CURVED CRACK UNDER THERMAL LOAD

L. DREILICH, D. GROSS *

The problem of a slightly curved and arbitrary loaded crack is studied on the basis of the complex method. The formulation leads to a singular integral equation which is solved approximately by use of asymptotic expansions. A general formula for the stress intensity factor is derived and compared with known results for special cases. For a crack in a homogeneous stress field the stability of the crack growth direction is investigated.

The thermoelastic problem is solved for a straight crack and steady state heat source.

1. INTRODUCTION

The problem of a slightly curved crack of arbitrary shape in an infinite plane was first studied by Banichuk (1) and Goldstein and Salganik (2) on the basis of the complex method. They assumed, that the stress functions can be described in the sense of the pertubation method by asymptotic expansions. Cotterell and Rice (3) used the same method to evaluate the stress intensity factors and to predetermine the path of a semiinfinite crack. On the same basis higher order solutions were derived by Karinhaloo et.al. (4).

Unlike the former investigations the present study starts with the description of a crack by a dislocations distribution. This approach leads to a singular integral equation for the density of dislocations which is solved approximately by means of asymptotic expansion. The calculated stress intensity factors are used to discuss the stability of the direction of the crack path.

So far only a few analytical solutions are known for thermally loaded cracks. One of them is the problem of a straight crack in an infinite plane with homogeneous heat flow which was solved by Sih (5). Florence and Goodier (6) gave a solution for a crack with heated faces. There is no analytical solution for a crack in an arbitrary temperature field or for a

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* Dipl.-Ing. L. Dreilich, Prof. Dr.-Ing. D. Gross,
Technische Hochschule Darmstadt, Institut für Mechanik,
Hochschulstr. 1, D-6100 Darmstadt, Federal Republik of Germany

thermally loaded curved crack. In this paper a solution is given for a straight crack in an inhomogeneous temperature field being produced by a stationary heat source.

2. BASIC EQUATIONS

The heat conduction is described by the Fourier equation. In the stationary case it reduces to the potential equation $\Delta T = 0$ for the temperature T. Using the complex variable $z = x + iy$ the solution can be written as

$$T(x,y) = \mathrm{Re}\{\Theta'(z)\} , \qquad (1)$$

where Θ' is an analytic function. Primes denote derivatives with respect to the argument. Considering the crack as an insulator, the heat flow vanishes across R (Fig.1)

$$\mathrm{Im}\{e^{i\vartheta(x)}\ \Theta''(z)\} = 0 , \qquad (2)$$

where ϑ stands for the angle between the crack contour and the x-axis at the point $t = x + i\,\lambda(x)$.

The plane thermoelastic problem can be described by three functions $\varphi(z)$, $\psi(z)$ and $\Theta(z)$. The stresses and displacements follow from the generalized Kolosov equations:

$$\begin{aligned} \sigma_y + \sigma_x &= 2[\varphi'(z) + \overline{\varphi'(z)}] , \\ \sigma_y - \sigma_x - i\,2\tau_{xy} &= 2[\bar z\ \varphi''(z) + \psi'(z)], \qquad (3) \\ 2\mu(u+iv) &= \varkappa_1\ \varphi(z) - z\ \overline{\varphi'(z)} - \overline{\psi(z)} + \varkappa_2\ \Theta(z) , \end{aligned}$$

Bar denotes the conjugate to the appropriate term. The material constants $\varkappa_1$, $\varkappa_2$ differ for plane stress and plane strain

$$\varkappa_1 = \begin{cases} (3-\nu)/(1-\nu) \\ 3 - 4\nu \end{cases} , \quad \varkappa_2 = 4\mu\alpha \begin{cases} 1 + \nu & \text{plane stress} \\ 1 & \text{plane strain} . \end{cases}$$

The quantities ν, μ and α are the Poisson's ratio, shear modulus and the coefficient of thermal expansion respectively.

Consider now a curved crack R defined by the continuous function $\lambda(x)$, Fig.1. The derivatives of $\lambda(x)$ are assumed to be continuous too.

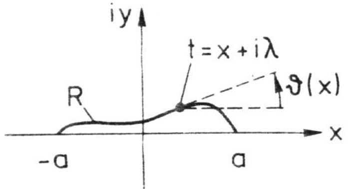

Figure 1

Curved crack in infinite plane

The crack surface is subjected to an arbitrary complex stress vector $\sigma + i\tau$ which follows from the solution of the undisturbed problem (plane without crack). Fig.2 showes the normal component σ and the tangential component τ of the stress vector at a point t, for example at the upper crack face.

