POTENTIAL USE OF FRACTURE MECHANICS IN THE FATIGUE DESIGN OF FILLET WELDED JOINTS

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The object of this paper is to describe an approach based on linear elastic fracture mechanics to the calculation of the fatigue life of two fillet welded joints - a cruciform joint with non-load carrying weld and a tubular joint. Particular attention is focused on the cruciform joint and on a probabilistic study of several random variables such as fatigue life, defect size, exponent of the propagation law, etc...

INTRODUCTION

The fatigue fracture of fillet welded joints usually results from the propagation of cracks initiating from existing defects such as undercuts at the toes of the welds (cf. fig. 1). It is this type of weld defect which is considered in this paper. The use of fracture mechanics concepts for the fatigue life calculation of fillet welded joints allows a better comprehension of the influencing parameters and the crack propagation mechanism.

The practicability and reliability of this approach comes up against an important and essential difficulty related to the size of the initial crack considered as being existent in the welded joint.

There are nevertheless, cases in which this approach may be a precious design tool, notably :

1. when the initial defect size is known and may be taken as the initial crack size;

2. when the relative influence of various parameters such as the geometry of the joint, plate thickness, stress range, etc. on the fatigue life of the joint has to be determined;

3. when carrying out a sensitivity analysis of the influence that the defect size exerts on the fatigue life;

4. when carrying out a probabilistic study of certain random variables such as fatigue life, defect size, exponent of the propagation law, etc.

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EVALUATION OF THE RELATIVE FATIGUE LIFE

The present study is founded on the application of the PARIS relation whose validity has been repeatedly verified by fatigue tests carried out under constant amplitude loading; this relation is written as :

$$da/dN = D(\Delta K)^n \qquad (1)$$

with : D and n - two constants depending on the material in which the crack propagation occurs.

$\frac{da}{dN}$ - Rate of crack propagation per cycle

ΔK - variation of the stress intensity factor (K).

By integrating the equation (1), the following expression is obtained for the joint life :

$$N_R - N_I = \frac{1}{D} \int_{a_i}^{a_f} \frac{da}{(\Delta K)^n} \qquad (2)$$

with : N_R - number of cycles to failure

N_I - number of cycles to crack initiation

a_i, a_f - Length of respectively the initial crack and the final crack resulting in failure.

The following assumptions have been made :

- The crack length from which the so-called "crack propagation" begins, is taken as the initial defect size.
- The length of the final crack corresponds to a through crack.

An appropriate calculation of ΔK in the case of a non-load carrying cruciform joint (figure 2) leads to the following expression for the life duration (1).

$$N_R = N_I + \frac{1}{D} \cdot C_o^{-n} \cdot S_r^{-n} \cdot T^{1-n/2} \cdot I_c(\frac{a_i}{T}, C_o, n) \qquad (3)$$

with :

$$I_c(\frac{a_i}{T}, C_o, n) = \alpha(C_o - 1) + \beta(\frac{a_i}{T})^\gamma \qquad (4)$$

$\alpha = 0{,}124 \exp(0{,}28n)$

$\beta = 0{,}459 \exp(-0{,}657\, n)$

$\gamma = -\,0{,}122\, n^{1{,}291}$

parameters depending only on the exponent n

C_o - factor depending on the residual welding stresses.

(When $C_o = 1$, the influence of the residual stresses is neglected).

Equations (2) and (3) permit the calculation of the number of cycles to failure N_R, provided that certain assumptions related to the specification of the values for n, D, C_o and N_I are taken into account in the calculation.

Since these values generally present a noticeable scatter, calculations of this type may be subject to sizeable errors. To overcome this difficulty, the ratio N_R/N_R^* is used.

with N_R^* life of similar joint,

and S_r^*, the nominal stress range of this joint

T^*, its plate thickness, and

a_i^*, its size of the initial defect of the joint having a life duration N_R^*.

This relative evaluation of N_R may be appropriate only if adopting the following assumption :

- The variables D, n, C_o and $\frac{N_I}{N_R}$ have at least the same distribution law in each joint belonging to the sample.

