EFFECT OF MEAN STRESS ON THE NOTCH FACTOR IN FATIGUE

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The effect of the mean stress on the notch factors, defined not only as the ratio of stress amplitudes for the fatigue limits of unnotched and notched specimens but also as that of the corresponding maximum stresses, was derived analytically and compared with experimental results. For normalized and tempered steels and for heat-treated aircraft Al-alloys, agreement with test results was sufficiently good.

INTRODUCTION

It is well known that the notch effect is very strong in fatigue and is dependent on many factors such as the theoretical stress concentration, the notch radius, the type and size of notched specimen, the loading type, the kind of material, etc.

It is measured by the notch factor $K_f = \sigma_d/\sigma_{dn}$ and, as a rule, estimated from tests with symmetrical loading cycles ($\sigma_m = 0$). In this case σ_{dn} and σ_d are the fatigue limits for notched an unnotched specimens obtained in tension-compression, rotating-bending or reversed torsion ($R = -1$).

The ratio σ_d/σ_{dn} in pulsating-tension ($R = 0$) is, in general, different than that in tension-compression ($R = -1$), because of the effect of the mean stress, but the latter is mostly neglected in the absence of information about its magnitude and of verified analytical estimation methods.

For example, when the so-called Neuber-Topper rule $K_\sigma \times K_\varepsilon = K_f^2$ is applied in notch analyses for local strain calculations, the calculated or experimental K_f value used is that for $R = -1$, even when the loading spectrum is of the pulsating-tension type (as, e.g. for the lower aircraft wing surface).

In fatigue life calculations by the nominal stress method, it is sometimes assumed that the value $K_f = \sigma_a/\sigma_{an}$ in unsymmetrical stress cycles $\sigma = \sigma_m \pm \sigma_a$ is independent of σ_m, which means that the notch factor should be applied to the alternating stress only. In some other - more conservative - fatigue calculations, the notch factor is applied both to the static and the alternating stress. The notch factor K_{fo} for pulsating-tension

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(R = 0) should then be equal to its counterpart K_f for tension-compression (R = - 1), but this is, in general, not true for such ductile engineering materials as Al-alloys and normalized or tempered steels, as is shown in the tables given below. Following the relation between the notch factors for R = 0 and R = - 1, the effect of the mean stress on the notch factor will be discussed and defined analytically.

DERIVATION OF THE NOTCH FACTOR FOR THE CASE $\sigma_m > 0$ WITH THE AID OF GOODMAN'S $\sigma_a - \sigma_m$ RELATION

Figure 1 presents Goodman's $\sigma_a - \sigma_m$ relations, assumed linear both for notched and unnotched specimens. $\sigma_d \cong OD$ and $\sigma_{dn} = OD'$ are here the fatigue limits (at $N_B \cong 10^7$) for unnotched and notched specimens when $\sigma_m = 0$, and $OF = S_u$ is the tensile strength of standard material specimens. DF and D'F define the fatigue limits for cycles where $\sigma_m > 0$. Therefore, for unnotched specimens

$$AB = \sigma_a = \sigma_d \,(1 - \sigma_m/S_u)$$

and for notched specimens

$$AC = \sigma_{an} = \sigma_{dn} \,(1 - \sigma_m/S_u)$$

The notch factor, defined as a ratio of the stress amplitudes

$$K_f = \sigma_a/\sigma_{an} = AB/AC = \sigma_d/\sigma_{dn}$$

is according to Goodman's relation independent of the mean stress. By contrast, the notch factor defined as a ratio of the maximum stresses for unnotched and notched specimens

$$K_{fm} = \frac{OA+AB}{OA+AC} = \frac{\sigma_m+\sigma_d(1-\sigma_m/S_u)}{\sigma_m+\sigma_{dn}(1-\sigma_m/S_u)} = f(\sigma_m) \qquad (1)$$

is a function of the mean stress.

For $\sigma_m = 0$ $K_{fm} = \sigma_d/\sigma_{dn}$ and for $\sigma_m = S_u$ $K_{fm} = 1$. K_{fm} decreases with increase of the mean stress, tending to 1.

Fig. 1 yields also the relation between the notch factor K_{fo} for pulsating-tension (R = 0, $\sigma_{min} = 0$) and the notch factor K_f for tension-compression.

Since

$$QM = \frac{S_u \times \sigma_d}{S_u + \sigma_d} \qquad \text{and} \qquad PN = \frac{S_u \times \sigma_{dn}}{S_u + \sigma_{dn}}$$

$$K_{fo} = \frac{QM}{PN} = \frac{\sigma_d}{\sigma_{dn}} \times \left(\frac{S_u + \sigma_{dn}}{S_u + \sigma_d}\right) = K_f \left(\frac{S_u + \sigma_{dn}}{S_u + \sigma_d}\right) \qquad (2)$$

and according to Goodman's relation $K_{fo} < K_f$.

