FRACTURE ASPECTS OF CONCRETE

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Linear elastic fracture mechanics is not suitable for concrete. A different type of model has been developed, which is adopted to the properties of concrete and to the types of tensile fracture which can be expected in concrete structures. The model is general in the sense that it can describe all types of tensile fracture, from plastic fracture to different types of elastic brittle fracture. Application to practical design problems has given new insight of importance for design. It has also shown that fracture energy is a material property which in some cases may be more important than the strength of the material, for the strength of a structure.

INTRODUCTION

The first attempts to apply fracture mechanics to concrete were made about 25 years ago. Thus e.g. Kaplan (1) performed a number of tests on notched beams. The application of LEFM to the test results was not successful, and it has generally been concluded that LEFM cannot be applied to concrete, at least not to small specimens. If fracture mechanics is to be applied to concrete it is thus necessary to find some other method than LEFM.

Also for metals it is well known that LEFM often cannot be directly applied, but modifications have to be introduced in order to achieve realistic results. One example is the R-curve analysis. The question then can be raised whether these modifications are suitable for concrete as well. However, the reasons for the deviation from LEFM are quite different for metals and for concrete. For metals, the main problem is lateral deformations due to deviations from pure plane strain conditions and a transition from plane strain towards plane stress. This is a problem which can be treated

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by means of R-curves. For concrete the difference between plane strain and plane stress is not an essential problem, as non-elastic deformations are caused by the opening of microcracks, and not by yielding. For concrete, the problem is the large extension of the process zone, and this cannot be solved by means of the methods developed for metals.

Another difference between the application to metals and to concrete is that fracture mechanics for metals is used for analysing the stability of an existing crack, whereas the most interesting application to concrete concerns the tensile fracture behaviour of non-precracked structures.

The conclusion is that most of the methods in fracture mechanics, which have been developed for metals, are not suited for concrete, but that it is necessary to develop methods which are fundamentally different.

THE FICTITIOUS CRACK MODEL

In 1974 the development of a new model for the description of the fracture behaviour of concrete and related materials started, resulting in a first publication by Hillerborg et al. (2) in 1976. This model has later become known as the Fictitious Crack Model.

The basis for the model is the behaviour of concrete in a direct tensile test, where the complete stress-deformation curve is studied until final fracture, Figure 1. Before the maximum stress is reached, the elongation is uniform along the specimen, and it can thus be described by means of a stress-strain curve. As the deformation increases beyond the point of maximum stress, the stress starts descending. This is due to the formation of a damage zone. Within this damage zone the stress decreases as the deformation (and the damage) increases. Outside the damage zone, the stress decrease gives an unloading of the material and the corresponding deformation follows an unloading branch in the stress-strain relation.

The behaviour cannot be described by means of a simple stress-strain curve to the right of the maximum point in the stress-deformation diagram, as the deformation within the damage zone increases at the same time as the deformation outside this zone decreases.

The fundamental idea of the fictitious crack model is that the total deformation of the specimen is the sum of two parts: one part is the deformation corresponding to the stress-strain curve, the other part is the additional deformation w within the damage zone.

The total elongation of the specimen can thus be described as the sum of two parts, one part corresponding to the stress-strain curve, the other part is the additional deformation w.

$$\Delta L = \varepsilon L + w \tag{1}$$

In this Equation, ΔL is the total elongation of the original length L, and ε is the strain from the σ-ε-diagram. Before the damage zone has started to form, w=0 and ε is taken from the ascending branch of the σ-ε-curve. After the damage zone has started to form, ε is taken from the unloading branch of the σ-ε- curve, w from the σ-w-curve.

The area below the σ-w-curve corresponds to the energy absorbed in the cleavage process per unit area. This energy is denoted G_F and it is called fracture energy. This is a material property which has proved to be important for concrete.

It is assumed that the σ-w-curve is a material property, which is independent of the size of the specimen. This is a reasonable assumption for concrete, where the deformation within the damage zone is due to the opening of microcracks. This type of behaviour does not give rise to significant lateral deformations or stresses. There is in this case no significant difference between plane stress and plane strain.