By starting from equations (3) and (4) and neglecting the influence of the welding residual stresses ($C_o = 1$), the latter assumption leads to the relation;

$$\frac{N_R}{N_R^*} = (\frac{S_r}{S_r^*})^{-n} \cdot (\frac{T}{T^*})^{(1-\frac{n}{2}-\gamma)} \cdot (\frac{a_i}{a_i^*})^{\gamma} \qquad (5)$$

with $\gamma = -0{,}122\, n^{1{,}291}$

Equations (3) and (4) as well as (5) are therefore the basis for the approach to the four topics mentioned in the introduction.

STUDY OF SOME PARTICULAR CASES RELATED TO THE CRUCIFORM JOINT

The fracture mechanics concept outlined above has been successfully used for the analysis of various welded joints. In the following the application of the method to the cruciform joint is illustrated.

Life calculation of the joint

For a given size of initial defect, the life duration of the joint may be calculated by applying equations (3) and (4) provided that the following values are determined :

* N_I - life duration related to the initiation of an actual crack, a value often considered as negligible (3,4); but Yamada et al. (2) note that the crack initiation may represent up to 40 % of the total life in certain cases. For a joint with an actual crack detected during its life, the calculation of the remaining life duration will have N_I=0.
* n and D- the two parameters of the Paris Law, which depend a priori on the quality of the parent metal. Several authors concur in the belief that these two parameters are related (5), with the average value for n being about 3,75. There is considerable dispersion in the values of both parameters however.

C_o - is parameter, which may be taken as 1., when the influence of the welding residual stresses is not considered. A statistical study of a large number of samples would be necessary for assigning a value to this parameter.

A discerning selection of these values leads to a good agreement between experimental tests and theoritical analysis.

Relative influence of certain parameters. For the cruciform joint, the analysis concerns essentially the influence exerted by the wall thickness, the nominal stress range and the size of the initial defect on the life duration of the joint.

By applying the eq.(5) and for :

$$S_r = S_r^* \qquad a_i = a_i^*$$

the influence of the wall thickness is written as :

$$\frac{N_R}{N_R^*} = \left(\frac{T^*}{T}\right)^{(\gamma + \frac{n}{2} - 1)} \qquad (6)$$

This expression is in conformity with the test results of Booth (6), plotted in figure 3. As a matter of fact, if taking n as the slope of the S-N curve obtained by linear regression, i.e. n = 3,75, in each of the two samples with thicknesses T = 25mm and = 38 mm, the eq.(6) is written as :

$$N_R/N_R^* = 1.92$$

This result should be compared with $N_R/N_R^* = 2$ obtained from using the experimental results.

As regards the influence of the nominal stress range, for $T = T^*$ and $a_i = a_i^*$ eq.(5) leads to a result, which can be obtained similarly by the S-N curve approach, i.e :

$$\frac{N_R}{N_R^*} = \left\{\frac{S_r}{S_r^*}\right\}^{-n} \qquad (7)$$

And finally for $T = T^*$

$S_r = S_r^*$
eq.(5) is written as :

$$\frac{N_R}{N_R^*} = \left\{\frac{a_i}{a_i^*}\right\}^{\gamma} \qquad (8)$$

This equation permits one to analyse the influence of the initial defect size, and thus represents a precious tool for the study of problems related to the monitoring of crack propagation and "remaining" fatigue life estimation.

An approach to the evaluation of the statistical distribution of the initial defect size. The size of an initial defect in a joint is a random quantity. Both the location of a defect and the determination of its size set difficult problems from the experimental point of view. In order to avoid this difficulty, the present paper describes a simple method for the evaluation of the statistical distribution of defects in the fillet welded joints.

This method employs well defined and relatively well measured experimental quantities.

Starting from equations (3) and (4), the ratio a_i/T is expressed as follows :

$$\frac{a_i}{T} = \left[\frac{1}{\beta}\left\{(N_R - N_I).D.C_o^n.S_r^n.T^{(n/2-1)} - \alpha(C_o - 1)\right\}\right]^{1/\gamma} \qquad (9)$$

The thus calculated a_i/T ratios present an appreciable scatter due not only to the scatter of the experimental values N_R but also due to the values assigned to D and n, the two parameters of the Paris relation

To circumvent this disadvantage the procedure consists in acting upon the ratio a_i/a_{max}, with :

a_{max} - the maximum value of the defect in the sample conventionally taken as a function of the plate thickness T and the welding process.