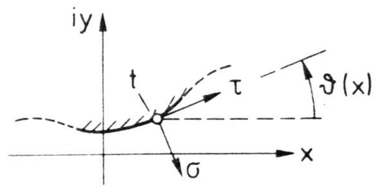

Figure 2

Stress components σ and τ

The representation of the crack under this load by the dislocation distribution leads to the singular integral equation for the dislocation density g (7):

$$\sigma + i\tau = 2\int_R \frac{g(t)}{t-t_0}\,dt - \int_R H(t,t_0)\left[\frac{g(t)dt}{t-t_0} - \frac{\overline{g(t)dt}}{\bar t-\bar t_0}\right], \qquad (4)$$

$$H(t,t_0) = 1 - e^{-i2\vartheta(x_0)}\ \frac{t - t_0}{\bar t - \bar t_0} .$$

Since the crack is closed at both ends, the side condition

$$\int_R g(t)\,ds = 0 \qquad (5)$$

with $ds = |dt|$, must be satisfied. The stress functions φ' and ψ' can be calculated from the dislocation density g as follows:

$$\begin{aligned} \varphi'(z) &= \varphi_0'(z) - \int_R \frac{g(t)}{t - z}\,dx , \\ \psi'(z) &= \psi_0'(z) - \left[\int_R \frac{\overline{g(t)}}{t - z}\,dt - \frac{d}{dz}\int_R \frac{\bar t\ g(t)}{t - z}\,dt\right] , \end{aligned} \qquad (6)$$

where φ_0' and ψ_0' are the stress functions of the undisturbed problem (plane without crack).

For the slightly curved crack $\left(\lambda/a \ll 1 \text{ and } \frac{d^{n+1}(\lambda/a)}{d(\lambda/a)^{n+1}} \ll \frac{d^n(\lambda/a)}{d(\lambda/a)^n} \text{ for } n \in N_0\right)$, the deviation λ from the straight crack, Fig.1, can be considered as a perturbation. Therefore the asymptotic expansions with respect to the perturbation parameter λ/a can be conceived. For example the Taylor expansions of ϑ and λ at the point x_0 bring the quantity H into the following form:

$$H(t,t_0) = 1 - e^{-i2\vartheta_0}\frac{t-t_0}{\bar t - \bar t_0} = 1 - \left(1 + \sum_{n=1}^{\infty}\frac{(-i2\vartheta_0)^n}{n!}\right)\frac{1+i\dfrac{\lambda-\lambda_0}{x-x_0}}{1-i\dfrac{\lambda-\lambda_0}{x-x_0}} =$$

$$= 2i\left(\vartheta_0 - \frac{\lambda-\lambda_0}{x-x_0}\right) + \ldots = -\,i(x-x_0)\lambda_0'' + \ldots \;.$$

From the undisturbed stress field being continuous, the load of the crack faces can be obtained using the equilibrium conditions

$$(\sigma+i\tau)\Big|_t = (\sigma_y - i\tau_{xy})\Big|_{\lambda=0} + i\lambda\frac{\partial}{\partial x}(\sigma_x+i\tau_{xy})\Big|_{\lambda=0} - i\lambda'(\sigma_y-\sigma_x-i2\tau_{xy})\Big|_{\lambda=0} + \ldots$$

If we neglect the terms of the order higher than λ' in the obtained expansions, then equations (4), (5) and (6) reduce to:

$$\sigma_y - i\tau_{xy} - i\lambda'(\sigma_y-\sigma_x-i2\tau_{xy}) + i\lambda(\sigma_x+i\tau_{xy})' = 2\int_{-a}^{a}\frac{g(x)}{x-x_0}\,dx \qquad (7)$$

$$\int_{-a}^{a} g(x)\,dx = 0 \qquad (8)$$

$$\varphi'(z) = \varphi_0'(z) - \left[\int_{-a}^{a}\frac{1+i\lambda'}{x-z}\,g(x)\,dx - \frac{d}{dz}\int_{-a}^{a}\frac{i\lambda}{x-z}\,g(x)\,dx\right]$$

$$\psi'(z) = \psi_0'(z) - \left[\int_{-a}^{a}\frac{1-i\lambda'}{x-z}\,\overline{g(x)}\,dx - \frac{d}{dz}\int_{-a}^{a}\frac{x(1+i\lambda')}{x-z}\,g(x)\,dx + \right.$$

$$\left. + \frac{d}{dz}\int_{-a}^{a} i\lambda\,\frac{g(x)-\bar g(x)}{x-z}\,dx + \frac{d^2}{dz^2}\int_{-a}^{a}\frac{ix\lambda}{x-z}\,g(x)\,dx\right]. \qquad (9)$$