It is further assumed that the σ-ε- and σ-w-curves are material properties, which are independent of the stresses in other directions as long as these stresses are small compared to the strength. This assumption makes it possible to apply the model to more general cases than the direct tensile tests, i.e. to the material within a structure. The assumption is of course an approximation, but at least as long as the stresses in other directions are small it may be acceptable. In principle, the σ-ε- and σ-w-curves can be assumed to depend on the stresses in other directions, but this would highly complicate the numerical analyses.

The damage zone has a cetain length in the direction of the tensile stress. This length is small in comparison with the other dimensions of concrete specimens, and at least when the specimen approaches fracture most of the deformation w seems to occur in a single crack. Therefore the fracture zone can be assumed to have zero initial length, and thus act like a crack opening with a width w, while it is still carrying stresses according to the σ-w-curve. Such an assumed crack, which still can transfer stresses, has been called a fictitious crack, and this explains the name of the model.

It is also possible to assume a non-zero length (in the stress direction) of the damage zone. This does not change the fundamental ideas of the model. An example of this type of application has been given by Bazant and Oh (3). In finite element application, the length of the fracture zone (in the stress direction) is then normally assumed to be equal to the length of one element. The choise of length is mainly a question of efficiency in the numerical calculation, and it depends on the type of available finite element program. The choice has no significant influence on the results of the analyses.

The fictitious crack model can be said to assume a crack extension with a cohesive zone. From this point of view it has a resemblance with the well-known models of Barenblatt and Dugdale, and sometimes it is referred to as a model of the Barenblatt-Dugdale type. However it involves two essential new ideas, which make it differ from these previous models:

- The σ-w-curve is assumed to be a material property, which can be determined by means of a direct tensile test.
- The model can be applied also to structures without initial cracks.

The σ-ε-curve of a material is fully defined if we know the shape, the tensile strength f_t, and the modulus of elasticity E. In the same way, the σ-w-curve is fully defined if we know the shape, the tensile strength f_t and the fracture energy G_F.

It can be demonstrated that the ratio between the steepness of the σ-ε-curve and the steepness of the σ-w-curve is important, and that this ratio can be expressed by means of the material porperty

$$\ell_{ch} = EG_F/f_t^2 \qquad (2)$$

This material property is called characteristic length. It is not a directly measurable length, but just a value which is calculated through the combination of three measurable material properties. The ratio between the size of a structure and ℓ_{ch} gives a dimensionless number, which characterizes the toughness of the structure and thus also its fracture behaviour. Examples are given below.

THE TENSILE PROPERTIES OF CONCRETE

In order to make realistic analyses it is necessary to use realistic values of material properties, i.e. in this case realistic σ-ε- and σ-w-curves. On the other hand, it is for practical reasons necessary to make simplifications in order to limit the numerical calculations. Straight-line approximations highly simplify the calculations, and thus such approximations should be used if they do not introduce too great errors.

In this connection, it should be noticed that it is much more essential to simplify the σ-ε-curve than the σ-w-curve, as the former is valid for the whole specimen, whereas the latter is only valid for the damage zone.

Fortunately, the σ-ε-curve for concrete is normally rather linear all the way to the maximum point. Therefore, a purely elastic behaviour can be assumed outside the damage zone without the introduction of too great errors. All analyses which have been published so far have been based on this assumption. It must however be remembered that this simplification sometimes may have an essential influence, and calculations ought to be performed where the influence of a more correct σ-ε-curve is studied.

The first attempts to determine the shape of the descending branch, i.e. the σ-w-curve, were made by Hughes and Chapman (4) in 1966 and by Evans and Marathe (5) in 1968. Due to experimental difficulties these curves were unsafe and incomplete, though of great value, as they pointed to the possibility of measuring the complete descending branch.

The first complete and systematic tests in order to determine the σ-w-curve for concrete were performed by Petersson (6). His results have later been confirmed by other laboratories. Today, the shape of the curve is rather well known, and it seems that this shape does not vary much from one concrete or mortar to the other, see Figures 2 and 3. Thus the same shape may be assumed for all qualities without introducing too great errors.

Once the shape of the σ-ε-curve has been established, the curve is completely defined by means of the two parameters f_t and G_F.

