PREDICTION OF THRESHOLDS USING CYCLIC PROPERTIES OF MATERIALS

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A model and an analytical formula for predicting the true threshold value, $\Delta K_{c,th}$, for fatigue crack growth (FCG) in the absence of environmental and closure effects is presented. The model is based on the tensile and cyclic properties of the material, the cyclic plastic zone size and the minimum crack advance (interatomic spacing). It is found that calculated values are in good agreement with experimental results from various steels and one aluminium alloy.

Based on the same approach an equation for predicting near-threshold da/dN-values is assessed and successfully used in the analysis of near-threshold FCG data on a Ck45 steel in the as-rolled condition and with two different heat treatments.

INTRODUCTION

Theoretical methods of estimating ΔK_{th}-values and near-threshold (Regime A) FCG rates have attracted the attention of many researchers during the last twenty years (1-12). It has been demonstrated (9,12,13) that a relationship exists between da/dN-values and the crack tip strain range. The FCG process is almost entirely controlled by cyclic plastic strain occurring at the crack tip (13). Therefore models to predict threshold and FCG rates based on the cyclic properties of the material, such as those presented by Majumdar and Morrow (1), Radon (9,10) and Glinka (12), seem to be very promising.

Based on the stress and strain distribution ahead of the crack tip and restricting the FCG to the loading part of the fatigue cycle when the level of plastic strain ahead of the crack tip reaches the fracture strain, ε_f, of the material, the following

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expression was previously derived by Radon (9):

$$\frac{da}{dN} = \frac{2^{1+n'}(1-2\nu)^2\ (\Delta K_{eff}^2 - \Delta K_{c,th}^2)}{4\ (1+n')\pi\ \sigma_{yc}^{1-n'}\ E^{1+n'}\ \varepsilon_f^{1+n'}} \tag{1}$$

It should be emphasized that the model used for the derivation of Eq. 1 applies only to the threshold region (Regime A i.e. da/dN–values less than ~10^{-5} mm/cycle). The model also takes account of the fact that the crack is growing in a field of local plasticity under plane strain (Pε) conditions. In fact, the present authors have concluded in another work (14) that the cyclic plastic zone at the crack tip is generated and developed under Pε conditions.

To predict FCG rates using Eq. 1, a number of material constants and the appropriate values of ΔK corresponding to the loading amplitudes need to be evaluated, namely:

n' : the cyclic strain hardening exponent

ν : the Poisson's ratio

σ_{yc} : the cyclic yield stress

E : the Young's modulus

ε_f : the true fracture strain

ΔK_{eff} : the effective stress intensity factor range

$\Delta K_{c,th}$: the critical (true or intrinsic) threshold value i.e. the threshold value in the absence of crack closure.

All these parameters must be correctly evaluated by means of experimental data. An analysis carried out by Radon (10) using available data from three different steels (BS 4360-50D,HT80 and a C-Mn steel) showed that FCG rates obtained by Eq. 1 compared well with the experimental values. Unfortunately this analysis could not be extended to other materials because some of the parameters (specially n' and σ_{yc} and also ε_f) are not usually reported.

It is known (15,16) that the fatigue threshold, ΔK_{th}, may depend on stress ratio R, specimen thickness and environment (temperature included). $\Delta K_{c,th}$ - value in Eq. 1 represents the minimum lower bound of ΔK_{th} and, for this reason, is the safest value to be used in fatigue design. The value of $\Delta K_{c,th}$ can be determined experimentally by testing at high stress ratios (R>0.7) because, in such cases, the crack tip closure is neglegible and the applied stress intensity range is equal to the effective one. However, near-threshold FCG tests can be unnecessary if there is a reliable theoretical method of calculating $\Delta K_{c,th}$. In an earlier work based on energetic considerations (10), the following expression was proposed:

$$\Delta K_{c,th} = \left[\frac{4\pi E \gamma_s}{2-(\sigma_{uts}\,\varepsilon_f)/((1+n')\sigma_{yc})} \right]^{1/2} \tag{2}$$

The calculated values of $\Delta K_{c,th}$ using this equation gave good agreement with the experimental results obtained in various steels (BS 4360-50D, HT 80, C-Mn steel, HY 130, SM 50A and SM 58Q).However, Eq. 2 is not very suitable since experimental determination of γ_s (specific surface free energy) is difficult and may lead to errors.