3. SOLUTION OF THE INTEGRAL EQUATION. STRESS INTENSITY FACTOR

A solution of the integral equation (7) is assumed to be of the form

$$g(x) = \sum_{n=0}^{\infty} A_n \frac{T_n(x/a)}{\sqrt{1-(x/a)^2}}\,, \qquad (10)$$

which satisfies the side condition (8) for $A_0 = 0$. Inserting this into (7) and making use of the orthogonality conditions one gets the expansion coefficients

$$A_n = \frac{1}{\pi^2}\int_{-1}^{1}\left[\sigma_y - i\tau_{xy} - i\lambda'(\sigma_y-\sigma_x-i2\tau_{xy}) + i\lambda(\sigma_x+i\tau_{xy})'\right] \qquad (11)$$

$$\sqrt{1-(x/a)^2}\,U_{n-1}(x/a)\,d(x/a)\,.$$

T_n and U_n are the Chebyshev polynomials of the first and second kind respectively. The functions (9) follow then to be

$$\varphi'(z) = \varphi_0'(z) - \pi\sum_{n=1}^{\infty} A_n\left\{{}^1H_n(z) - \frac{1+i\lambda'(z/a)}{\sqrt{(z/a)^2-1}}\,T_n(z/a) - \right.$$

$$\left. - \frac{d}{dz}\left[{}^2H_n(z) - \frac{i\lambda(z/a)}{\sqrt{(z/a)^2-1}}\,T_n(z/a)\right]\right\},$$

$$\psi'(z) = \psi_0'(z) - \pi\sum_{n=1}^{\infty}\left\{A_n\left[{}^3H_n(z) - \frac{1-i\lambda'(z/a)}{\sqrt{(z/a)^2-1}}\,T_n(z/a)\right] - \right. \qquad (12)$$

$$- A_n\frac{d}{dz}\left[{}^4H_n(z) - \frac{1+i\lambda'(z/a)}{\sqrt{(z/a)^2-1}}\,z\,T_n(z/a)\right] +$$

$$+ (A_n - \bar A_n)\frac{d}{dz}\left[{}^2H_n(z) - \frac{i\lambda(z/a)}{\sqrt{(z/a)^2-1}}\,T_n(z/a)\right] +$$

$$\left. + A_n\frac{d^2}{dz^2}\left[{}^5H_n(z) - \frac{i\lambda(z/a)}{\sqrt{(z/a)^2-1}}\,z\,T_n(z/a)\right]\right\}.$$

$H_n(z)$ are the principle parts of the expansions of the parenthetical functions at the point $z/a \to \infty$. For example ${}^4H_n(z)$ is the principle part of $\dfrac{1+i\lambda'(z/a)}{\sqrt{(z/a)^2-1}}\,z\,T_n(z/a)$.

The complex stress intensity factors are given by the singular parts of the stress functions (12) at the crack tips

$$K^{\pm} = K_I^{\pm} + iK_{II}^{\pm} = \pm\sqrt{\pi a}\,\left[2 + i\lambda'(\pm a)\right]\sum_{n=1}^{\infty}\pi\bar A_n(\pm 1)^n\;. \qquad (13)$$

The sign (+) denotes here the right and (-) the left crack tip. The coefficients (11) when combined with the identity $\sum_{n=1}^{\infty}(\pm 1)^n U_{n-1}(x/a) = \pm\frac{1}{2}\frac{a}{a\mp x}$ furnish the following expression for the stress intensity factor

$$K^{\pm} = \frac{1}{\sqrt{\pi a}}\int_{-a}^{a}\left[\left(1 + \frac{i}{2}\lambda'(\pm a)\right)\left(\sigma_y+i\tau_{xy}\right) + i\lambda'\left(\sigma_y-\sigma_x-i2\tau_{xy}\right) + \right. \qquad (14)$$

$$\left. + i\lambda\left(\sigma_x+i\tau_{xy}\right)'\right]\sqrt{\frac{a\pm x}{a\mp x}}\,dx\;.$$

This solution is appropriate for a general, curved crack contour.

As an example let us consider a circular arc crack in a homogeneous stress field, Fig.3.

