APPLICATION OF A GLOBAL-LOCAL BLENDED TYPE FINITE ELEMENT IN THERMAL FRACTURE

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In this paper, a new special finite element is presented in order to reproduce the stress singularity near the crack tip. Two alternative methods for quasistatic crack growth simulation have been developed. The applicability of this formulation in problems with thermal stresses is shown, and numerical results are presented.

INTRODUCTION

Problems of thermal fracture are relevant in many industrial applications. Thermal loads modify fracture analysis in two ways. Firstly, material properties are different in each part of the structure. On the other hand, many ordinary formulations used with the Finite Element Method are not able to solve fracture problems when thermal strains are present. It is well known that the Rice's J integral (1) does not work and has to be modified including an area integral next to path integral (2)-(5).

In this paper, two different methods for propagation analysis are shown. These methods are founded on a new special finite element (6), with global-local blended functions, that confer a good capacity in modelling the crack tip.

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Finally, results of an example are discussed. That example concerns to the evolution of the stress intensity factor, K_I, when crack grows in a structure under a strong thermal gradient.

CRACK MODELLING AND GROWTH SIMULATION

A new special finite element. Crack modelling

This new special finite element, like the Benzley's one (7), uses a global-local formulation, that was developed by Mote (8). The displacement field is interpolated by the expression

$$u_i = N_{ij}d_j + H_{ij}\beta_j \tag{1}$$

where:

u_i is the displacement field
N_{ij} the local shape functions
H_{ij} the global shape functions
d_j, β_j unknown parameters

Expression (1) must be modified when is used to solve a bidimensional fracture (mode I) problem and d_j is wanted to represent nodal displacements. Then (1) has to be changed by

$$u_1 = u = N_{1j}d_j + K_I(F_{I1} - N_{1j}F^j_{I1})$$
$$u_2 = v = N_{2j}d_j + K_I(F_{I2} - N_{2j}F^j_{I2}) \tag{2}$$

$j=1,2,...n$ (number of nodes)

where K_I is the stress intensity factor, F_{Ii} the theoretical linear fracture mechanics functions, and F^j_{Ii} the value of F_{Ii} at "j" node.

The special finite element presented in this paper is an eight-noded rectangular one, and the crack tip is always placed at an intermediate node, like 6 of Figure 1.a.

The shape functions N_{ij} are the ordinary ones of isoparametric finite elements, except those corresponding to 5,6,7 nodes -Figure 1.b-, which are -see Figure 2-

$$N_5 = N_{15} = N_{25} = \frac{(1-\xi)(1-\eta)}{4} - \frac{1}{2}N_4 - \frac{1-\xi_6}{2}N_6$$

$$N_7 = N_{17} = N_{27} = \frac{(1+\xi)(1-\eta)}{4} - \frac{1}{2} N_8 - \frac{1+\xi_6}{2} N_6 \qquad (3)$$

$$N_6 = N_{16} = N_{26} = \frac{(1+\xi)(1-\eta)}{2(1+\xi_6)} \qquad -1<\xi<\xi_6$$

$$N_6 = N_{16} = N_{26} = \frac{(1+\xi)(1-\eta)}{2(1-\xi_6)} \qquad \xi_6<\xi < 1$$

Local coordinates (ξ,η) come from general ones (x,y) by a homogeneous linear transformation.

With these global-local blended shape functions, nodal constraints of nodes 6 and 7 can reproduce exactly kinematic restrictions of cracks -see Figure 3.a-, while ordinary shape functions of isoparametric finite elements only are able to reproduce them when the crack tip is placed at nodes 5 or 7, but not in an intermediate position -see Figure 3.b-.

The lack of compatibility between the special interpolation defined by means of equation (2) and the isoparametric one of the standard finite elements has been overcome by defining a special class of transition elements surrounding the special finite element which contains the crack tip. Thus, crack is simulated as shown in Figure 4, in which special finite element is signified by A, transition finite elements, by B, and ordinary isoparametric finite elements, by C. The displacement field at B elements is interpolated by the expressions,

$$u_1 = u = N_{1j}d_j + K_I S \; (F_{I1} - N_{1j}F^j_{I1}) \qquad (4)$$

$$u_2 = v = N_{2j}d_j + K_I S \; (F_{I2} - N_{2j}F^j_{I2})$$

Here, S represents a shift function, such that it equals 1 on the boundary adjacent to the special element A, and equals 0 on boundaries adjacent to standard elements (type C elements) -see Figure 4-, and N_{ij} are now ordinary isoparametric shape functions.