In the present paper, a new model for the calculation of $\Delta K_{c,th}$ avoiding the estimation of γ_s, is proposed and checked against experimental data recently obtained by the authors from another steel (BM 45) and also against data previously reported (12) and obtained from other two steels (BS 4360-50D and 10Ni) and one aluminium alloy (2219-T831). Instead of energetic considerations, the new model is based on a kinematic approach i.e. the concept of minimum crack advance. A simplification of Eq. 1 will also be presented. Values of $\Delta K_{c,th}$ calculated by the new model will be used in the simplified form of Eq. 1 and the theoretical da/dN-values will be compared with the experimental results obtained in the BM 45 steel in three different metallurgical conditions.

MODEL FOR THE PREDICTION OF $\Delta K_{c,th}$-VALUES

Previous work (9) has shown that the strain range at the distance x ahead of a tensile fatigue crack in a work-hardening elastic-plastic material can be expressed by:

$$\Delta\varepsilon_{(x)} = 2\varepsilon_{yc} \left[\frac{(1-2\nu)^2\ \Delta K^2}{4(1+n')\pi\sigma_{yc}^2\, x} \right]^{1/1+n'} \tag{3}$$

where ε_{yc} is the cyclic yield strain of the material.

Eq. 3 is however physically unrealistic in the vicinity of the crack tip because when $x \to 0, \Delta\varepsilon \to \infty$. If it is assumed that a finite crack tip radius ρ will exist at the crack tip, then Eq. 3 gives the following more realistic strain distribution:

$$\Delta\varepsilon_{(x)} = 2\varepsilon_{yc} \left[\frac{(1-2\nu)^2\ \Delta K^2}{4(1+n')\pi\sigma_{yc}^2\,(x+\rho)} \right]^{1/1+n'} \tag{4}$$

To the first approximation our present proposal may be formulated as follows:

Experimental or average da/dN-values in the threshold region are frequently less than one interatomic spacing per cycle (non-continuum FCG process previously discussed in ref. 15). However, on the atomic scale, the minimum physically meaningful crack advance is one interatomic spacing b_0 (or the minimum Burgers vector of the crystalline structure). Hence one should distinguish between average da/dN-values and local crack advance per cycle. However, this is not contradictory to the other concept that one atomic bond may only be broken after some number of cycles. Then, it is possible to assume, as previously suggested by Chonghua and Minggao (8), that the crack tip radius ρ at the threshold is equal to b_0. Thus, for near-threshold situations in metallic structures, Eq. 4 becomes:

$$\Delta\varepsilon_{(x)} = 2\varepsilon_{yc}\left[\frac{(1-2\nu)^2\ \Delta K^2}{4(1+n')\pi\sigma_{yc}^2(x+b_0)}\right]^{1/1+n'} \tag{5}$$

When $\Delta K=\Delta K_{c,th}$ the necessary condition for crack growth is that an atomic bond is broken which means that the strain range at the distance $x = 2b_0$ must be equal to the true fracture strain range of the material, $2\varepsilon_f$, i.e.:

$$\Delta\varepsilon = 2\varepsilon_f \quad \text{for } x = 2b_0 \ \wedge\ \Delta K = \Delta K_{c,th} \tag{6}$$