The tensile strength f_t is a conventional material property, but the fracture energy G_F is a new property, for which we have no standard test method. A tentative RILEM Recommendation for the determination of G_F has been recently published (7). According to this method, the fracture energy is determined by means of a three-point bend test on a notched beam. The principle of the test is that the work performed during a test to complete separation is divided by the area of the ligament. It is thus assumed that no energy is absorbed outside the tensile fracture zone. In other words, it is assumed that no part of the absorbed energy belongs to the σ-ε-curve, nor to compressive deformations. The range of errors which are introduced through this approximation, still has to be investigated.

The orders of magnitude of the different parameters for ordinary concrete are, see e.g. Hillerborg (8).

$f_t = 2 - 4$ MPa

$E = 20 - 40$ GPa

$G_F = 65 - 200$ N/m

$\ell_{ch} = 0.1 - 1$ m

When numerical calculations are performed by means of finite elements, the suitable choice of assumption for the shape of the σ-w-curve depends on the type of program used. If a curved shape is assumed, it is necessary to perform iterations (or the computer approximates the curve by straight parts). If a stepwise linear shape is assumed, it is possible to perform the calculation by means of increments without iterations. This incremental method is suitable if the σ-w-curve is a single straight line or consists of a few linear parts; but as the number of linear parts increases, it will lead to a great increase in the computer time.

For the results presented below, two different approximations for the shape of the σ-w-curve have been used, viz. a single straight line (SL) or a bilinear shape (C), see Figure 4. The bilinear shape was chosen by Petersson (6) in order to give a good approximation for the measured curve for concrete. It has therefore been denoted by C for concrete.

EXAMPLES OF PRACTICAL RESULTS

Notched and unnotched beams of plain concrete.

The very first application of the model was to unnotched beams in bending (2). This analysis showed that the flexural strength of concrete (modulus of rupture, MOR) can be expected to be about 50 percent higher than the tensile strength, and that the flexural strength decreases as the depth of the beam increases. Both these results agree well with the experience from many tests.

Figure 5 from (6) shows the theoretical variation of the flexural strength. In the Figure two curves are shown, corresponding to the different assumptions regarding the shape of the σ-w-curve. It can be seen that the influence of this choice is not very important regarding the general conclusions which can be drawn from Figure 5, but that it can mean a difference of about 7 percent in the value of the flexural strength. Thus, in many cases it is sufficient to use the simplest possible approximation, the single straight line. It must however be noticed that this may not always be true. In some cases the shape of the σ-w-curve may play an important role.

Figure 6 illustrates the type of calculations, upon which Figure 5 is based. A damage zone is assumed to start in the bottom of the beam after the tensile stress has reached the tensile strength. As the damage zone grows, the acting load first increases up to a maximum point which gives the formal flexural strength, whereupon it starts decreasing. The left lower diagram shows the relation between the damage zone length a_s and the bending moment M, the right lower diagram, the relation between the center deflection δ and M.

Results of analyses of notched beams are given in Figure 7, which is also based on results from (6). For notched beams the stength approaches that which is predicted by LEFM when the depth increases. For concrete beams of a normal depth LEFM always gives bad predictions.

For small beams, the strength values in Figures 5 and 7 approach the values according to the theory of plasticity. The lines in the Figures show the values corresponding to an unlimited compressive strength, i.e. compressive failure has not been taken into account. For a real material the values are somewhat lower.

For large beams, the stength values approach those according to the theory of elasticity, i.e. elastic brittle fracture and LEFM respectively.

Thus the model is general in the sense that it covers all the region from the theory of plasticity to the theory of elasticity, and in the latter case normal brittle failure as well as conventional fracture mechanics. The most important property of the model is its ability to analyse tensile fracture in the intermediate region between these theoretically pure cases.

Three-point bend tests on notched beams have somtimes been used in order to try to determine the parameters K_c or G_c in LEFM. The errors which can then be expected can be analysed by means of the fictitious crack model. Figure 8 from (6) shows the results of this analysis. The curve marked D corresponds to a σ-w-curve with a constant stress, suddenly falling to zero, thus the assumption used by Dugdale. This curve may be expected to be approximately valid for metals.