EXPERIMENTAL AND PREDICTED NOTCH FACTORS FOR DIFFERENT σ_m VALUES

Some experimental results for different loading types are presented in Table 1. It can be seen that for all three considered loadings (torsion, bending and axial loading) K_f increases and K_{fm} decreases with increase of σ_m. These results, however, are not representative, since they refer to sharp V-notches with very small notch radius and notch depth, which are avoided in structural elements.

By contrast, Table 2 presents test results for specimens with holes - a type of notch common in structures. In this case the stress concentration is well defined and limited ($K_t = 2.45$). As can be seen from the Table, the effect of σ_m on K_{fm} can be quite accurately predicted with Eq. (1) for steel and Al-alloy specimens with central notches.

Table 1: Effect of mean stress on fatigue limits and notch factors of steel specimens with sharp V-notches of small size [1].

Material and UTS in MPa	Type of loading	Notch para-meters	Mean stress σ_m MPa	Stress amplitude σ_a MPa	σ_{an}	Notch factors K_f	K_{fm}
CrNi-steel	Torsion	Round	0	310	210	1.48	1.48
S_u=1080		d=12mm	100	310	210	1.48	1.32
		V-notch	200	310	205	1.51	1.26
		r=0.05mm	300	300	195	1.54	1.21
		t=0.02mm	400	280	180	1.56	1.17
CrNi-steel	Reversed	Flat	0	560	300	1.87	1.87
S_u=1200	Bending	V-notch	100	540	290	1.86	1.64
		r=0.05mm	200	520	275	1.89	1.52
		t=0.02mm	300	500	250	2.00	1.45
			400	480	230	2.09	1.40
			500	460	205	2.24	1.36
			600	440	180	2.45	1.33
Carbon steel	Axial	Round	0	225	140	1.61	1.61
S_u=769	Loading	d=16mm	100	220	120	1.83	1.45
		V-notch	200	210	105	2.01	1.35
		r=t=0.1mm	300	204	86	2.32	1.30

* $K_f = \sigma_a/\sigma_{an}$, $K_{fm} = (\sigma_m+\sigma_a)/(\sigma_m+\sigma_{an})$

EXPERIMENTAL AND PREDICTED NOTCH FACTORS FOR PULSATING-TENSION

Table 3 presents fatigue limits and notch factors for flat steel specimens with cut-outs for R = 0 (pulsating-tension) and R = -1 (tension-compression). Eq. (2) permits prediction of K_{fo} for pulsating-tension if the value of K_f for tension-compression is given. Comparison of the predicted K_{fo} values for R = 0 with the experimental shows quite good agreement for the Ck45 and 42CrMo4 steels, while for St52 the experimental values are overestimated.

Table 2: Effect of mean stress on fatigue limits and notch factors of flat steel specimens with central hole [1]

Material and UTS in MPa	Notch para-meters	Mean stress σ_m MPa	Stress amplitude MPa K_t=1	K_t=2.45	Notch factors (experi-mental) K_f	K_{fm}	Predicted K_{fm}
St52	d=17mm	0	182	120	1.52	1.52	-
S_u=591	d/W=0.24	100	170	110	1.55	1.29	1.26
		200	155	98	1.58	1.19	1.15
Spring steel	d=12mm	0	198	150	1.32	1.32	-
S_u=967	d/W=0.24	100	190	138	1.38	1.22	1.18
		200	180	126	1.42	1.17	1.12
		300	172	120	1.43	1.12	1.08
		400	159	110	1.40	1.09	1.06
		500	142	102	1.40	1.07	1.04

Table 3: Fatigue limits and notch factors of flat steel specimens in tension-compression and pulsating-tension

Source	Material	Type of notch	Stress ratio R	Stress amplitude MPa K_t=1	K_t=3.6	Notch factors exp.	pred.
LBF Darmst.	Ck45	Cut-out	-1	325	146	2.23*	-
TM83/78[2]	S_u=747MPa	r=2mm	0	261	130	2.01*	2.10
LBF Darmst.	42CrMo4	Cut-out	-1	506	196	2.58*	-
TM84/78[3]	S_u=1097MPa	r=2mm	0	371	174	2.13*	2.08
TH Darmst.	St52-3	Cut-out	-1	270	101	2.67*	-
ISS 17/71	S_u=590MPa	r=4mm	0	205	97	2.11+	2.47
[4]	$S_{0.2}$=450	Hole	-1	270	144++	1.88+	-
		d=24mm	0	205	138.5++	1.48+	1.74

*N_B=2z10^6 , +N_B=10^7 , ++K_t=2.36

Table 4 presents fatigue limits and notch factors for 6061, 2024 and 7075
Al-alloy sheet specimens with cut-outs. As can be seen, the K_{fo} values
predicted according to Eq. (2) are mostly in good agreement with the experi-
mental for 2020-T3 and 7075-T6 specimens. For 6061-T4 the estimates are
mostly conservative but less overestimated than according the the assumption
$K_{fo} = K_f$. The ratio $S_{0.2}/S_u$ is considerably smaller for 6061 than for 2024
and 7075 indicating a greater strain-hardening capability.*

Table 4: Fatigue limits ($N=10^7$) and notch factors of flat Al-alloy specimens in tension-compression and pulsating-tension