Thus, for threshold conditions, Eq. 5 gives

$$2\varepsilon_f = 2\frac{\sigma_{yc}}{E}\left[\frac{(1-2\nu)^2\ \Delta K_{c,th}^2}{4(1+n')\pi\sigma_{yc}^2(2b_0+b_0)}\right]^{1/1+n'} \tag{7}$$

and $\Delta K_{c,th}$ becomes:

$$\Delta K_{c,th} = \left[\frac{3b_0\ \varepsilon_f^{1+n'}\ E^{1+n'}\ \sigma_{yc}^{1-n'}}{(1-2\nu)^2/(4(1+n')\pi)}\right]^{1/2} \tag{8}$$

SIMPLIFIED FORMS OF EQUATIONS 1 AND 8

In plane strain conditions and using the Von Mises criterion the dimension of the cyclic plastic zone ahead of the fatigue crack tip r_c, can be given by (9):

$$r_c = \frac{(1-2\nu)^2}{4(1+n')\pi}\left(\frac{K_{max}}{\sigma_{yc}}\right)^2 \tag{9}$$

However, according to plastic zone studies carried out recently by the authors (13,14), the average dimension of the cyclic plastic zone, r_c, is expressed by a general relationship:

$$r_c = \alpha_c\left(\frac{K_{max}}{\sigma_{yc}}\right)^2 \tag{10}$$

where α_c is a nondimensional parameter experimentally obtained.

Combining Eqs. 9 and 10 gives:

$$\alpha_c = \frac{(1-2\nu)^2}{4(1+n')\pi} \tag{11}$$

and, thus, Eqs. 1 and 8 can be rewritten as:

$$\frac{da}{dN} = \frac{2^{1+n'}\alpha_c(\Delta K^2_{eff} - \Delta K^2_{c,th})}{\sigma_{yc}^{1-n'}\,E^{1+n'}\,\varepsilon_f^{1+n'}} \tag{12}$$

and

$$\Delta K_{c,th} = \left(\frac{3b_0\,\varepsilon_f^{1+n'}\,E^{1+n'}\,\sigma_{yc}^{1-n'}}{\alpha_c}\right)^{1/2} \tag{13}$$

EXPERIMENTAL WORK

Experimental data obtained in the BM 45 steel, a trade-mark of a Ck 45 steel (according to DIN 17200) were used to check the validity of Eqs. 12 and 13. This steel was tested in the as-rolled condition (ferritic-pearlitic microstructure having a small grain size: ASTM No.10.5) and also in two heat-treated conditions: quenched (austenitised at 850°C 30 min and subsequently quenched in oil at 20°C) and quenched + tempered (same treatment as for the quenched condition followed by tempering at 550°C 1 hour). Fatigue tests were carried out in air (room temperature) in a closed loop electrohydraulic machine at a cyclic frequency of 25 Hz (sinusoidal wave).

Cyclic Stress-Strain Curves

Cyclic stress-strain curves were obtained for the three different conditions of the BM 45 steel (as-rolled, quenched and quenched + tempered) using cylindrical specimens of 9 mm diameter. The method of connecting the tips of stable hysteresis loops from several companion specimens (cycled at completely reversed constant strain amplitudes) was used initially. Later, the multiple step method (17) was also applied. Since the results obtained by these two methods were very close only the multiple step method was selected to obtain the remaining cyclic $\sigma - \varepsilon$ curves for the quenched and quenched + tempered conditions because it requires fewer specimens. In Figs. 1, 2 and 3 the resulting cyclic $\sigma - \varepsilon$ curves are compared with the monotonic curves. Fig. 4 is a typical example of the differences in the shape of the stable hysteresis loops obtained in the same steel with different heat-treatment conditions.

