DEFORMATION OF CRACKED BODIES

V.M. Radhakrishnan.*

Deformation of a body containing a moving crack will be a function of both the applied stress and the crack length. Based on a phenomenological approach, an analysis is presented to discuss the cases where crack propagation takes place under constant load and constant strain rate.

INTRODUCTION

Deformation and its rate of a body containing a moving crack are controlled and contributed by both the load acting on the body and the crack length and its growth rate. A typical example where such a situation arises is the crack growth due to creep which is recognised as one of the major causes of failure of components in high temperature pressure vessels. Several parameters like stress intensity factor, reference stress, crack opening displacement and energy rate line integral C* have been tried to correlate the crack growth rate, da/dt, with varying degrees of success. Of these, C* has been used to a great extent recently. However, the line integral is applicable to (non-linear) elastic materials only, where the energy required to crack propagation is the surface energy S of the newly created surfaces. If two specimens with crack lengths a_1 and a_2 are loaded to the same deflection Δ_1, as shown in Fig. 1(a), the energy difference is only that due to the new surface formation. But when a crack in an engineering material moves from a_1 to a_2 due to the load

* Department of Metallurgical Engineering, I.I.T., Madras-600 036, India.

application (Fig. 1 b) the energy supplied will be not only for the new surface creation but also for plastic deformation near the crack zone. Though the initial and final conditions of the cases in (A) and (B) are the same, the energy requirements are different. It should also be noted that, unlike the Griffith approach of energy balance, the line integral approach is not based on any energy balance fracture criterion. For a realistic approach, the energy supplied to the material must be equated to the energy absorbed in (a) the body in general (b) the volume of material just ahead of the crack and (c) the creation of new surfaces. To achieve this and characterize the crackgrowth rate, the fracture zone at the tip of the crack has to be defined, its dependence on stress and crack length has to be evaluated and the energy absorbed has to be computed - a problem that cannot be easily solved without many assumptions. Hence it is doubtful how effectively the energy rate line integral can be extended to engineering materials.

In the following a phenomenological approach to crack growth in a deforming body under the action of the applied stress is discussed which is able to describe well the crack growth rate under both constant load and constant deflection rate conditions.

PARAMETER DEVELOPMENT

Consider a body containing a crack subjected to a load. The crack is growing and the load point deflection is contributed by both crack growth and creep deformation at the net section, as very often happens in bending type of loading. The Isochronous load-deflection curve, as shown in Fig.2., can be given by

$$\Delta = C_{(t)} \; (P/P_o)^{\alpha} \qquad (1)$$

where $C_{(t)}$ is the time dependent compliance. P_o is a constant. With time both the crack length $\underline{a}$ and the compliance increase. The change in compliance dC with change in crack length da at larger values of $\underline{a}$ will be more than at smaller values of $\underline{a}$. So the compliance can be given as

$$C_{(t)} = a_o \; (a/a_o)^{\gamma} \qquad (2)$$

as shown in Fig.3. a_o and γ are constants. The reasonableness of the assumption can be explained as follows. Considering equation (1) , the compliance is equal to the load point deflection at load $P = P_o$, where P_o is a reference load which can be taken to be small enough as to cause negligible creep deformation, so that

$$\Delta = C_{(t)}\Big|_{P=P_o} \qquad (3)$$

In such a case any increase in load point deflection will be caused by the increase in crack length, so that

$$\Delta \propto (a)^{\gamma} \qquad (4)$$

So it can be taken that the compliance $C_{(t)}$ is also dependent on the crack length $\underline{a}$ through a similar relation as given in eqn.(2). Now from eqn(1) we get

$$\dot{\Delta} = \gamma (a/a_o)^{\gamma-1} (P/P_o)^{\alpha} \dot{a} + a_o (a/a_o)^{\gamma} \alpha \frac{P^{\alpha-1}}{P_o^{\alpha}} \dot{P} \qquad (5)$$

In the case of brittle materials where the contribution to deformation by load P will be negligible, the exponent α in eqn (1) will tend to zero and the deflection rate will be governed by the crack length and its growth rate only.

DATA CORRELATION

Under constant load condition, the loading rate $\dot{P}$ will be equal to zero and so we get the crack growth rate $da/dt \propto R$ where the parameter R is given as

$$R = \dot{\Delta} / \left(\frac{P}{P_o}\right)^{\alpha} \left(\frac{a}{a_o}\right)^{\gamma-1} \qquad (6)$$

For bending type of loading

$$R = \dot{\theta} / \left(\frac{M}{M_o}\right)^{\alpha} \left(\frac{a}{a_o}\right)^{\gamma-1} \qquad (7)$$

where P and M are load and bending moment per unit thickness. The above relation has been shown to describe well the experimental data on 6061 Al alloy tested at different temperatures (1,2) with centre notch (CC) and deep notch CT type specimen geometries, as shown in Fig.4.

In the case of creep brittle materials which show negligible deformation, the load point deflection will be contributed by the crack growth only. In such a case, the value of the exponent in eqn.(1) will tend to zero and the crack growth rate will depend on load point deflection rate, as has been shown in the case of 6242 Ti alloy (3).