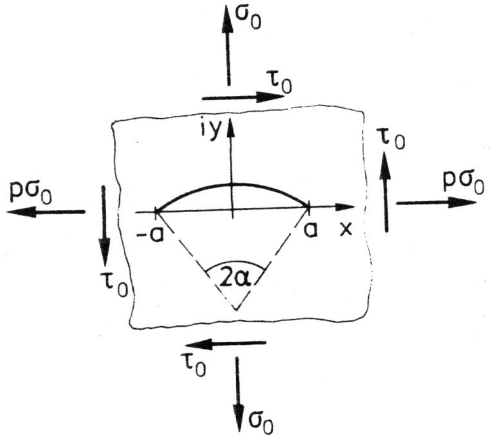

Figure 3

Circular arc crack in homogeneous stress field.

In this case the expression for the K-factor can be simplified. With the help of stress intensity factors $k_I = \sqrt{\pi a}\,\sigma_0$ and $k_{II} = \sqrt{\pi a}\,\tau_0$ for a straight crack of length 2a, the equation (14) can be splitted into the real and imaginary part:

$$\frac{K_I^{\pm}}{k_I} = 1 - \frac{k_{II}}{2k_I}\lambda'(\pm a) - \frac{2k_{II}}{\pi k_I}\int_{-1}^{1}\lambda'(x/a)\sqrt{\frac{1 \pm x/a}{1 \mp x/a}}\,d(x/a),$$

$$\frac{K_{II}^{\pm}}{k_{II}} = 1 + \frac{k_I}{2k_{II}}\lambda'(\pm a) + \frac{(1-p)k_I}{\pi\, k_{II}}\int_{-1}^{1}\lambda'(x/a)\sqrt{\frac{1 \pm x/a}{1 \mp x/a}}\,d(x/a). \qquad (15)$$

Hence for small α and in view of the relation $\lambda'(x/a) = -\alpha\, x/a + \ldots$ one obtains

$$\frac{K_I^{\pm}}{k_I} = 1 \pm \frac{3k_{II}}{2k_I}\alpha\,, \qquad \frac{K_{II}^{\pm}}{k_{II}} = 1 \mp (1-p)\frac{k_I}{k_{II}}\alpha\,. \qquad (16)$$

This result agrees with the exact solution given in (8), (3) up to the first order of α.

4. STABILITY OF CRACK GROWTH DIRECTION

Using the results of Sec.2 the stability of crack growth direction can be investigated. Hereby we restrict our consideration to one crack tip.

If we assume that the crack growth is determined by the condition $K_{II} = 0$, then equation (14) can be interpreted as an integro-differential equation for the function λ(x)

$$-\int_{-a}^{a}\tau_{xy}\sqrt{\frac{a+x}{a-x}}\,dx = \frac{1}{2}\lambda'(a)\int_{-a}^{a}\sigma_y\sqrt{\frac{a+x}{a-x}}\,dx + \int_{-a}^{a}\left[\lambda'(\sigma_y-\sigma_x) + \lambda\sigma_x'\right]\sqrt{\frac{a+x}{a-x}}\,dx\,. \qquad (17)$$

In the particular case of the crack in homogeneous stress field, Fig.5, equation (17) reduces to an inhomogeneous Volterra integral equation of second kind for the function $\sqrt{2a}\,\lambda'(2a)$. Employing the coordinate transformation $\xi = a + x$, this equation takes the form

$$-2\frac{k_{II}}{k_I}\sqrt{2a} = \sqrt{2a}\,\lambda'(2a) + 2\sqrt{2}(1-p)\frac{\sigma_0}{k_I}\frac{1}{\pi}\int_0^{2a}\frac{\sqrt{\xi}\,\lambda'(\xi)}{\sqrt{2a-\xi}}\,d\xi\,. \qquad (18)$$

The solution, shown in Fig.4, is

$$\lambda'(x=a) = -2\frac{k_{II}}{k_I}\frac{\pi}{8(1-p)}\left\{1 - e^{\frac{16}{\pi}(1-p)^2}\left[1 - \mathrm{erf}\left(\frac{4}{\sqrt{\pi}}(1-p)\right)\right]\right\}, \qquad (19)$$

where erf denotes the error function.

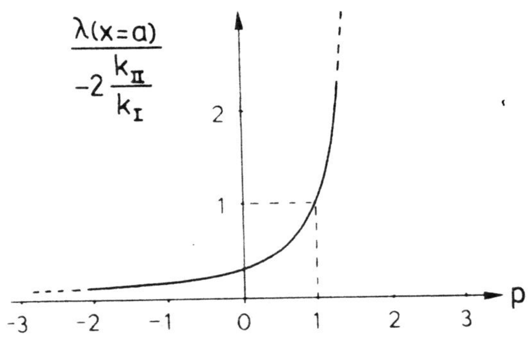

Figure 4

Solution of integral equation (18).