When the depth d increases, the value of G_c ought to approach G_F, as the size of the damage zone becomes small in coparison with the size of the specimen. Thus, the correct value of G_c ought to be equal to G_F. With this correct value we have thus

$$EG_F = EG_c = K_c^2$$

$$\ell_{ch} = EG_F/f_t^2 = (K_c/f_t)^2$$

From Figure 8, it can be seen that a reasonably good value of K_c or G_c can be expected for metals if

$$d > 5\ell_{ch} = 5(K_c/f_t)^2$$

This is in a good agreement with generally accepted rules. For concrete, a correspondingly correct value would require a depth of more than 10 ℓ_{ch}, which means a depth of several meters or even more than 10 meters. From this it is evident that this type of test is unsuitable for the determination of K_c or G_c. The same conclusion is valid also for other test of the same kind, such as the compact tension test, the double cantilever beam test, and the double torsion test.

From figure 7 it is evident that for beams of a normal size the net bending strength of the ligament above a notch is rather independent of the notch. Concrete is thus not very notch sensitive in normal applications.

Shear strength of reinforced beams

Gustafsson (9) has analysed the shear strength of concrete beams with longitudinal reinforcement, but without shear reinforcement by means of the fictitious crack model. The analysis is rather complex, including assumptions regarding the ratio between the elastic moduli of steel and concrete, bond-slip relations for the bond between steel and concrete, failure conditions in combinded compression and shear for concrete etc. The details will not be presented here, but only the results, which are given in Figure 9. In this Figure the influence of the depth of the beam, the reinforcement ratio ρ and the span to depth ratio L/d can be seen. The influence of these parameters are rather wellknown from test, but previously it has not been possible to give them a satisfactory theoretical explanation. This has been possible with the analysis presented in Figure 9. The theoretical influence of the studied parameters on the shear strength shows a good general agreement with test results. Further research is however needed before this complex type of failure can be accurately analysed in detail.

Unreinforced concrete pipes.

Concrete pipes in practical use show two main types of failure, called crushing failure and bending failure, Figure 10. The formal flexural strength of the concrete in a tested pipe can be calculated by means of the conventional theory of elasticity. It has then been found that the flexural strengths calculated from tests with the two different types of failure give quite different results.

Gustafsson (9) made a theoretical analysis of the two types of pipe failure by means of the fictitious crack model with the results shown in Figure 10, where f_f is the formal flexural strength calculated by means of the theory of elasticity. Comparisons with test results showed that the difference in the formal flexural strengths obtained in the two types of test could be fully explained, see (9) or Gustafsson and Hillerborg (10). The Figure also shows that the crushing strength is much more size dependent than the beam strength.

The results presented in Figure 10 are today used by concrete pipe manufacturers for the design of pipes.

Sensitivity analysis

The results in Figures 5, 7, 9, and 10 are presented with logarithmic scales on the axis. It can be demonstrated, see Hillerborg (11) or (12), that, with such scales, the sensitivity of the structural strength with regard to changes in material properties can easily be evaluated from the slope of the curve. The structural strength is defined as the formal stress at maximum load, f_f, f_{net}, and f_v respectively. The sensitivity is defined as the relative change in the structural strength divided by the relative change in the value of a material property.

If the slope of a curve in Figures 5, 7, 9 or 10 is denoted by -B (the slope is always negative, so B is positive), the sensitivity with regard to E and G_F is B, with regard to f_t it is (1-2B). It has to be noted that that value of B shall be used which would have been obtained if the logarithmic scales had been the same on the two axes. In all the Figures, the scale on the vertical axes has been extended 4 times in comparison with the scale on the horisontal axes. The measured slopes have to be divided by 4 in order to get B.

In the case of the bending of an unnotched beam the value of B is in the order of 0.15, which means that the sensitivity with regard to f_t is 0.7 and with regard to G_F it is 0.15. For concrete pipes, the value of B is about 0.1 for bending failure and 0.3 for crushing failure. For crushing failure, thus, the sensitivity is about 0.4 with regard to f_t and 0.3 with regard to G_F, thus about the same magnitude.