σ_{yc} and n'-values were calculated from the cyclic $\sigma - \varepsilon$ curves. σ_{yc} represents the 0.2% offset cyclic yield strength. For the calculation of n'-values 10 points (ε,σ) in the plastic region of each cyclic $\sigma - \varepsilon$ curve were taken; the corresponding values $\bar{\varepsilon},\bar{\sigma}$ (true strain, true stress) were computed and, using the Hollomon relation $(\bar{\sigma} = k\bar{\varepsilon}^{n'})$, a linear regression analysis (by the least

TABLE 1 - Mechanical properties

Symbol (Unit)	Material					
	BM45 as-rolled	BM45 quenched	BM45 q.& temp.	BS4360 50D	10Ni	2219-T851
σ_y (MPa)	335	815	580	386	1309	358
σ_{yc} (MPa)	345	815	690	312	1106	334
E (GPa)	207	207	207	210	207	71
n'	0.21	0.08	0.13	0.177	0.109	0.121
ε_f	0.80	0.40	0.71	0.72	0.34	0.35
σ_{uts} (MPa)	640	1040	755	560	1357	455

TABLE 2 - Chemical composition by percent weight

Chemical Element	Material			
	BM45	BS4360-50D	10Ni	2219-T851
C	0.45	0.180	0.12	-
Mn	0.74	1.40	0.28	0.25
Si	0.20	0.36	0.07	0.088
S	0.027	0.003	0.006	-
P	0.027	0.018	0.008	-
Ni	0.06	0.095	10.29	-
Cr	< 0.03	0.11	2.03	< 0.0001
Mo	-	0.02	1.03	-
Al	-	0.035	-	balance
Nb	-	0.039	-	-
Cu	-	0.16	-	6.28
Co	-	-	8.07	-
Fe	balance	balance	balance	0.25

squares method) was applied to find the best k and n'-values in the equation:

$$\log \bar{\sigma} = \log k + n' \log \bar{\varepsilon} \tag{14}$$

The correlation coefficients obtained with Eq. 14 were above 0.98.

The values of σ_{yc} and n' are given in Table 1 together with other parameters obtained from monotonic tensile tests. ε_f-values were calculated from tensile tests using the relationship : $\varepsilon_f = \ell n\,(1-\Psi)^{-1}$, where Ψ is the reduction of area of the material.

Table 1 also shows other data previously reported (12) and derived from BS 4360-50D steel, 10Ni steel and 2219-T851 aluminium alloy. The chemical compositions are given in Table 2.

Near-Threshold FCG Tests

Fatigue crack propagation tests were carried out for the three different conditions of the BM 45 steel using CT specimens (thickness = 9 mm). Dimensions and ΔK - values were defined as recommended by the ASTM Method E647-81. The complete set of experiments (reported in ref. 13) included results for three different stress ratios ($R = 0.1$, 0.5 and 0.8) but to check the validity of Eqs. 12 and 13 only the data obtained with $R = 0.8$ were used because in this case the applied ΔK can be taken as equal to ΔK_{eff}. Near-threshold FCG rates and the corresponding $\Delta K_{c,th}$-values were attained using both the load-shedding technique (decreasing ΔK at constant R) and the increasing R technique (constant K_{max}).

CALCULATIONS VS. EXPERIMENT

The values of σ_{yc}, E, n' and ε_f indicated in Table 1 were used in Eq. 13 to estimate the theoretical threshold ($\Delta K_{c,th}$). Following previous reviews (13,14) of the experimental work of a large number of researchers, $\alpha_c = 0.012$ was chosen for all six different microstructures. The values of b_0 were, respectively, 2.48×10^{-7}mm for the steels and 2.86×10^{-7}mm for the aluminium alloy.

In Table 3, the theoretical thresholds given by Eq. 13 are compared with the experimental values. Since very good agreement was found, the model used for the derivation of Eq. 13 seems to be successful.

