THE DEVELOPMENT OF A SINGLE SPECIMEN UNLOADING COMPLIANCE TEST SYSTEM

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This paper describes an investigation to assess the errors which can arise in crack growth resistance curves measured using the unloading compliance method. The results of the study are presented for both the compact and single edge notch bend specimen geometries. These results should not only enable the experimentalist to assess the performance of a computerised unloading compliance test system, but also provide the necessary information to select combinations of specimen size, a/W ratio, and transducer working range to ensure the required level of performance.

INTRODUCTION

One of the most popular methods of measuring a crack growth resistance curve (R-curve) from a single specimen is the unloading compliance technique. This method relies on the principle that when a cracked body which has undergone some plastic straining is unloaded it behaves in an elastic manner and its compliance (reciprocal of the load, displacement gradient) is a function of crack length. Therefore by repeatedly unloading the body the crack lengths at various amounts of plastic strain can be estimated. At each unloading the fracture parameter of interest and the crack length (from the unloading compliance) are calculated, thus enabling an R-curve for the material to be constructed from a single specimen. This method requires careful experimentation and sophisticated testing equipment to realise its full potential. For this reason there has been an increasing trend to develop computerised unloading compliance test systems which provide a fully automated test procedure.

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This paper describes an investigation to determine how the performance of a computerised unloading compliance system is affected by:

1. Inaccurate estimation of the material's Young's modulus.
2. Inaccurate estimation of the effective thickness of sidegrooved specimens.
3. The original a/W ratio of the test specimen.
4. The resolution of the load and displacement channels.
5. Transducer signal noise.
6. The frequency of logging data.
7. Transducer non-linearity.

The results are presented for both the single edge notch bend (SENB) and compact specimen geometries. These results should not only enable the experimentalist to assess the performance of an unloading compliance system for different sized specimens of different materials, but also provide the necessary information to select combinations of specimen size, a/W ratio and transducer working range to ensure the required level of performance.

GENERAL COMPLIANCE RELATIONSHIPS

Since the majority of unloading compliance tests are performed on either SENB or compact specimens, this paper concentrates primarily on these two specimen geometries.

The compliance relationships for the compact and SENB specimen geometries which were used throughout this investigation are given by Equations 1 and 2 respectively. These relationships were originally proposed by Saxena and Hudak (1) and Joyce et al (2).

Compact specimen

$$a/W = 1.000196 - 4.06319\mu + 11.242\mu^2 - 106.043\mu^3 + 464.335\mu^4 - 650.677\mu^5 \qquad [1]$$

where $\mu = \dfrac{1}{\left[EB\dfrac{V}{P}\right]^{\frac{1}{2}} + 1}$ (V = Load line displacement)

SENB Specimen

$$a/W = 0.998265 - 3.81662\mu - 1.80596\mu^2 + 32.3104\mu^3 - 44.156\mu^4 - 52.6788\mu^5 \qquad [2]$$

where $\mu = \dfrac{1}{\left[EB\dfrac{V}{P}\right]^{\frac{1}{2}} + 1}$

(V = mouth opening displacement measured at specimen surface)

Note, when testing sidegrooved specimens the specimen thickness, B, should be reduced by an effective thickness, B_{eff} in the elastic compliance relationships.

The variation of non-dimensional compliance (EB V/P) with a/W ratio is presented in Fig. 1 for the compact and SENB specimen geometries.

The Effect of Specimen Size and a/W Ratio

The main objective of an elastic-plastic unloading compliance test is to generate a crack growth resistance curve where the fracture resistance (usually expressed in terms of CTOD or J) is plotted against crack growth. However since the crack growth is determined from the change in the specimen a/W ratio, which in turn is calculated from the measured change in the specimen compliance, it is clear that for a fixed increment of crack extension the sensitivity of the unloading compliance technique will increase with decreasing specimen size. In reality there is a limiting specimen size below which the accuracy of the measured compliance decreases because of the problems associated with accurately measuring the very small changes in load and displacement encountered during an unloading.

It is also evident from Fig. 1 that for a fixed increment of crack growth the sensitivity of the unloading compliance will increase with increasing initial a/W ratio. For example for crack growth resulting in a 0.1 increase in the specimen a/W ratio, the change in compliance for an initial a/W ratio of 0.6 is more than twice that for an initial a/W ratio of 0.5.