Consider now a sample of fatigue tests :

- having the same plate thickness T, and
- with all the test specimens being manufactured using the same welding process.

For this particular case, the following assumption is made :
- D, n, T, Co, a_{max} and N_I/N_R are constant over the whole sample.

By adopting this assumption and applying the equations (3) and (4), the following expression is obtained :

$$\left\{\frac{a_i}{a_{max}}\right\}^{\gamma} = \frac{N_R.S_r^n}{N_R^*.S_r^{*n}} \cdot (\tau + 1) - \tau \qquad (10)$$

with : $$\tau = \frac{\alpha(C_o - 1)}{\beta(a_{max}/T)^{\gamma}}$$

$N_R^*.S_r^{*n} = \inf(N_R.S_r^n)$ .... in the sample.

A discerning selection of the sample allows one to investigate the statistical distribution of the variable a_i/a_{max}.

Numerical applications. Among the experimental results (7), two samples of tests specimens have been selected for application of the equation (10). The characteristics of these two samples are :

Sample 1 :

- Automatic inert-gas arc welding, basic electrode,
- plate thickness T = 12 mm,
- sample size = 197 tests.

Sample 2 :

- Manual welding, rutile electrode, horizontal position,
- plate thickness T = 10 mm,
- sample size = 109 tests.

Figure 4 shows the evaluated statistical distribution of a_i/a_{max} in both samples, for $C_o = 1$.

Discussion of the results. - As the value of a_{max} is a constant for the whole sample the statistic distribution of the ratio a_i/a_{max} and of a_i are similar. As concerns the determination of a_{max}, it is still difficult at the present time to make a definite statement , but cértain authors give global values for a_{max} based on measurement carried out on typical welds. Signes et al. (8) have found that the values of a_{max} are comprised between 0,1 and 0,5 mm (as an example, one may take a_{max} = 0,5 mm).

Watkinson et al. (9) give a value of a_{max} = 0,4 mm for manual arc welds. Finally, in order to investigate the uncertainty of the fatigue resistance of welded joints, Engesvik and Moan (10) have used truncated distribution laws of the initial defect a_i, such as $a_i \in$ (0,075, 0,4 mm) i.e. a_{max} = 0,4 mm.

- The distribution of the ratio a_i/a_{max} is closely related to the value of n; figure 4 shows this dependant relation for the two samples under consideration. The choice of a value for n should therefore be based on the experimental finding according to which both initiation and propagation of the crack occur in the heat affected zone (HAZ). As the size of this zone and the mechanical properties depend on the welding process, it is difficult to propose a value for n. Nevertheless, some authors (5.10) consider the distribution for n to be normal, and it is this of distribution which is adopted in the next paragraph.

- An examination of the results of figure 4 indicates that they may be represented by a lognormal or Weibull-type distribution. Note that when the histogram is drawn with relatively large intervals, it could be approximated by a Laplacien distribution.

Analysis of the probabilistic interaction of certain parameters.
In equation (5), the variable $(N_R/N_R^*) = \mathsf{N}$ may be determined with the variables n, $(T/T^*) = \zeta$, $a_i/a_i^* = \mathfrak{a}$, and $S_r/S_r^* = \mathsf{S}$.

Tests have shown that some of these variables are random variables. Knowing their distribution laws allows one to determine the law for and as a consequence, the uncertainties relative to the ratio of life durations for two states of cracking in a given welded joint.

The distribution law of the variable n is Gaussian, mean 3,76 and standard deviation ± 0,87; cf. figure 5a (5). As concerns the distribution law of the random variable $\mathfrak{a}$, the statistical results of 829 measurements for butt and fillet welds, given in (4), have been used. In 504 cases, the defects were not perceptible or measurable, they can thus be considered as being lesser than 0,05 mm. For the remaining measurable defects, the histogram in figure (5.b) has been obtained, with a_i^* = 1 mm, which can be represented by a log-normal law with an average value of 0,089 and a standard deviation of 0,0887. Other distribution laws for n and $\mathfrak{a}$ could also be adopted if desired.