For the shear strength of beams, Figure 9, the value of B may be in the order of 1/3, which means that the sensitivity with regard to G_F is the same as that with regard to f_t, i.e. the fracture energy G_F is an equally important property as the tensile strength f_t, for the strength of the structure.

The tensile strength is approximately proportional to the square root of the compressive strength. This means that a 10 percent change in the compressive strength only gives a 5 percent change in the tensile strength. The sensitivity with regard to the compressive strength is thus only half that with regard to the tensile strength, i.e. approximately (0.5-B).

A consequence of this is that the sensitivity of the shear strength with regard to the fracture energy is as a rule higher than the sensitivity with regard to the compressive strength. This is a very important, and even sensational conclusion. In connection with shear tests, the compressive strength has always been measured and reported, as it has been looked upon as the most fundamental material property, whereas the fracture energy has never been measured. Shear strength formulas in design codes are also mainly based on the compressive strength, whereas the fracture energy is a material property which has so far never been taken into consideration. This new material property ought to be taken into account in future research and in design codes. This is of special importance where concretes of new or unusual compositions are concerned, e.g. very high strength concrete or concrete with lightweight or other unusual aggregates.

Other materials than plain concrete

The fictitious crack model cannot only be applied to concrete, but to all materials where the failure mainly depends on the opening of microcracks, which coalesce into macrocracks. A typical example is rock, which behaves very much like concrete, even though its characteristic length as an average seems to be smaller.

The model is also well suited for fibre reinforced concrete, Hillerborg (13), and to other fibre reinforced materials with a brittle matrix. Bäcklund (14) and Aronsson (15) have sucessfully applied it to carbon fibre reinforced epoxy and to glass fibre reinforced polyester. They used the name Damage Zone Model instead of Fictitious Crack Model. They did not measure the shape of the σ-w-curve, but just assumed a straight line for most of their calculations.

Gustafsson (9) has demonstrated how the model can be applied to fracture of wood along the grains, and further research is going on along the same line. It will seem that this is a promising approach for the study of the influence of holes, shrinkage etc on the carrying capacity of timber structures.

When it comes to metals, the application is far more difficult, as large lateral strain occur before final fracture. The model ought in principle to be applicable, but the numerical difficulties are very large. The finite element program has to cope with large deformations in a three-dimensional analysis with stresses and strains which in a complicated manner are interrelated in the three dimensions.

It ought to be possible to describe the final cleavage fracture in metals by means of a σ-w-curve, but the shape of this curve is presumably quite different from that for concrete. The result of a first attempt to determine a σ-w-curve for steel is shown in

Figure 11.

CONCLUSIONS

1. The Fictitious Crack Model is general in the sense that it is applicable to all types of tensile fracture and thus covers the domain between plastic fracture, elastic brittle fracture (fracture as soon as the tensile strength is reached), and linear elastic fracture mechanics (crack stability and growth).

2. The model is easy to apply to concrete because the stress-strain curve is nearly linear up to the maximum stress, and because the fracture behaviour is such that no account has to be given to the three-dimensional stress state.

3. The model is also easy to apply to other materials which have a similar fracture behaviour, e.g. rock, many fibre reinforced materials and cleavage fracture of wood. The model is in principle applicable also to metals, but this requires much more sophisticated numerical methods.

4. The main application of the model to concrete concerns ordinary non-precracked structures. It has given a possibility to understand and theoretically analyse types of fracture, where design rules hitherto have been based on tests. This gives a highly increased reliability of the design rules, and it may lead to material saving.

5. It has been demonstrated that in some cases the most essential material property for the strength of a concrete structure is not the concrete strength, but its fracture energy. This essential material property has earlier not been recognised for concrete. It can thus be regarded as a new material property, which should be measured by means of some type of standard test.