Finally, the theoretical $\Delta K_{c,th}$-values given by Eq. 13 were used in Eq. 12 to check its validity by comparison with experimental da/dN - results. Eq. 12 was obtained for each metallurgical

TABLE 3 - Comparison between theoretical and experimental values of $\Delta K_{c,th}$ for various materials

Material	$\Delta K_{c,th}$ ($MN\ m^{-3/2}$)	
	Eq. 13	Experimental
BM45 as-rolled	3.58	3.5 ($R = 0.8$)
BM45 quenched	2.39	2.3 ($R = 0.8$)
BM45 q.& tempered	3.54	3.6 ($R = 0.8$)
BS 4360-50D	2.96	3.0 ($R = 0.7$)
10 Ni	2.75	2.4 ($R = 0.8$)
2219-T851	0.94	1.0 ($R = 0.8$)

condition of the BM 45 steel using the data shown in Table 1 and $\alpha_c = 0.012$. The growth rates predicted by Eq. 12 are compared with the experimental results in Figs. 5, 6 and 7. The experimental results plotted in the figures were obtained at $R = 0.8$, since, as previously mentioned, the applied ΔK under such high stress ratio can be taken as equal to ΔK_{eff}. A very satisfactory correlation between the calculations and the experiment was found thus confirming the validity of the model at least for this type of steel. However, it should be stressed that this model is only valid for the near-threshold region (Regime A). Any extrapolation of Eq. 12 for Regime B (Paris-law region) may not produce satisfactory predictions. Further experiments in the threshold region on various materials, in particular on high strength steels are recommended.

CONCLUSIONS

1) A model to predict the threshold value in the absence of environmental and closure effects $\Delta K_{c,th}$ based on tensile and cyclic properties of the material, cyclic plastic zone parameter and a minimum crack advance (interatomic spacing b_0) was developed. Using this model the following equation was derived:

$$\Delta K_{c,th} = \left(\frac{3b_0 \; \varepsilon_f^{1+n'} \; E^{1+n'} \; \sigma_{yc}^{1-n'}}{\alpha_c} \right)^{1/2}$$

The predicted $\Delta K_{c,th}$-values for a range of steels and one aluminium alloy were very close to the experimental results obtained in fatigue under high stress ratios in air.

2) A simplification of an equation previously derived to estimate da/dN - values in the near-threshold region was developed as follows:

$$\frac{da}{dN} = \frac{2^{1+n'} \alpha_c \left(\Delta K_{eff}^2 - \Delta K_{c,th}^2 \right)}{\sigma_{yc}^{1-n'} \; E^{1+n'} \; \varepsilon_f^{1+n'}}$$

The prediction of near-threshold FCG rates for three different microstructures of the medium carbon BM 45 steel using the above equation also showed a satisfactory agreement with the experimental results.

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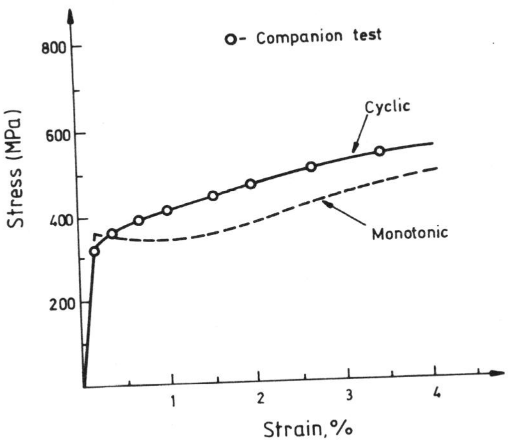

Fig. 1 σ - ε curves of the BM 45 steel in the as- -rolled condition.

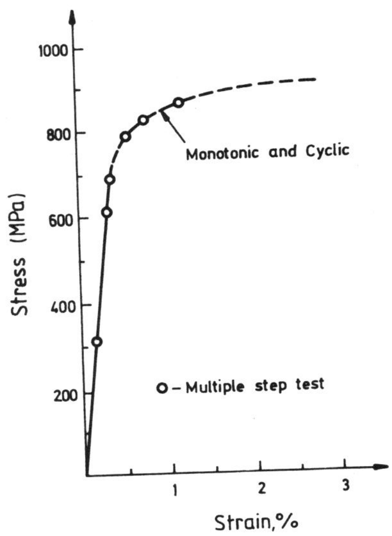

Fig. 2 σ - ε curves of the BM 45 steel in the quenched condition.