If we consider $\lambda_0' = -2\frac{k_{II}}{k_I}$ as an initial angle, then one can see that $\lambda'(a) > \lambda_0'$ for $p > 1$ and $\lambda'(a) < \lambda_0'$ otherwise. It is seen in Fig.5 that the crack tip angle has the tendency to decrease with decreasing p (stable). When p increases the deviation from the straight crack increases too (unstable).

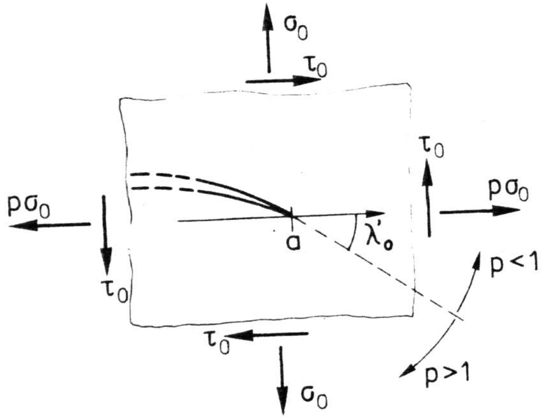

Figure 5

Stability of crack growth direction of right crack tip.

5. CRACK WITH HEAT SOURCE

5.1 Heat conduction

Consider now a straight crack of length 2a in a plane with stationary heat source of intensity C, Fig.6. The position of the source is given by $\zeta = \xi + i\eta$. For the problem at hand, the crack is assumed to be an insulator.

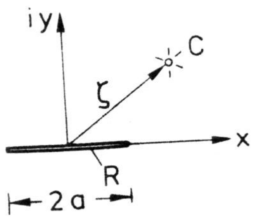

Figure 6

Stationary heat source and crack in infinity plane.

There are several methods to solve this Neumann problem (1), (2). It is possible, for instance, to introduce a single layer potential. Such an approach leads to a singular Fredholm integral equation of the first kind for the potential density μ:

$$- \operatorname{Im}\left\{\frac{C}{\zeta - x_0}\right\} = \int_{-a}^{a} \frac{\mu(x)}{x - x_0}\, dx \; . \qquad (20)$$

Expanding the function μ into a seriesof Chebyshev polynomials of the first kind

$$\mu(x) = \sum_{n=1}^{\infty} B_n \frac{T_n(x/a)}{\sqrt{1-(x/a)^2}} \qquad (21)$$

with the coefficients

$$B_n = -\frac{2C}{\pi a^{n+1}} \operatorname{Im}\left\{(\zeta - \sqrt{\zeta^2 - a^2})^n\right\} , \qquad (22)$$

the solutions can be written as follows

$$\Theta'(z) = C \ln \frac{z-\zeta}{b_0} + C \sum_{n=1}^{\infty} \frac{2i}{n a^{2n}} \operatorname{Im}\left\{(\zeta - \sqrt{\zeta^2 - a^2})^n\right\}(z - \sqrt{z^2 - a^2})^n , \qquad (23)$$

where b_0 is the radius of a circle around ζ on which the temperature vanishes in the case of undisturbed temperature field (without a crack).

4.2 Thermoelastic Problem

The analysis of the continuity of the displacement field (3) shows that the displacement vector is discontinuous for a cycle along the curves L_1 and L_2 on account of the function $\Theta(z)$, Fig.7.

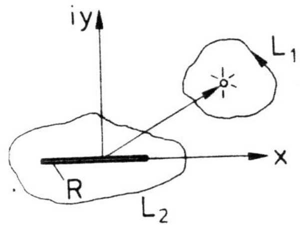

Figure 7

Discontinuity of displacement vector for cycle along L_1 and L_2.

These discontinuities must be compensated by the appropriate functions φ_1 and ψ_1. It has to be noted here, that φ_1 and ψ_1 shall produce no resultant force and no resultant moment. We get:

$$\varphi_1'(z) = -\frac{\varkappa_2 C}{1+\varkappa_1}\left[\ln \frac{z-\zeta}{b_1} + i\,\frac{\operatorname{Im}\{\zeta - \sqrt{\zeta^2 - a^2}\}}{\sqrt{z^2 - a^2}}\right] ,$$

$$\psi_1'(z) = \frac{\varkappa_2 C}{1+\varkappa_1}\left[\frac{\zeta}{z-\zeta} + i\,\operatorname{Im}\{\zeta - \sqrt{\zeta^2 - a^2}\}\,\frac{z^2 - 2a^2}{(z^2-a^2)^{3/2}}\right] , \qquad (24)$$

where b_1 is the radius of a circle around ζ on which the stress vector vanishes in the case of undisturbed problem (no crack). The functions φ_1 and ψ_1 produce a continuous displacement field except along the crack, but they involve stresses $(\sigma_y + i\tau_{xy})_1\big|_R \neq 0$ at the crack face as a consequence.