Crack growth simulation

To simulate the crack growth a mesh displacement technique has been adopted, in two different ways. The first one simulates the crack like Figure 4 shows: the crack tip is always placed at the middle point of the lower side in the special element A. When crack grows, element A moves with the tip, and elements B distort adapting their size to elements C, which do not change. When some elements B increase their size up to a given average, the mesh is regenerated as it is shown in Figure 5.

The second way allows the crack to move along the side of the special element. Evidently while it is happening, the mesh does not change with exception of the node in which the crack tip is located. The mesh regeneration takes place when the tip reaches the corner of element A. Figure 6 shows an example of that process.

Before ending, it is worth noting that when the crack tip reaches the corner of A element, the global-local blended shape functions must be substituted by a global-local formulation with ordinary shape functions.

THERMAL LOADS

The Constitutive Law of a locally isotropic linear elastic medium is

$$\sigma_{ij}=\lambda u_{r,r}\delta_{ij}+\mu(u_{i,j}+u_{j,i})-\gamma T\delta_{ij} \tag{5}$$

where λ,μ are Lame's constants, $\gamma=\alpha(3\lambda+2\mu)$ –α linear thermal dilatation constant–, and T temperature increment from thermal equilibrium one. In a general case, λ,μ and α are temperature functions.

Let us admit that temperature distribution is regular, then it is easy to understand that thermal effects, represented by $\gamma T\delta_{ij}$ in equation (5) do not affect the singular stress field at crack tip neighbourhood, and can be taken into account by means of the polynomial functions of the global-local formulation.

Using the assumption given by equation (2) in a standard finite element formulation, the stiffness matrix for the special element, V^e, becomes

$$[K]^e = \left[\begin{array}{c|c} [K^{nn}] & \{K^{nI}\} \\ \hline \{K^{nI}\}^T & K^{II} \end{array}\right] \tag{6}$$

where

$$K_{ij}^{nn} = \int_{V^e} D_{klmn}\, N_{ki,l}\, N_{mj,n}\, dV \tag{7}$$

$$K_i^{nI} = \int_{V^e} D_{klmn} N_{ki,l} G_{m,n} \, dV \tag{8}$$

$$K^{II} = \int_{V^e} D_{klmn} G_{k,l} G_{m,n} \, dV \tag{9}$$

where D_{klmn} is the elastic tensor, and G_i represents the term in brackets of (2).

The equivalent thermal loads in the special finite element are evaluated as follows:

$$\{F\}^e = \begin{bmatrix} \{F^n\} \\ F^I \end{bmatrix} \tag{10}$$

and

$$F_i^n = \int_{V^e} D_{klmn} N_{ki,l} \varepsilon_{mn}^o \, dV \tag{11}$$

$$F^I = \int_{V^e} D_{klmn} G_{k,l} \varepsilon_{mn}^o \, dV \tag{12}$$

where

$$\varepsilon_{mn}^o = \gamma T \delta_{mn} \tag{13}$$

It is worth noting that the functions $G_{i,j}$ show a $r^{-\frac{1}{2}}$ singularity at the neighbourhood of the crack tip, and hence a special integration technique is required to evaluate stiffness matrix and thermal loads. Neverthe-less that singularity is not an important difficulty be-cause it must be taken into account only for the special element A.

Besides, the local blended shape functions, defined in equations (3), produce a discontinuity of the deriva-tives $N_{ij,k}$. Due to it, integrals (7),(8),(11) have to be evaluated dividing the special finite element in sub-domains where those derivatives are continuous.

RESULTS

The special finite element and two ways of modelling crack growth have been used to solve the quasistatic ther-mal propagation of the specimen shown in Figure 7, sub-jected to a constant temperature gradient and with an initial crack ($a_o/w = 0.2$).

Evidently, crack propagation and arrest depend on K_{Ic} and K_{Ia} local values; but for the scope of this paper, it is enough to get the evolution of K_I when crack grows -for instance from a/w=0.2 to a/w=0.75-. Wilson and Yu (2) studied this same case, using an invariant integral, J*. They got that the adimensional parameter

$$K_I^* = \frac{K_I(1-\nu)}{E\alpha T_o(\pi a)^{\frac{1}{2}}} \quad (14)$$

when a/w=0.5, was between 0.4975 and 0.5114 according to integration paths.