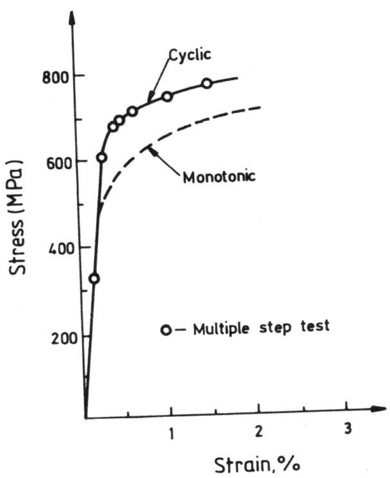

Fig. 3 σ - ε curves of the BM 45 steel in the q.& tempered cond.

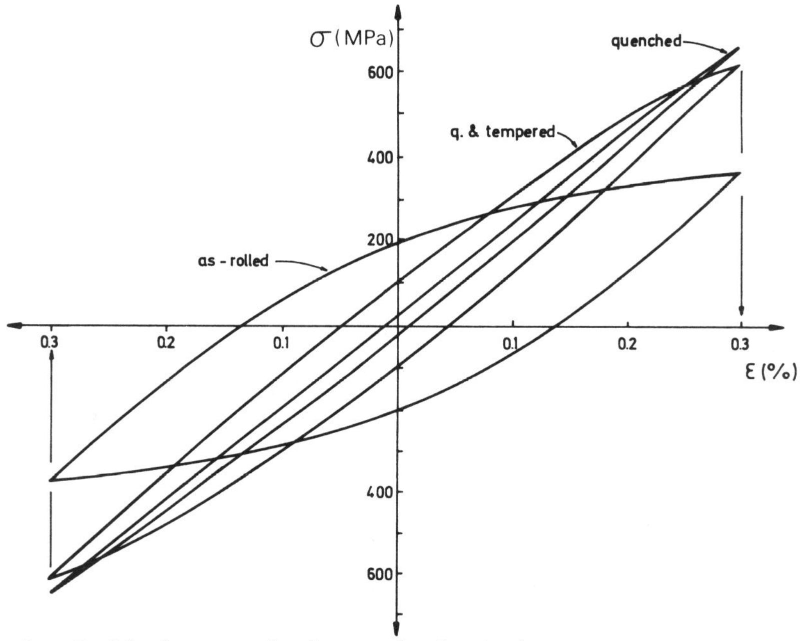

Fig. 4 Stable hysteresis loops obtained with completely reversed constant strain amplitude ($\Delta\varepsilon = 0.6\%$) for three conditions of BM 45.