Many experiments have also been carried out under constant deflection rate controlled conditions. Two such cases for type 316 stainless steel tested at 594°C are given in Figs. 5 and 6, the raw data taken from references (4,5). In the first case it is a centre crack type of specimen. The total deformation over a gage length of 100 mm was only around 2.5 mm. Since the total deflection was rather small, 316 type stainless steel can be considered as creep brittle material at 594°C in the presence of a dominant crack. In the second case, it is a CT type of specimen. Here also the total deflection was around 4 mm. It can be seen that the load drops in both the cases after reaching a maximum, but the crack length continues to increase. It can be observed that there are well defined regions where $\dot{a}$, $\dot{\Delta}$, $\dot{P}$ remain constant. The respective values of these rates are given by the side of each curve. The full lines in Fig. 5 are due to the present author. The units are the same as given in the reference. From eqn. (5) the crack growth rate is given by

$$\dot{a} = \frac{\dot{\Delta}}{\gamma}\left(\frac{a_o}{a}\right)^{\gamma-1}\left(\frac{P_o}{P}\right)^{\alpha} - \frac{\alpha}{\gamma}\frac{a}{P}\dot{P} \qquad (8)$$

Since the material 316 type stainless steel at 594°C shows negligible deformation in the presence of a dominant crack, the exponent α can be taken to be zero in which case a correlation between $\dot{\Delta}$ and $\dot{a}$ should be obtained. Fig. 7 shows the relation between $\dot{a}$ and $\dot{\Delta}$. The fit appears to be good; nevertheless there is some scatter. This indicates that the material at 594°C may be in the transition region from creep brittleness to creep ductility condition. A value of $\alpha = 0.5$ is tried with the constants $a_o = 1$ mm and $\gamma = 1$ and the result is shown in the same figure. The description appears to be very good, specially so, when the crack growth data for two geometries obtained at two laboratories could be correlated with the proposed parameter.

CONCLUDING REMARKS

The phenomenological approach to crack growth problem is based only on the load-deformation relation of the body in the presence of a dominant crack which is growing. It does not assume any criterion for fracture at the crack tip. The relation developed appears to be applicable to cases of crack growth under constant load creep conditions. In such a situation, at any given time equation(1) will be valid with a positive value of the exponent α. The loading rate dP/dt will be zero. In the case of constant deflection rate controlled condition, one has to be careful in interpreting the eqns. (1) and (5). The crack length increases with decreasing loading rate. So dP/dt will be negative. Hence, in the Isochronous deformation - load relation, the value of the exponent may turn to be negative. With proper interpretation of the loading conditions, eqn. (5) can then be successfully used to describe the deformation rate of a body in which a dominant crack is propagating under the action of the applied load.

As pointed out earlier, the other parameters do not correlate well the crack growth rate under creep conditions. The scatter is very large. Even the energy rate integral C* which is often tried to correlate crack growth rates shows systematic scatter (6) and the scatter band is of one order of magnitude. In the absence of a well defined and developed fracture criterion, the above analysis appears to give a reasonable approach to describe the crack growth rates in deforming bodies.

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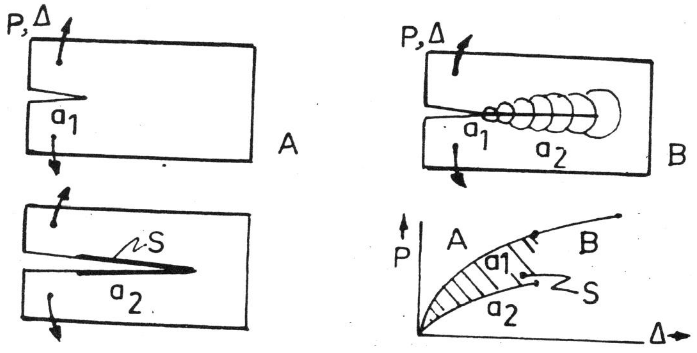