Therefore a second set of functions φ_2 and ψ_2 must be superposed, so that the boundary condition $(\sigma_y + i\tau_{xy})_1\big|_R + (\sigma_y + i\tau_{xy})_2\big|_R = 0$ is satisfied.

If the stress vector at the boundary is expanded in a series of Chebyshev polynomials of the second kind

$$(\sigma_y + i\tau_{xy})_1\Big|_R = \sum_{n=0}^{\infty} B_n(\zeta)\,U_n(x/a) \,, \tag{25}$$

then φ_2' and ψ_2' have the following form

$$\varphi_2'(z) = \frac{-1}{2\sqrt{z^2-a^2}} \sum_{n=0}^{\infty} \frac{\overline{B_n(\zeta)}}{a^n}\left(z - \sqrt{z^2-a^2}\right)^{n+1} ,$$

$$\psi_2'(z) = \frac{-1}{2\sqrt{z^2-a^2}} \sum_{n=0}^{\infty} \frac{B_n(\zeta)}{a^n}\left(z - \sqrt{z^2-a^2}\right)^{n+1} - \left(z\,\varphi_2'(z)\right)' . \tag{26}$$

The coefficients are given by

$$B_0(\zeta) = \frac{2\varkappa_2 C}{1+\varkappa_1}\left[\frac{1}{2} + \ln\frac{a}{2b_1} - \ln\left|\frac{\zeta-\sqrt{\zeta^2-a^2}}{a}\right| - \frac{\zeta-\bar\zeta}{a^2}\left(\zeta-\sqrt{\zeta^2-a^2}\right) + \frac{1}{2}\,\mathrm{Re}\left\{\frac{1}{a^2}\left(\zeta-\sqrt{\zeta^2-a^2}\right)^2\right\}\right] ,$$

$$B_n(\zeta) = \frac{2\varkappa_2 C}{1+\varkappa_1}\left[-\frac{\zeta-\bar\zeta}{a}\left(\frac{\zeta-\sqrt{\zeta^2-a^2}}{a}\right)^n + \mathrm{Re}\left\{-\frac{1}{n}\left(\frac{\zeta-\sqrt{\zeta^2-a^2}}{a}\right)^n + \frac{1}{2+n}\left(\frac{\zeta-\sqrt{\zeta^2-a^2}}{a}\right)^{n+2}\right\}\right], \text{ for } n \geq 1. \tag{27}$$

The appearing series converge for all $z \in [-a,a]$.

The general solution for the thermoelastic problem is obtained by superposition of the two sets of stress functions $\varphi = \varphi_1 + \varphi_2$ and $\psi = \psi_1 + \psi_2$. From this the complex stress intensity factor follows

$$K^{\pm} = \frac{2\varkappa_2 C}{1+\varkappa_1}\sqrt{\pi a}\left[-\frac{1}{2} - \ln\frac{a}{2b_1} + \ln\left|\frac{\zeta-\sqrt{\zeta^2-a^2}}{a}\right| \pm \frac{1}{a}\left(\zeta-\sqrt{\zeta^2-a^2}\right) + \frac{\zeta-\bar\zeta}{a}\,\frac{\zeta-\sqrt{\zeta^2-a^2}}{a \mp \left(\zeta-\sqrt{\zeta^2-a^2}\right)}\right] . \tag{28}$$

In Figs. 8, 9 the $K_I^{\pm}$ and $K_{II}^{\pm}$ are traced versus the position of heat source. Independently of η, $K_{II}^{\pm}$ equals zero if the source is at the position $\xi = a$.

The obtained solution is a basic solution, analogous to basic solutions for a crack under a single force or under a single dislocation load (9), (10). Using this solution it is possible to construct other special solutions, for instance, by integration, Figs. 10, 11. In other cases this basic solution can be applied to formulate boundary value problems in form of integral equations, which can serve as a starting point for further analytical or numerical treatments.