The Effect of Young's Modulus and Specimen Thickness

To enable the specimen a/W ratio to be determined using the unloading compliance method, both the specimen thickness and the elastic modulus of the material at the test temperature must be known in addition to the compliance of the test piece.

When testing non-ferritic materials or weldments the elastic modulus of the material at the appropriate temperature is not always known. Moreover the elastic modulus associated with a fracture toughness specimen is affected by the stress state in the specimen's ligament, ie. it may vary between E (plane stress) and $E' = E/1-\nu^2$ (plane strain). For these reasons it is normal practice to calculate an 'effective' value of Young's modulus (E_{eff}) which will produce agreement between the measured initial crack length and that estimated from unloading compliance.

Whilst it is a reasonably easy task to measure the thickness of a plane sided fracture toughness specimen, errors in the predicted a/W ratio can also arise from inaccurate estimates of the effective thickness of sidegrooved specimens. Although expressions have been proposed to define the effective thickness of sidegrooved SENB and compact specimens it is unlikely that these relationships are exact, as they take no account of the geometry of the sidegrooves (ie. root radius). However as proposed by Steenkamp (3) since both the effective thickness and the elastic modulus appear on the same side of the compliance calibration expression the problem of determining the effective thickness of a sidegrooved specimen can be avoided by defining an effective value of EB; denoted $(EB)_{eff}$. The value of $(EB)_{eff}$ can be calculated from:

$$(EB)_{eff} = \frac{C_{ao}}{(V/P)_o} \qquad [3]$$

where C_{ao} = non-dimensional compliance based on the measured initial crack length and calculated using the appropriate compliance expression.

$(V/P)_o$ = initial compliance of test specimen measured in the elastic regime.

The errors in the estimated a/W ratio (ie. a/W estimated – a/W actual) caused by underpredicting or overpredicting the value of $(EB)_{eff}$ are plotted as a function of the actual a/W ratio in Figs 3 and 4 for the compact and SENB specimen geometries respectively. It is evident from Figs 3 and 4 that apart from producing errors in the predicted a/W ratio, inaccurate estimates of $(EB)_{eff}$ will also produce errors in the predicted crack growth. The general effect of $(EB)_{eff}$ on the predicted crack growth is summarised in Table 1, for specimen a/W ratios between 0.5 and 0.8.

TABLE 1 The Effect of Inaccurate Estimates of $(EB)_{eff}$ on Predicted Crack Growth ($0.5 < a/W < 0.8$)

$(EB)_{eff}$		Predicted Crack Growth. Compact Specimens		Predicted Crack Growth. SENB Specimens	
20%	O	5– 8%	U	4–7%	U
10%	O	2– 4%	U	2–4%	U
10%	U	3– 5%	O	2–4%	O
20%	U	6–11%	O	3–9%	O

O = Overestimated U = Underestimated

It is evident that if $(EB)_{eff}$ is overestimated the crack growth will be underestimated and vice versa; the degree of over or underestimation increasing fairly uniformly with increasing initial a/W ratio.

FACTORS AFFECTING THE PERFORMANCE OF AN UNLOADING COMPLIANCE TEST SYSTEM

General

A fundamental requirement of any unloading compliance system is that it should be capable of accurately measuring the compliance of a fracture toughness specimen at each unloading during a test. In the case of a computerised unloading compliance test system employing a data acquisition unit, the accuracy with which the specimen compliance can be determined depends on the errors associated with analogue-to-digital conversion (including the effect of electrical noise) and the accuracy of the load and displacement transducers.

Errors Arising from Analogue-to-Digital (A/D) Conversion

The errors associated with A/D conversion can be assessed using the following expressions originally proposed by Van der Sluys and Futato (4) which describe the individual load and displacement points measured during an unloading:

$$P_i = R_p \text{ Integer } \left[N_p \frac{i}{n} + 0.5 + \text{noise} \right] \quad i = 1,2,3 \text{ --- } n \qquad [4]$$

$$V_i = R_v \text{ Integer } \left[N_v \frac{i}{n} + 0.5 + \text{noise} \right] \quad i = 1,2,3 \text{ --- } n \qquad [5]$$

where n = number of data pairs recorded during an unloading

b = number of bits in A/D converter

R_p = digital resolution of load signal

$= \dfrac{\text{load range}}{2^{b-1}}$

R_v = digital resolution of displacement signal

$= \dfrac{\text{displacement range}}{2^{b-1}}$

$N_p = \delta P/R_p$

$N_v = \delta V/R_v$

δP = total change in load during unloading

δV = total change in displacement during unloading

In the above expressions N_p and N_v can be considered as the number of digital signal resolutions encountered in an unloading ie. if $N_p = 100$ then the resolution of the load channel is 1/100th of the load range (δP) associated with the unloading. Henceforth N_p and N_v will be referred to as the 'effective' load and displacement resolutions.