The other two variables ζ and S on which depends N are deterministic in most cases. If that is not the case, their introduction as random variables presents no difficulties (*).
For the present numerical application, they are considered as being deterministic, and are taken as being unity.
Equation (5) is written as :

$$\mathsf{N} = \mathsf{S}^{-n} \cdot \zeta^{1-\frac{n}{2}-\gamma} \cdot \mathfrak{a}^{\gamma} \tag{11}$$

This relation is represented approximately as a linear function by expansion in a Taylor series corresponding to the mean values $\bar{\mathfrak{a}}$ and $\bar{n}$ of the basic variables $\mathfrak{a}$ and n, with the nonlinear terms being neglected. Thus :

$$\mathsf{N} = \bar{\mathsf{N}} + (n-\bar{n})\overline{\left\{\frac{\partial \mathsf{N}}{\partial n}\right\}} + (\mathfrak{a}-\bar{\mathfrak{a}})\overline{\left\{\frac{\partial \mathsf{N}}{\partial \mathfrak{a}}\right\}} \tag{12}$$

where :

$\bar{\mathsf{N}}$ = average of N calculated from $\bar{n}$ and $\bar{\mathfrak{a}}$.

$\overline{\left\{\frac{\partial \mathsf{N}}{\partial n}\right\}}, \overline{\left\{\frac{\partial \mathsf{N}}{\partial \mathfrak{a}}\right\}}$ = averages of the partial derivatives $\frac{\partial \mathsf{N}}{\partial n}$ and $\frac{\partial \mathsf{N}}{\partial \mathfrak{a}}$ calculated from $\bar{n}$ and $\bar{\mathfrak{a}}$

(*)For the joints submitted to random loadings (wind, wave etc.), the variable S contributes to the probabilistic evaluation of N.

The Mean First Order Second Moment Method (MFOSM) (11) allows to determine by applying eq. (12), the distribution function of N and its probability density. Figure 6 shows the probability density of N for the particular case when $\zeta = S = 1$, and when ξ and n are two independant variables.

TUBULAR JOINTS

By using a simplified approach, the fatigue life of tubular joints (12) is written as :

$$N_R - N_I = \frac{1}{D} \cdot S_r^{-n} \cdot (T_m)^{\frac{1-n}{2}} \cdot (n+1) \cdot I_t\left(\frac{a_i}{T_m}, n, C_o\right) \cdot (SCF)^{-n} \qquad (13)$$

with (cf. figure 7) :

S_r = nominal stress range in the chord,

SCF = stress concentration factor at chord-brace junction due to the global geometry.

T_m = chord wall thickness.

As for the case of the cruciform joint, one assumes that the integral I_t. can be written as :

$$I_t\left(\frac{a_i}{T_m}, n, C_o\right) = \bar{\beta}\left(\frac{a_i}{T_m}\right)^{\bar{\gamma}} + \bar{\alpha}(C_o - 1) \qquad (14)$$

The ratio N_R/N_R^* is again considered for similar reasons. Consider a sample of fatigue tests whose test specimens have all been manufactured of the same steel grade and by the same welding process. This allows one to assume that the variables D, n, C_o and N_I/N_R have at least the same distribution law in each joint belonging to the sample and that, when neglecting the influence of the welding residual stresses (C_o = 1), the ratio N_R/N_R^* may be written as :

$$\frac{N_R}{N_R^*} = \left(\frac{S_r}{S_r^*}\right)^{-n} \cdot \left(\frac{SCF}{SCF^*}\right)^{-n} \cdot \left(\frac{T_m}{T_m^*}\right)^{\frac{1-n}{2}-\bar{\gamma}} , \left(\frac{a_i}{a_i^*}\right)^{\bar{\gamma}} \qquad (15)$$

This equation, to be compared with equation (5) for cruciform joints, can be used along with equation (13) for the study of the four topics mentions in the introduction for the case of the tubular joint.