SYMBOLS USED

E = modulus of elasticity

G_c = critical energy release rate according to LEFM

G_F = fracture energ-, area under σ-w-curve

K_c = critical stress intensity factor according to LEFM

L = length or span of a specimen

ΔL = length change

M = bending moment

a = length of notch or crack

a_s = extension of damage zone

b = width of a beam

f_f = formal flexural strength

f_{net} = formal flexural strength of ligament

f_t = tensile strength

f_v = formal shear strength

ℓ_{ch} = characteristic length = EG_F/f_t^2

w = additional deformation within damage zone

δ = beam deflection

ε = strain

ρ = reinforcement ratio

σ = stress

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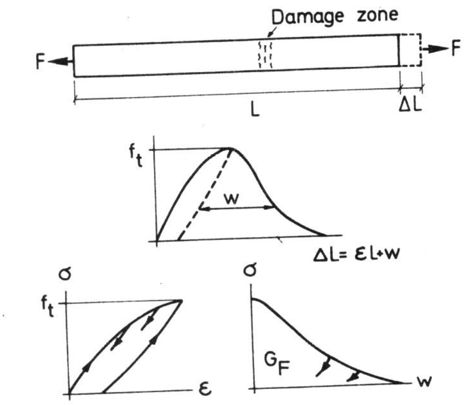


Figure 1 Deformations in a tensile test

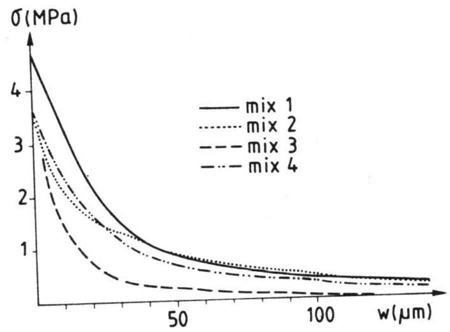


Figure 2 Examples of σ-w-curves for concrete

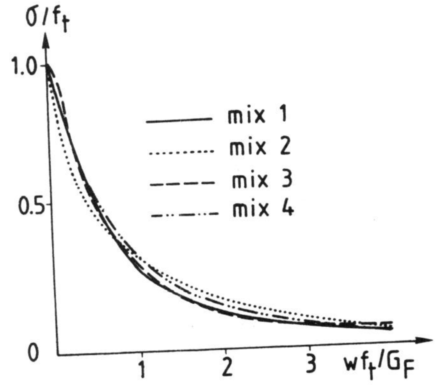


Figure 3 Dimensionless representation of curves in Figure 2

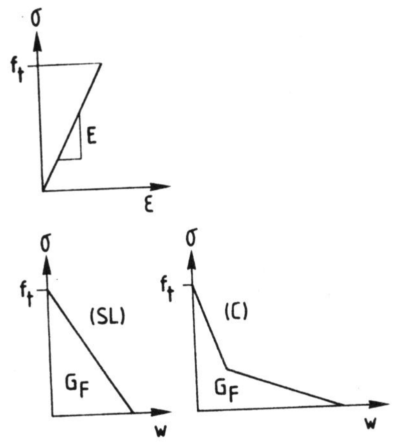


Figure 4 Simplified σ-ε- and σ-w-curves used in analyses

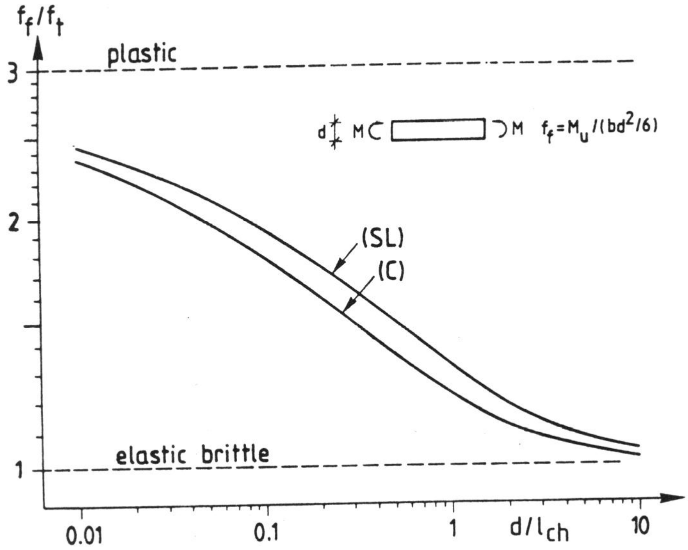
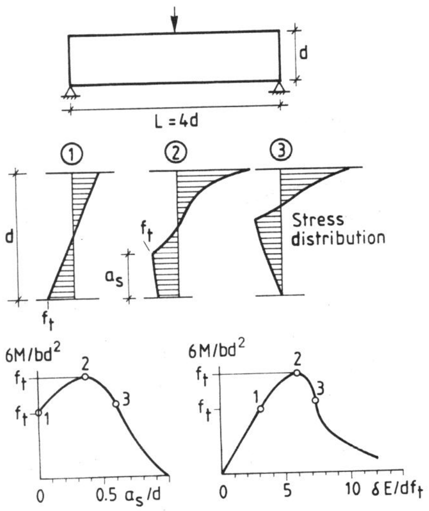