To quantify the errors associated with A/D conversion, a sensitivity study was undertaken. This study consisted of converting an imaginary continuously varying signal with a true slope equal to one into a set of imaginary load and displacement data using Equations 4 and 5 respectively. The noise was simulated by a random non-integer number in the range -d to +d where d is an integer multiple of the appropriate channel resolution. A linear regression analysis was performed on the converted data to determine the slope of the best fit straight line. This procedure was then repeated 1000 times for each combination of effective load and displacement resolution studied, and the minimum (Smin) and maximum (Smax) slopes were determined. The maximum range of error of the calculated slope, ΔS, was then calculated using the expression:

$$\Delta S = (Smax - Smin)/2$$

(Note the maximum percentage error in the measured compliance of a fracture toughness specimen caused by A/D conversion is given by $100\Delta S$).

It is important to appreciate that this model is only concerned with the errors associated with A/D conversion and consequently it is assumed that:

1. The compliance expressions are exact.
2. There are no errors associated with either the specimen thickness, elastic modulus, and the original analogue signals produced by the load and displacement transducers.

Determining the effective load and displacement resolutions. Since the majority of unloadings during a test occur between the general yield load (P_Y) and the limit load (P_L) a lower bound estimate of the range of load corresponding to an X% unloading can be determined from the following expressions.

a) Compact Specimens

$$\delta P_x = \frac{X}{100} \frac{B(W - a)^2 \sigma_{YS}}{2W + a} \qquad [6]$$

b) SENB Specimens

$$\delta P_x = \frac{X}{100} \frac{B(W-a)^2}{3W} \sigma_{YS} \qquad [7]$$

The effective resolution of the load channel for an X% unloading is therefore given by:

$$N_p = \delta P_x / R_p \qquad [8]$$

The calculation of the displacement range encountered during an unloading is more complex as it is a function of the specimen compliance. The displacement range for an X% unloading at general yield conditions can be determined from the expression:

$$\delta V_x = \frac{X}{100} \frac{P_Y C}{EB} \qquad [9]$$

where C = non-dimensional compliance of the specimen

The effective resolution of the displacement channel for an X% unloading is therefore given by:

$$N_v = \delta V_x / R_v \qquad [10]$$

To simplify the calculation of δV Figs 4 and 5 show the variation of $\delta V.E/\sigma_{YS}W$ with a/W for 10, 15 and 20% unloadings at general yield, for the compact and SENB specimen geometries respectively.

Results of the sensitivity study. The initial stages of the sensitivity study were concerned with determining the errors associated with A/D conversion for different combinations of effective load and displacement resolutions. A typical set of results for a SENB specimen with an a/W ratio of 0.6 are presented in Fig. 6 for a transducer signal noise level of ± 2 resolutions. The results were calculated assuming 40 data pairs were available for the compliance calculation. It is clear that the errors in a/W associated with A/D conversion are basically dependent on the lower effective resolution. This is particularly significant in unloading compliance testing where the effective load resolution is generally very much larger than the effective displacement resolution. This arises because with most modern test machines the operator can select an output signal range for the load signal which is not more than twice the calculated limit load of the specimen. Consequently for a typical 20% unloading performed during an unloading compliance test the load range corresponds to at least 10% of the full range of the A/D converter. In comparison the displacement range produced by a 20% unloading may only represent 1% of the total displacement range over which the A/D converter is calibrated. For this reason the remaining results of the sensitivity study assume an N_p/N_v ratio of 10. However, it is clear that the results are representative of any N_p/N_v ratio greater than approximately 2.

The effect of transducer signal noise. The maximum percentage errors in the measured compliance resulting from different levels of transducer signal noise are plotted as a function of N_v in Fig. 7. It is clear that for a given value of N_v the error in the compliance is approximately proportional to the level of signal noise.