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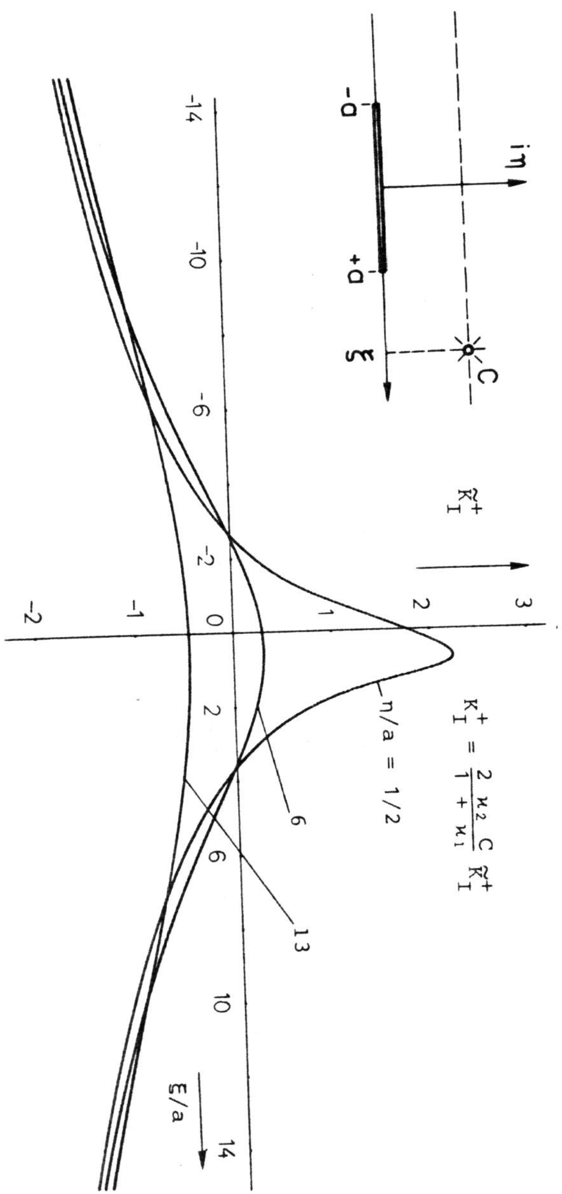