Figure 5 Theoretical variation in flexural strength f_f



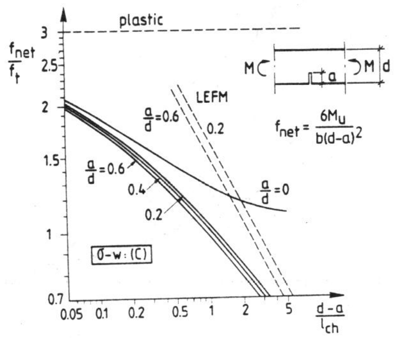


Figure 6 Analytical results for beam with d/ℓ_{ch} = 0.5

Figure 7 Theoretical variation in net bending strength of ligament

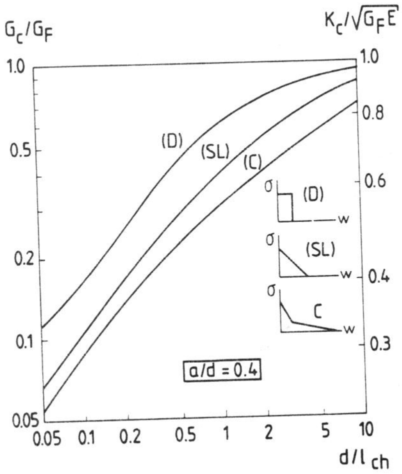


Figure 8 Theoretical variation in measured LEFM parameters

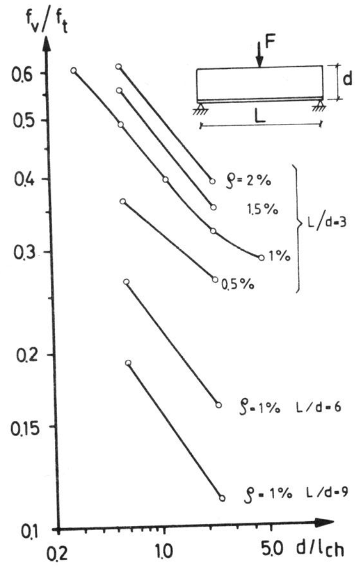


Figure 9 Theoretical variation in shear strength

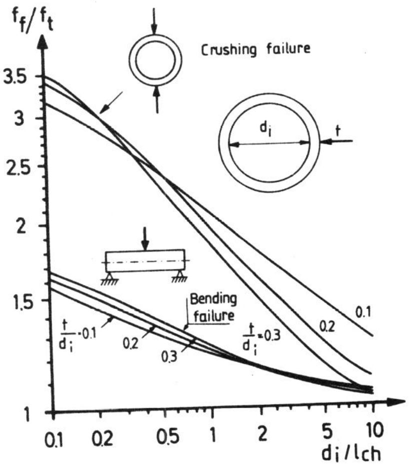


Figure 10 Theoretical variation in strength of a pipe

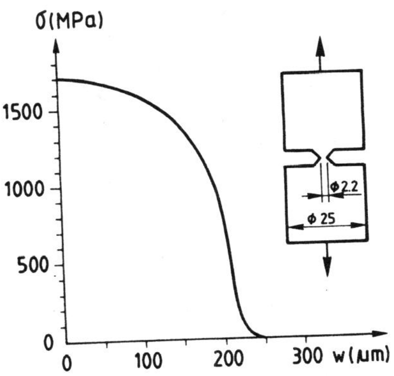


Figure 11 A measured σ-w-curve for cleavage fracture of steel