The effect of logging frequency. In addition to studying the effects of effective load and displacement resolution, and transducer signal noise on the measured compliance, the effect of logging frequency was also investigated. The results of this study are summarised in Fig. 8. The results are presented for the cases of 20, 40 and 100 data points being recorded during an unloading. It is clear that the range of errors associated with A/D conversion decrease with increasing logging frequency.

Discussion of results of sensitivity study. It is important to appreciate that the maximum percentage errors in the measured compliance arising from A/D conversion presented in Figs 6-10 are the maximum range of error, and that if a series of unloadings were performed at exactly the same load and displacement levels the measured compliance (C_m) would lie within the range:

$$C_m = C_a \pm \Delta C$$

where C_a = true compliance of specimen

$$\Delta C = 100\Delta S \times C_a$$

The maximum percentage error in measured compliance can be converted to the corresponding error in the predicted a/W ratio using Figs 9 and 10 for the compact and SENB specimen geometries respectively.

The Effect of Load and Displacement Transducers

Apart from errors arising from A/D conversion the level of accuracy to which the compliance of a fracture toughness specimen can be measured by an unloading compliance test system will depend on the quality of the load and displacement transducers. Since the unloading compliance technique relies on measuring small changes in the specimen compliance throughout a test the most important quality of a transducer is linearity. The normal definition of linearity is the maximum deviation between an actual transducer reading and the reading predicted by a straight line drawn between upper and lower calibration points, expressed as a percentage of the working range of the transducer.

Obviously since the majority of unloadings in an unloading compliance test are performed over the same load range the linearity of the load transducer is not as critical as that of the displacement transducer. With this in mind an experimental investigation was undertaken to measure the linearity of several high quality clip gauges and determine the extent to which any non-linearity might affect the compliance measurements made throughout an unloading compliance test.

The tests were performed using two different calibration procedures. The first method consisted of selecting upper and lower calibration points corresponding to the displacement range of interest and assuming the gauge had a perfectly linear response between these limits. The second procedure involved taking a series of calibration points over the displacement range and determining the best fit 3rd order polynomial expression. A typical set of results for a linearity test are shown in Fig. 11. It is clear that for the gauge studied the linear and polynomial calibration procedures result in maximum non-linearities of approximately 0.7% and 0.1% respectively.

Although a maximum non-linearity of 0.7% might be considered adequate for conventional fracture toughness tests it can lead to significant errors in the crack length predictions made during an unloading compliance test. Fig. 12 shows the results of the linearity tests presented in Fig.11 re-analysed to show the variation of the slope of the linearity graphs with increasing displacement. Bearing in mind that a perfectly linear gauge would have a slope of 1, then it is evident that for the gauge studied the linear calibration method would result in the compliance of the specimen at the beginning of a test being overestimated by approximately 3%. Moreover, as the test progressed the degree of overestimation would decrease fairly uniformly with increasing displacement until at the maximum displacment the specimen compliance would be underestimated by approximately 3%. This would result in the crack extension being underestimated. In comparison the variation of the slope produced by the polynomial calibration procedure is negligible (the errors in compliance can be converted to corresponding errors predicted a/W ratio using Figs. 9 and 10).

It is interesting to note that the linear calibration results presented in Fig. 12 could produce apparent negative crack growth. The phenomenon of negative crack growth is generally confined to the early stages of an R-curve (ie before ductile crack initiation) and is characterised by the predicted crack length initially decreasing with increasing measured toughness until a point is reached where both toughness and predicted crack length increase with respect to each other. For example consider a compact specimen with an initial a/W ratio of 0.5 which when tested does not exhibit any stable crack extension below load line displacements of 1mm. Based on the results presented in Figs. 9

and 11 the predicted a/W ratios at 0 and 1mm load line displacement would be approximately 0.506 and 0.502 resulting in an apparent negative crack growth of 0.004W. It should be noted that non-linear clip gauges may also cause errors in the calculated value of $(EB)_{eff}$.

CONCLUDING REMARKS

A detailed assessment of the unloading compliance technique has been undertaken. It has been shown that errors in predicted crack length can arise from the following sources:

1. Inaccurate estimates of the elastic modulus (E) of the material and/or the effective thickness (B_{eff}) of sidegrooved specimens.

2. Errors associated with A/D conversion and transducer signal noise.

3. Non-linear load and displacement transducers.

Although the errors associated with A/D conversion and transducer signal noise generally determine the quality of the unloading compliance data in terms of scatter they should not produce significant errors in the predicted crack growth. In comparison errors arising from sources 1 and 3 can result in significant errors in the predicted crack growth.