Figure 1 Crack growth in elastic and engineering materials

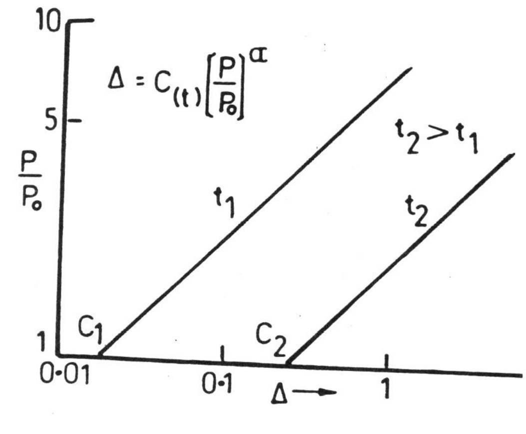

Figure 2 Isochronous P-Δ relation

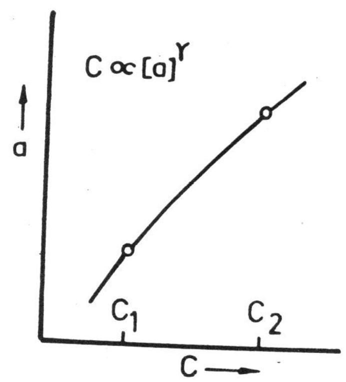

Figure 3 Compliance vs. a

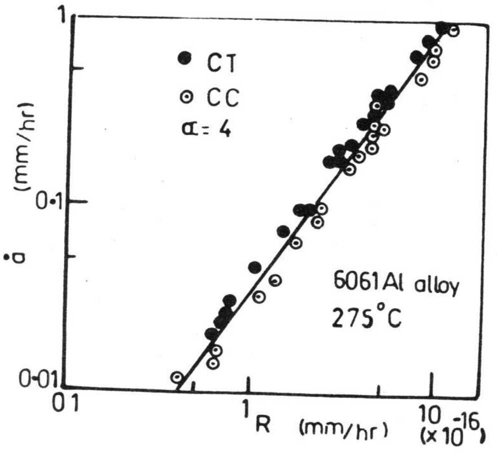

Figure 4 ȧ vs. the parameter R

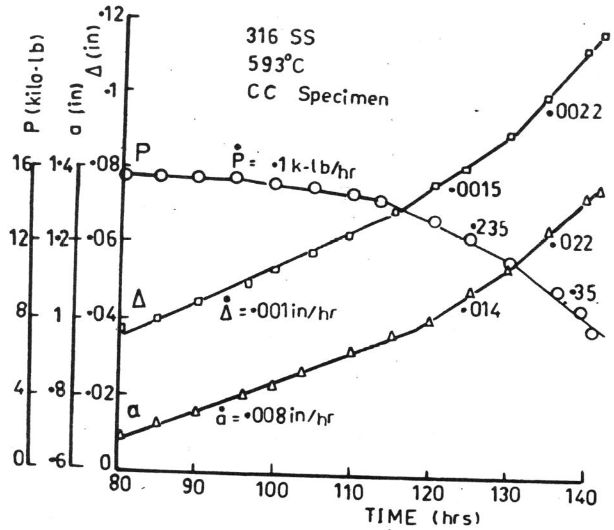

Figure 5 Variation of P, Δ, a with time

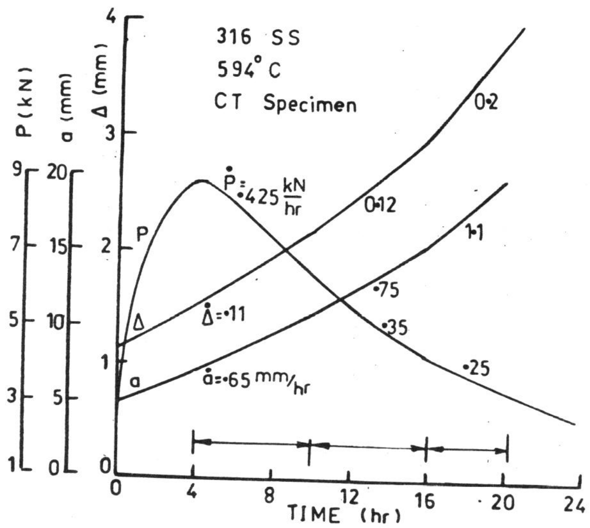

Figure 6 Variation of P, Δ, a with time

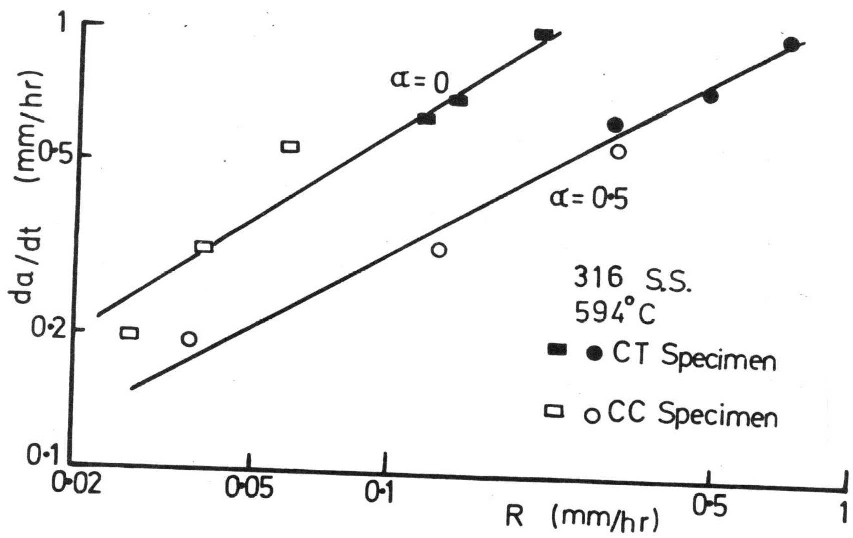

Figure 7 ȧ vs. the parameter R