Figure 8 Stress intensity factor K_I^+ for a heat source

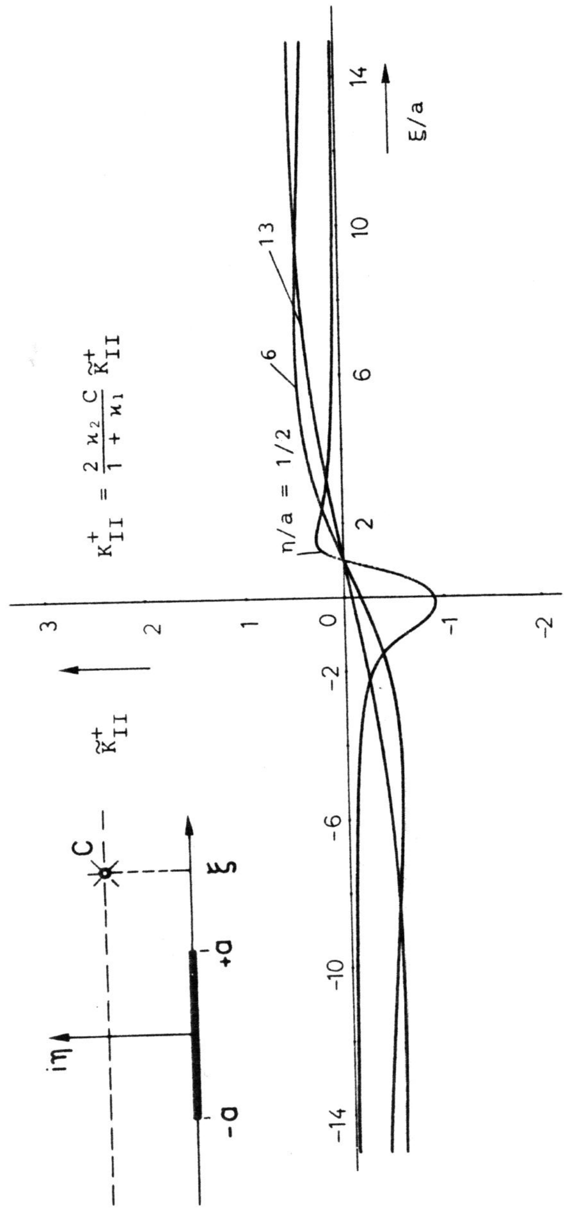

Figure 9 Stress intensity factor K_{II}^+ for a heat source

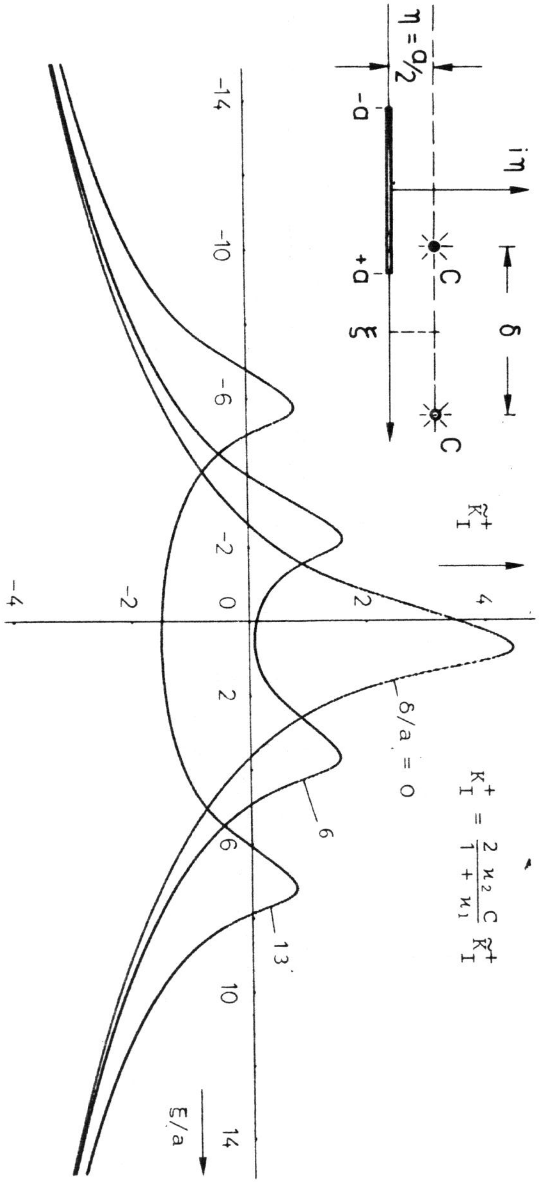

Figure 10 Stress intensity factor K_I^+ for two heat sources

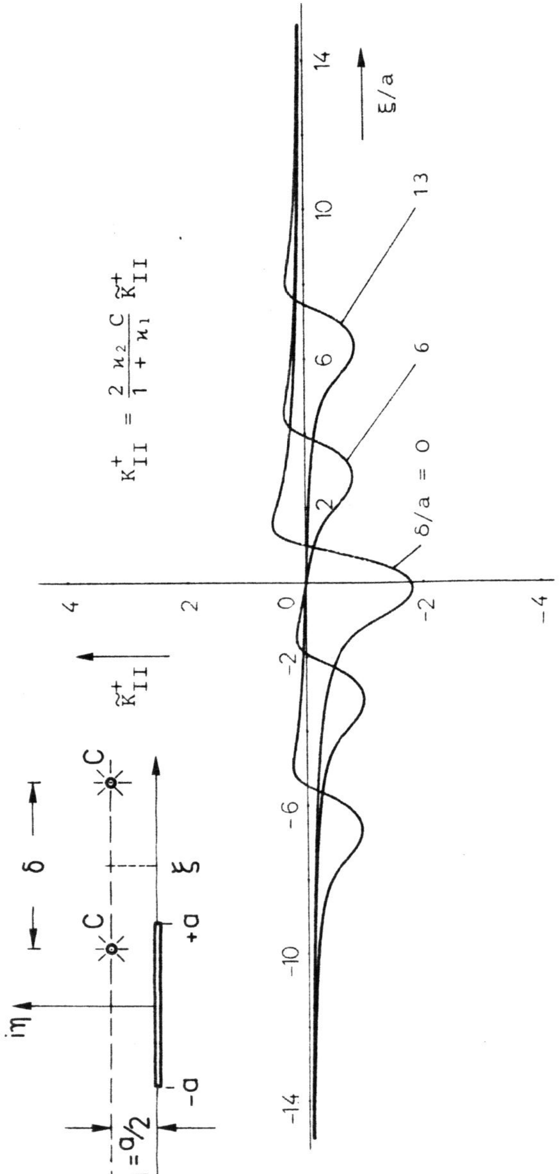

Figure 11 Stress intensity factor K_{II}^+ for two heat sources.