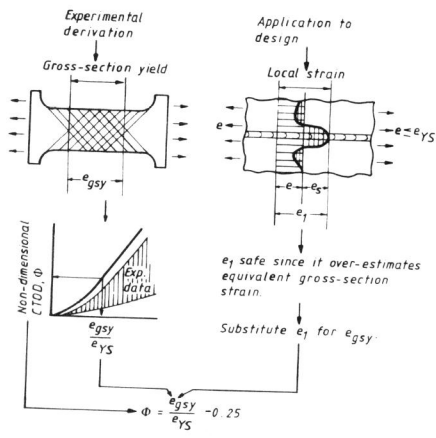


Fig.1. How the near-infinite size CTOD design curve was derived.

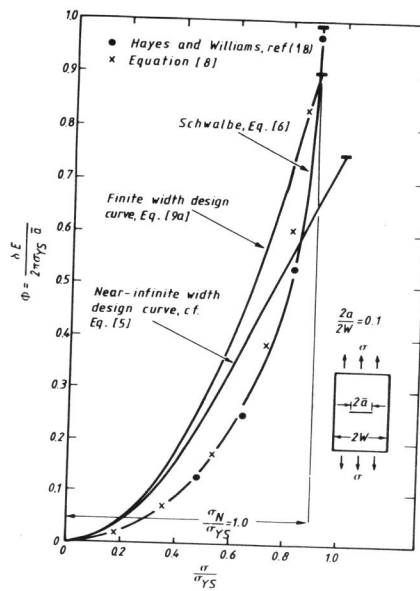


Fig.2. Relationships for plain material ( $\bar{a}/W = 0.1$ ).

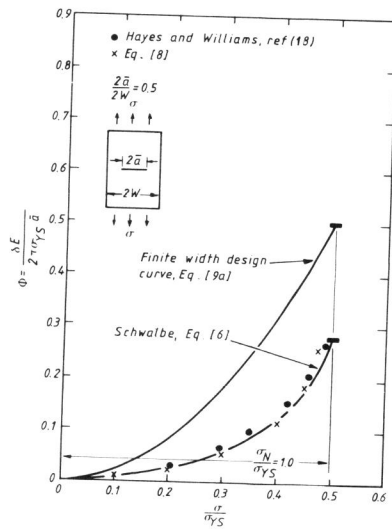


Fig.3. Relationships for plain material ( $\bar{a}/W = 0.5$ ).

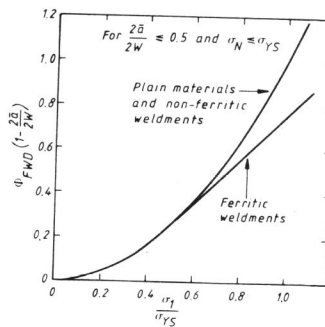


Fig.4. Recommended (1985) CTOD design curve.

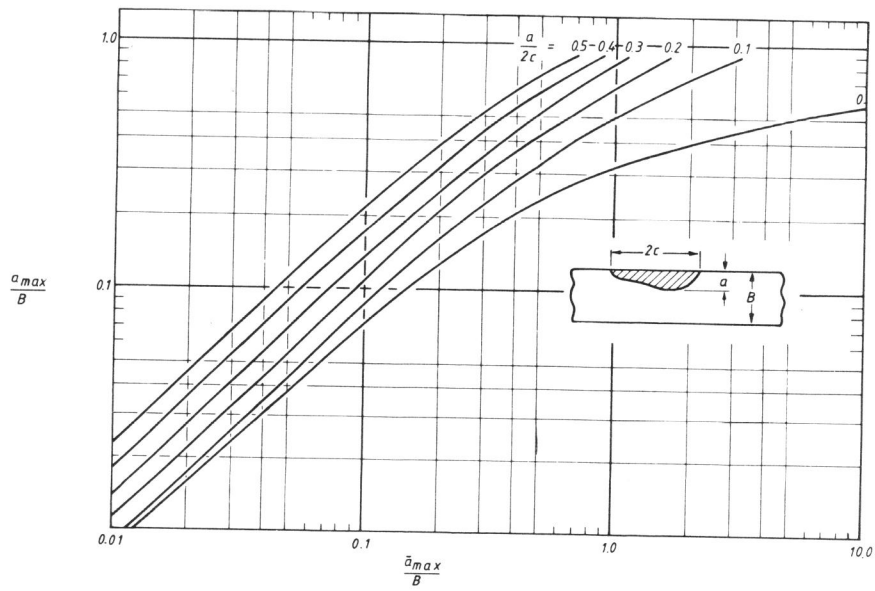


Fig.5. Relationships between surface and through-thickness cracks in flat plates.

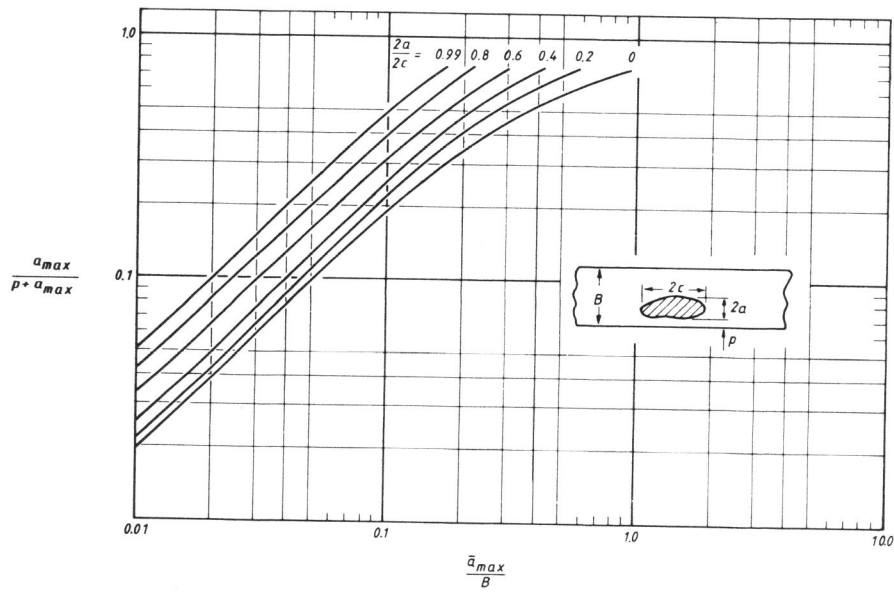


Fig.6. Relationships between dimensions of buried and through-thickness cracks in flat plates.