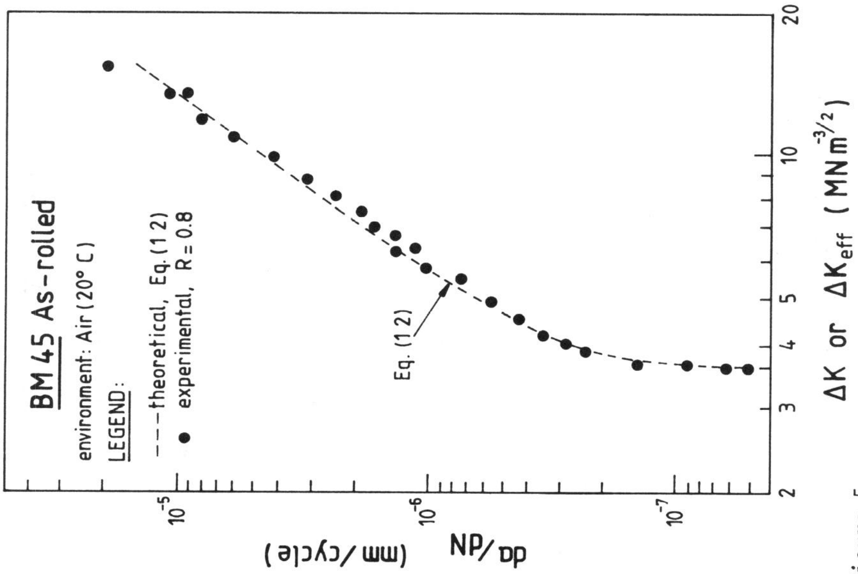

Figure 5

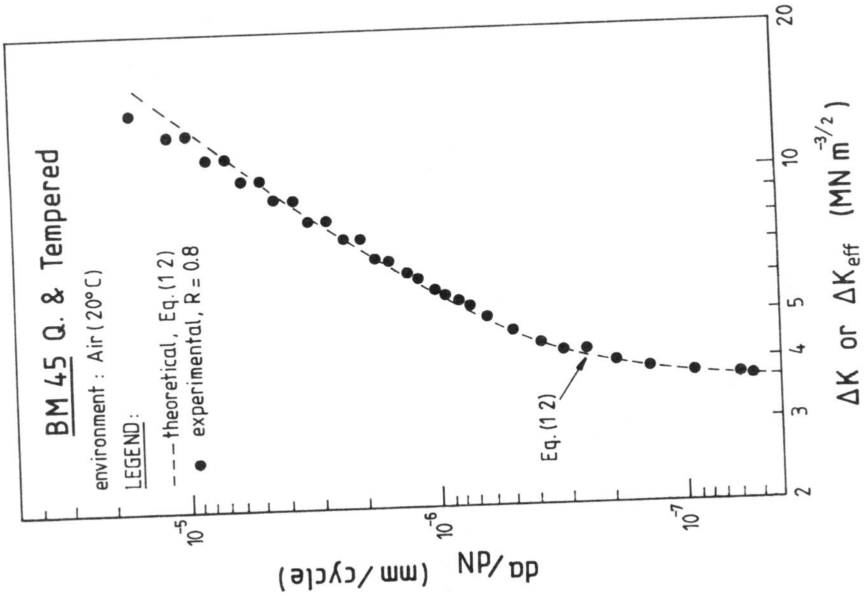

Figure 7

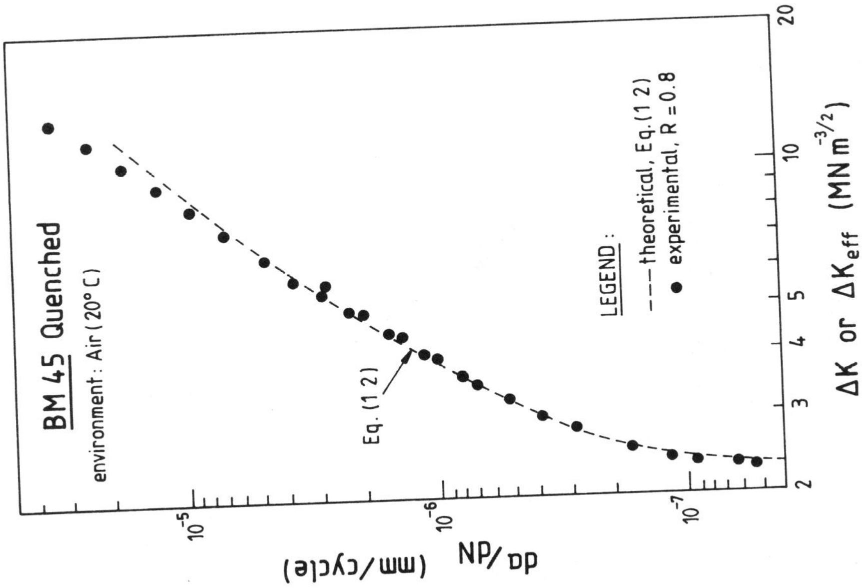

Figure 6