Finally it has been demonstrated that if transducer non-linearity is not adequately accounted for by the calibration procedure, apparent negative crack growth may occur in single specimen unloading compliance R-curves.

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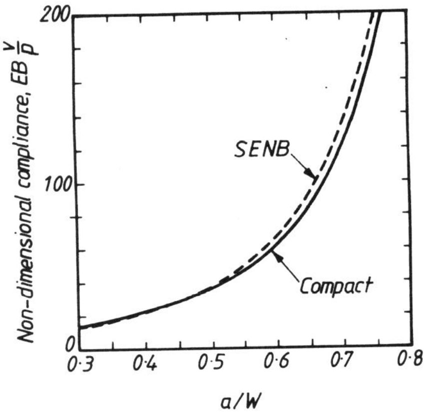

Fig.1. Variation of non-dimensional compliance with a/W ratio for compact and SENB specimen geometries (equations [1] and [2]).

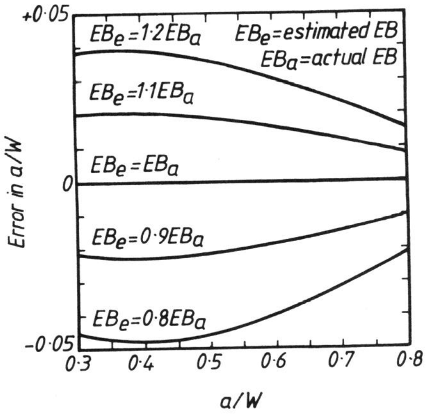

Fig.2. Errors in predicted a/W ratio resulting from inaccurate estimates of EB, for the compact specimen geometry.

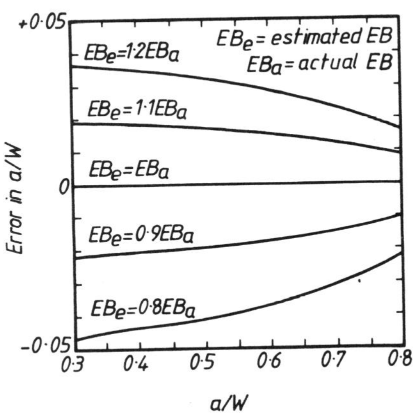

Fig.3. Errors in predicted a/W ratio resulting from inaccurate estimates of EB, for the SENB specimen geometry.

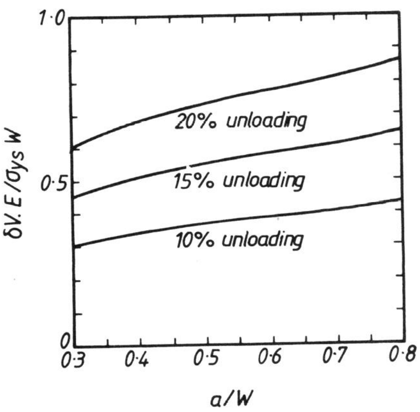

Fig.4. Variation of $\delta V.E/\sigma_{ys}W$ with a/W for compact specimen geometry.

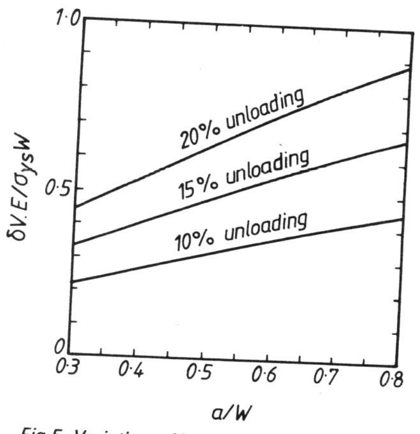

Fig.5. Variation of $\delta V.E/\sigma_{ys}W$ with a/W for SENB specimen geometry.

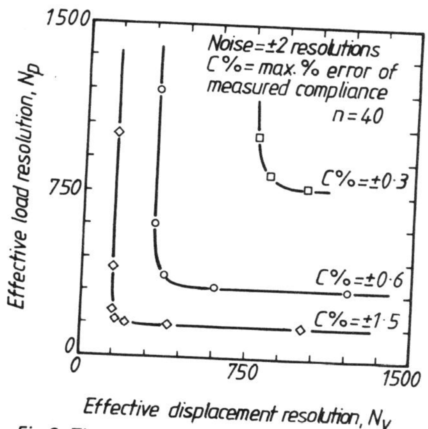

Fig.6. The effect of load and displacement resolution on the accuracy of the measured compliance.