Numerical application. The results of fatigue tests carried out on tubular joints in the framework of a research program "Marine Technology" of the ECCS have been exploited to constitute a sample of 73 fatigue tests with the follow characteristics (13) :

- the failure mode is the same (through crack at the weld toe in the chord) for all the test specimens; see figure 7.
- the fatigue tests have been carried out in air under constant amplitude loading.

According to equation (15), the figure 8 shows the distribution of a_i/a_i^* for n = 3.

CONCLUSIONS

The principal outcomes of the evaluation of the fatigue life in fillet welded joints, based on a fracture mechanics approach as presented here, are the possibility :

- of determining the fatigue life of the joint,
- of taking into account variables which are ignored by the usual S-N curves and of determining their influence on the fatigue life of the joint, and finally, using the results for the monitoring of crack propagation and "remaining" life evaluation,
- of studying the distribution of initial defects based on the fatigue test results with stress ranges at constant amplitude, provided that the sample of tests is correctly selected,
- of analysing the failure probability of a fillet welded joint using the knowledge of the statistical distribution of parameters which may influence the fatigue life.

Finally, it is clear that the approach provides a promising way for the analysis of other types of fillet welded joints.

SYMBOLS

a_i = initial crack length

a_f = through crack length

da/dN = rate of crack propagation

N_I = Number of cycles to crack initiation

N_R = Number of cycles to failure

D, n = two constants in the Paris relation

ΔK = variation of the stress intensity factor.

S_r = stress range

T = Plate thickness in cruciform joint

T_m = chord wall thickness in tubular joint

α, β, γ = Parameters depending only on (n), in cruciform joint

C_o = factor depending of the residual welding stresses

SCF = stress concentration factor in tubular joint

$\bar{\alpha}, \bar{\beta}, \bar{\gamma}$ = Parameters depending only on (n), in tubular joint

$S = S_r/S_r^*$

$\zeta = T/T^*$

$\xi = a_i/a_i^*$

$N = \frac{N_R}{N_R^*}$

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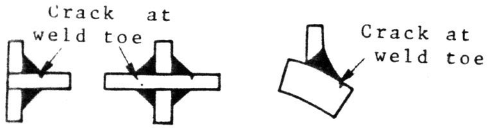