Material UTS MPa	Notch parameters r mm	K_t	T.C. R = - 1 σ_a MPa	K_f	P.T. R=0 $2\sigma_a$ MPa	K_{fo}^+	Predicted Kf_o
6061-T4	- -	1.00	104	1.00	130	1.00	-
S_u=310 [5]	0.5	2.88	56.5	1.84	113	1.15	1.63
	1	4.6	35	2.97	52.5	2.48	2.48
	2	3.6	40	2.60	69	1.88	2.20
	3	2.0	67.5	1.54	105	1.24	1.40
	4	2.52	56.5	1.84	94	1.38	1.63
	12.5	2.07	60	1.73	97.5	1.33	1.55
	20	2.07	62	1.68	92	1.41	1.51
2024-T3	- -	1.00	147	1.00	234	1.00	-
S_u=476 [5,6]	0.4	2.83	79	1.86	143	1.64	1.66
	1.6	2.83	70	2.10	119	1.97	1.84
	3.2	2.83	63	2.33	104	2.25	2.02
	12.7	2.12	74.5	1.97	136	1.72	1.74
7075-T6	- -	1.00	147	1.00	246	1.00	-
S_u=574 [5,6]	0.4	2.83	91.5	1.61	158	1.56	1.49
	1.6	2.83	68	2.16	125	1.97	1.92
	3.2	2.83	64.5	2.28	118	2.08	2.02
	12.7	2.12	77	1.91	144	1.71	1.72

+Notch factor for R=0

DISCUSSION AND CONCLUSIONS

As can be seen from Tables 2, 3 and 4, the K_{fm} and K_{fo} notch factor values
calculated with Eqs. (1) and (2) are mostly in good agreement with the
experimental in spite of the fact that the $K_f = \sigma_a/\sigma_{an}$ values are not fully
independent of σ_m and that the experimental $\sigma_a - \sigma_m$ dependence is not
perfectly linear. For the 6061 specimens the experimental K_{fo} values are
mostly smaller than the predicted, while for 2024 and 7075 specimens agree-
ment is sufficiently good. This is probably associated with the higher
ductility and stronger local strain-hardening for the notched 6061 specimens.
The lower value of K_{fo} for R = 0 compared with K_f for R = -1, is also
probably associated with stronger strain-hardening in pulsating tension com-
pared with tension-compression, where the fatigue limit stress is smaller.
A similar effect was observed when the notch factor was considered as a

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* For the steel St52 and the Al-alloy 6061-T4 the yield limit is relatively low and the relations $\sigma_a - \sigma_m$ are far from being linear.

function of the fatigue life N [7]. With decrease of the latter (which means simultaneous increase of the alternating stress), K_f decreases because of the stronger strain-hardening of the notched specimens at higher stresses.

The following conclusions may be drawn from the recent investigation:

1. The notch factor K_{fm} decreases with increase of σ_m and $K_{fo} < K_f$ for the considered engineering materials with good ductility.
2. The notch factors K_{fm} and K_{fo} may be calculated with Eqs. (1) and (2) for the materials considered.

LIST OF SYMBOLS

S_u = ultimate tensile strength, UTS

$S_{0.2}$ = yield limit

d = specimen diameter or diameter of a hole

r = notch or hole radius

t = notch depth

W = specimen width

N = fatigue life

N_B = basic number of cycles for fatigue limit estimation

σ_d = fatigue limit of unnotched specimens

σ_{dn} = fatigue limit of notched specimens

σ_m = mean stress of the stress cycle

σ_a = stress amplitude (sometimes for an unnotched specimen only)

σ_{an} = stress amplitude for a notched specimen

σ_{min}, σ_{max} = minimum and maximum stress of the stress cycle

R = stress ratio $\sigma_{min}/\sigma_{max}$

K_f = notch factor for R = -1 , ratio σ_a/σ_{an}

K_{fm} = notch factor for $\sigma_m > 0$, ratio $(\sigma_m+\sigma_a)/(\sigma_m+\sigma_{an})$

K_{fo} = notch factor for R = 0

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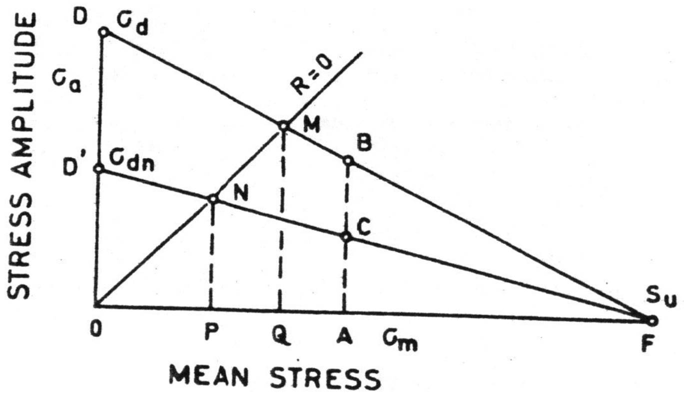

FIG. 1 $\sigma_a - \sigma_m$ DIAGRAM