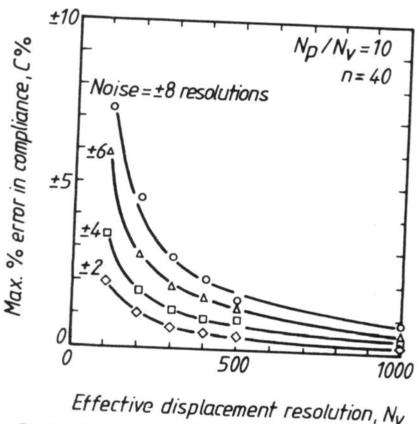

Fig.7. The effect of transducer signal noise on the accuracy of the measured compliance.

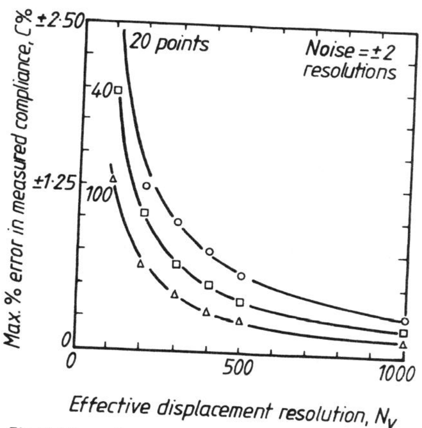

Fig.8. The effect of logging frequency on the accuracy of the measured compliance.

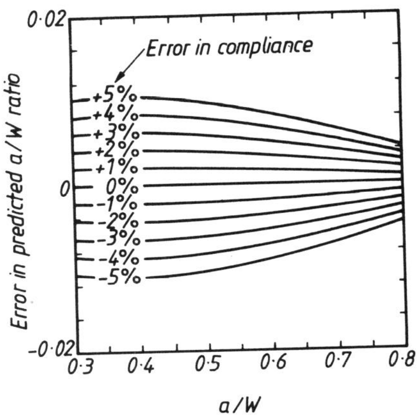

Fig.9. Error in predicted a/W ratio arising from errors in measured compliance for a compact specimen (equation [1]).

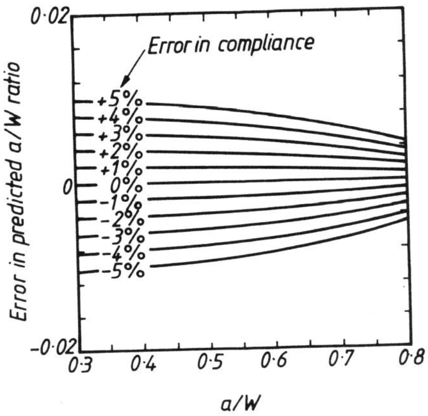

Fig.10. Error in predicted a/W ratio arising from errors in measured compliance for SENB specimen (equation [2]).

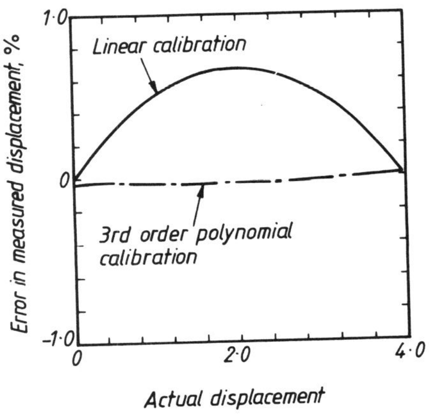

Fig.11. Results of linearity test on clip gauge.

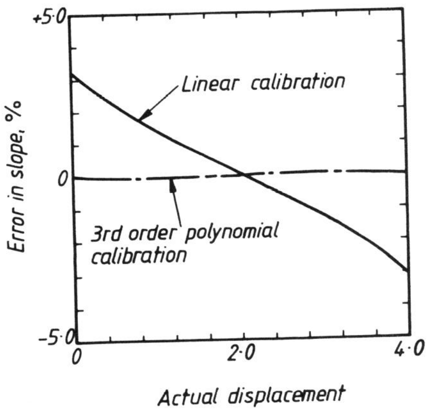

Fig.12. Variation of the slope of linearity graphs.