For symmetry, only the upper half of the structure has been modelled. The finite element definition of this problem is depicted in Figure 8. A mesh with 28 eight-noded elements has been used and crack growth has been simulated by constant steps of length $\Delta a/w=1/140$. It is worth saying that in the first way of simulating crack growth -special finite element moves together with crack tip-, finite element mesh is regenerated when some B finite elements change their size more than 50%; and that in the second way -crack tip moves along the special finite element A- we got the best numerical precision in leaving out blended formulation when the crack tip is closer than $\pm 0.05e$ to the vertex of the finite element A (e=special finite element length).

Results obtained with these techniques are shown in Figure 9, in which labels 1 and 2 refer to the two different ways used for simulating the crack extension. In particular, for a/w=0.5 a value $K_I^*=0.5008$ has been obtained, which is in very good agreement with the results of reference (2) indicated above.

Even though in this example the material properties were constant, the computer implementation has been conceived to accept temperature dependent properties.

CONCLUSIONS

A new special finite element with global-local formulation has been presented. Local shape functions of "blended" type are introduced to model correctly the crack tip. Two alternative ways of simulating the quasistatic crack extension have been developed, and the application to thermal stress fracture problems has been presented. Numerical results show a high efficiency in evaluating stress intensity factors.

SYMBOLS

a = crack length

d_j = nodal displacements

D_{ijkl} = elastic tensor

E = Young's modulus

F_{Ii} = theoretical functions of LEFM for displacement field

$\{F\}$ = force vector

H_{ij} = global functions

K_I = stress intensity factor for mode I

K_I^* = adimensional parameter related to K_I

$[K]$ = stiffness matrix

N_{ij} = local shape functions

S = shift function

T = temperature increment measured from the reference state

$u_i (i=1,2)$ = displacements

α = linear thermal expansion coefficient

β_j = unknown parameters associated with the global functions

ε_{ij}^o = thermal strain tensor

ξ, η = local coordinates

λ, μ = Lame's constants

ν = Poisson's ratio

σ_{ij} = stress tensor

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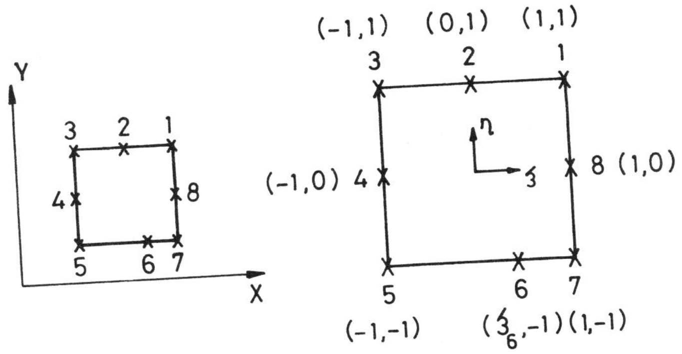

Figure 1. Quadrilateral element mapped into a square, with node 6 arbirarily placed.

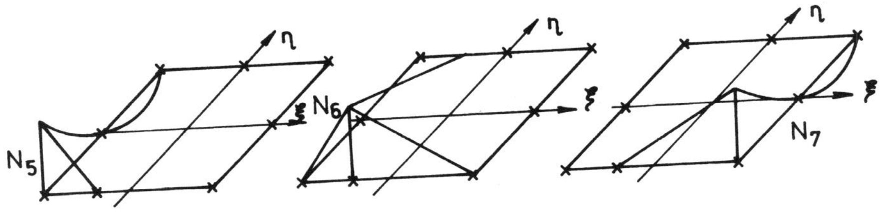

Figure 2. Blended shape functions for nodes 5,6 and 7.

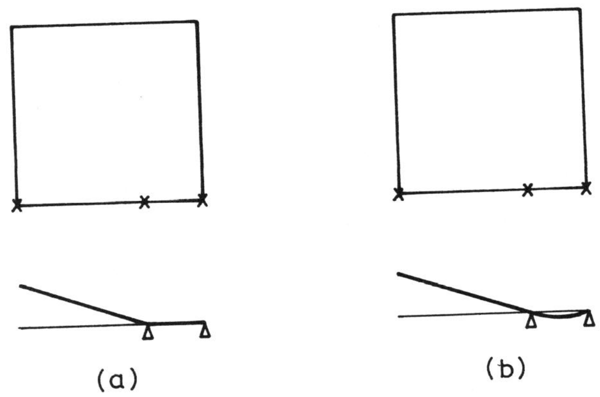

Figure 3. Nodal constraints: a) Isoparametric
b) Blended

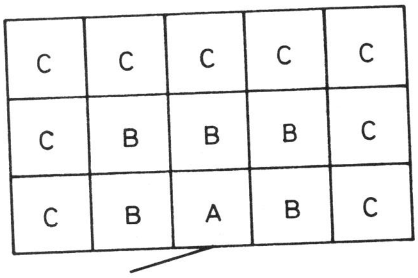

Figure 4. Special element (A) and transition elements (B).