Figure-1 Joints Presenting a Weld Toe Defect

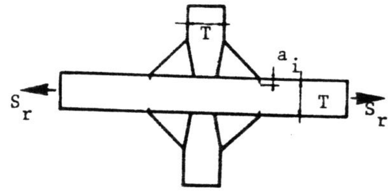

Figure-2 Cruciform non-Load Carrying Weld Joint

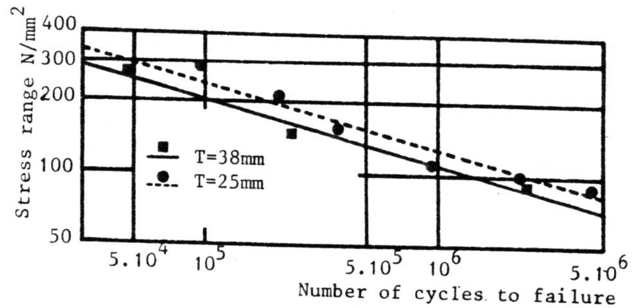

Figure-3 Thickness Influence On Fatigue Life

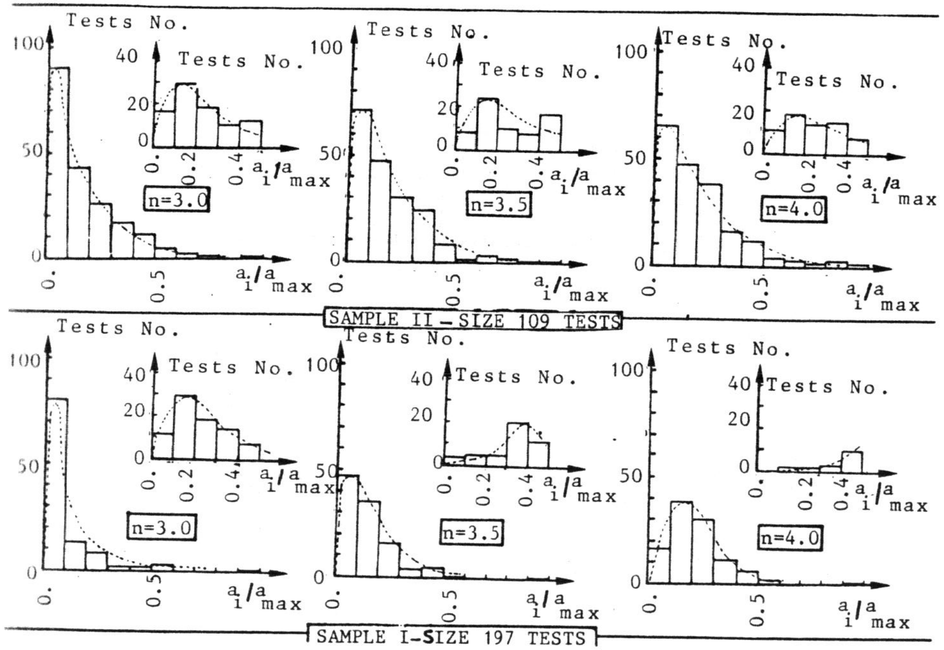

Figure-4 Derived Statistical Distribution of Defect Size in Cruciform Joint

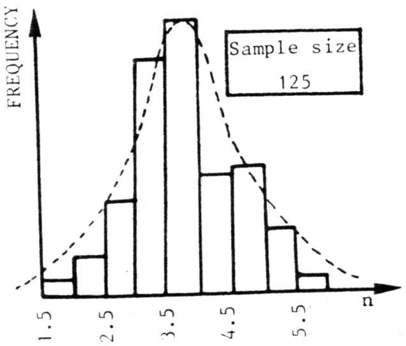

Figure-5-a- Distribution of The Exponent n in The PARIS Relation

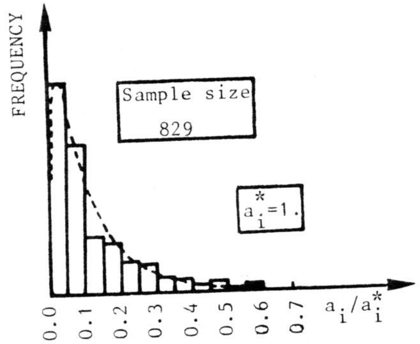

Figure-5-b- Experimental Defect Size Distribution

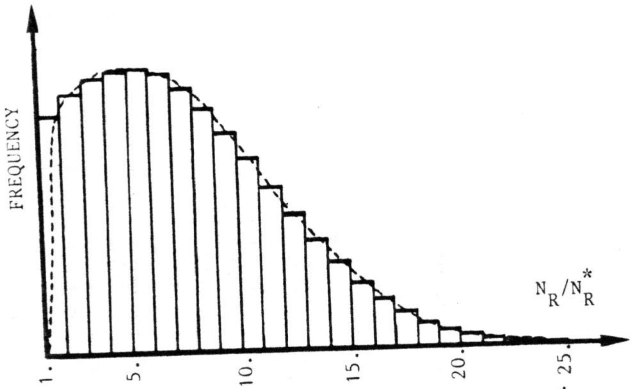

Figure-6 Derived Distribution of The Ratio N_R/N_R^*

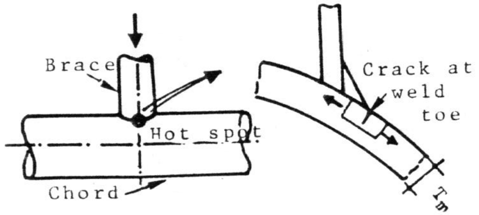

Figure-7 Tubular Joint

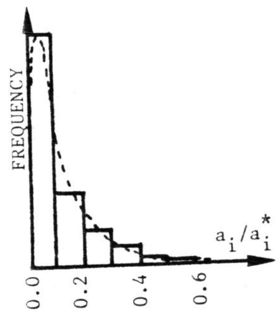

Figure-8 Derived Statistical Distribution of The Defect Size in Tubular Joint