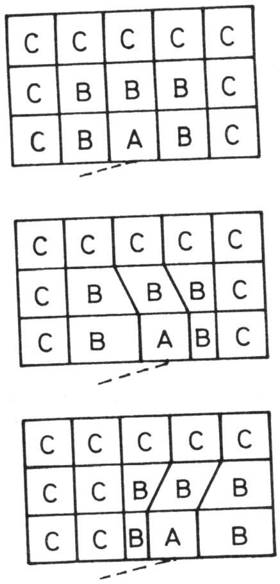

Figure 5. Moving element procedure.

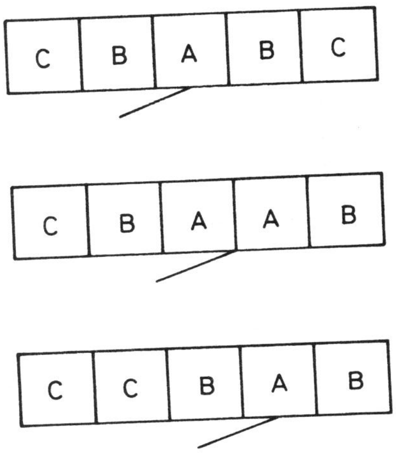

Figure 6. Moving intermediate node procedure.

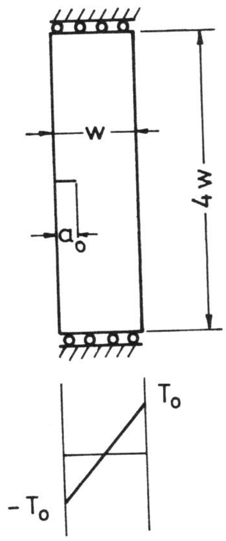

Figure 7. Edge cracked strip subjected to thermal loading.

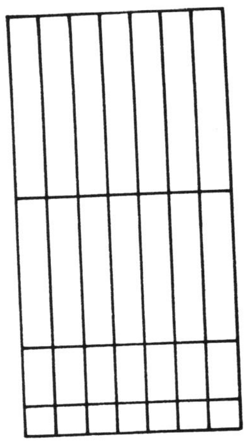

Figure 8. Finite element mesh of the upper half of the strip.

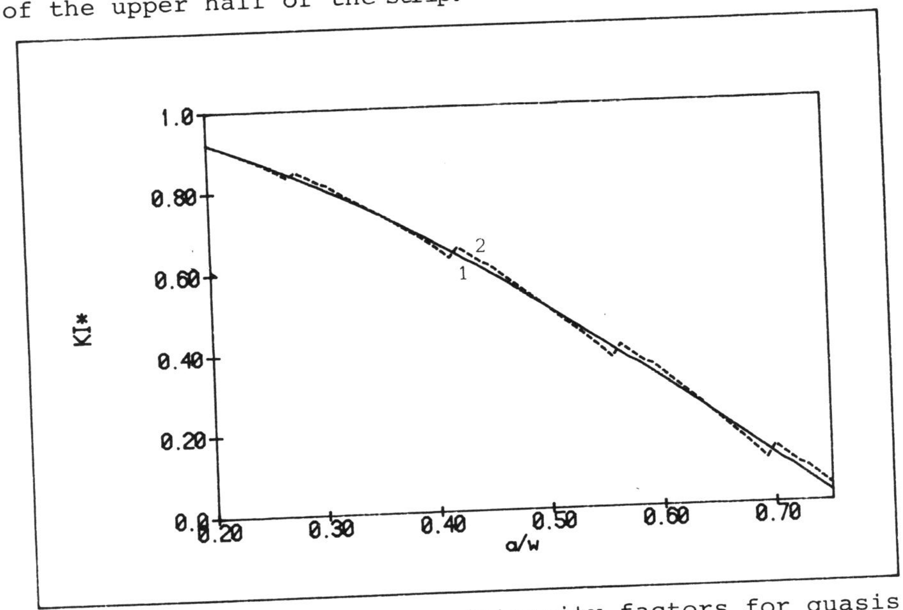

Figure 9. Normalized stress intensity factors for quasistatic propagation: (1) moving element, (2) moving intermediate node.