QUASISTATIC EXTENSION OF AN INTERFACE CRACK

K.P.Herrmann* and U.Strathmeier**

Micromechanical investigations concerning the quasistatic thermal crack growth in the interfaces of self-stressed unidirectionally reinforced composite structures with a relatively high fiber volume fraction have been performed. Thereby circular unit cells of two different two-phase composites (Al-matrix/C-fiber and Al-matrix/Glass fiber, respectively) have been considered in order to study the influence of an instationary temperature distribution on the behaviour of a quasistatic extending interface crack. The resulting mixed boundary-value problems of the plane thermoelasticity were solved numerically by using standard finite element programs.

INTRODUCTION

The investigation of the thermal cracking of unidirectionally reinforced composite structures is important for those compound materials subjected to thermal loading during service, for example certain components in spacecraft and nuclear reactor technology. Therein previous experiments show that different failure mechanisms dominate in the low- and high-fiber concentration ranges of unidirectionally reinforced composites. Thereby, for low-fiber concentration structures, cracks are generally first observed in the matrix material where the fibers act to restrain crack growth. On the contrary, in the range of high-fiber concentration, debonding becomes the dominate failure mechanism, that means cracks often following the fiber - to matrix interfaces. For a quantitative treatment of the interaction of composite microcomponents with microcracks extending in the heterogeneous microstructure the so-called micromechanical stress analysis is especially useful. This concept has been used by several authors in order to study finite curved interface cracks between dissimilar media loaded either at infinity with a uniform stress field or where the interfacial crack is opened by internal pressure. A comprehensive survey of the state-of-the-art was given by Piva and Viola (1) reporting also about the cancellation of the oscillatory anomalies of the elastic stress and displacement fields near the tip of an interface crack. The thermal fracture behaviour of fiber reinforced composites applying the micromechanical stress analysis has been studied in several papers by Herrmann et al (2-5).

In this paper, using circular unit cells of two different composite structures with circular fibers in a hexagonal array (cf. Fig.1), the quasistatic thermal crack growth of curved interface cracks due to transient thermal

*Institute of Mechanics, Department of Mechanical Engineering, Paderborn University, Federal Republic of Germany
**BBC Mannheim

stress fields has been investigated by means of the finite element method. Fiber and matrix of these microcomponents consist of homogeneous, isotropic and linearly elastic materials where the thermoelastic material properties vary discontinuously at the fiber-matrix interface S_{fm} from the values E_f, ν_f, α_f of the fiber to the values E_m, ν_m, α_m of the matrix. The material properties of the two composite structures Aluminum matrix/Carbon fiber and Aluminum matrix/Glass fiber used in the numerical calculations are shown in Table 1.

Table 1 - Thermoelastic Material Properties of the Composite Structures Aluminum matrix/Carbon fiber and Aluminum matrix/Glass fiber.

	Matrix	Fiber	Fiber
Notation	Aluminum	Carbon	ABC-Glass
E	69651	235440	115000
ν	0.34	0.27	0.284
α	24.0	3.0	8.4
κ	94.08	10.34	0.562
λ	56.88	40.0	0.314

Further, the quasistatic thermal crack growth in the interface of a circular unit cell according to Fig.2 is caused by a thermal shock. Therein, the cracked two-phase solid, having for $t = 0$ the constant temperature $T = 0^oC$ of the unstressed initial state is subjected at $t > 0$ to a temperature $T_B = 100^oC$ on the external surface S_m of the matrix material.

Formulation and Solution of the Corresponding Boundary-Value Problems

The determination of the thermal stress field acting in the cross section of a cracked unit cell according to Fig.2 and caused by a thermal shock mentioned above requires as a first step the solution of the following boundary-value problem of the instationary heat conduction equation for the plane inhomogeneous domain $A_f \cup A_m$ containing a curved interface crack $S_c^+ \cup S_c^-$ in the interval $|\phi| \leqq \phi_o$ (cf. Fig.2 for notation). Besides, the conditions of perfect contact at the uncracked part of the interface S_{fm} are assumed. Then the boundary-value problem reads:

$$\frac{\partial T_i}{\partial t} - \kappa_i \left\{ \frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r}\frac{\partial T_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T_i}{\partial \phi^2} \right\} = 0 \qquad (1)$$

$$T_i(r,\phi,0) = T_o = 0 \qquad ; \quad (i=f,m) \qquad (2)$$

$$T_m(r_m,\phi,t) = T_B \qquad ; \quad (|\phi| \leqq \pi) \qquad (3)$$

$$T_f(r_f,\phi,t) = T_m(r_f,\phi,t) \qquad (4)$$

$$\lambda_f \left\{ \frac{\partial T_f(r,\phi,t)}{\partial r} \right\}_{r=r_f} = \lambda_m \left\{ \frac{\partial T_m(r,\phi,t)}{\partial r} \right\}_{r=r_f} \qquad ; \quad (\phi_o \leqq |\phi| \leqq \pi) \qquad (5)$$

$$\lambda_i \left\{ \frac{\partial T_i(r,\phi,t)}{\partial r} \right\}_{r=r_f} = 0 \qquad ; \quad (|\phi| < \phi_o) \; , \; (i=f,m) \qquad (6)$$

The solution of the boundary-value problem (1)-(6), the instationary temperature distribution $T_i(r,\phi,t)$, (i=f,m) has been obtained by using a finite element program described by Marsal (6) together with the mesh-work generator program FEMGEN. The figures 3 - 7 give the instationary temperature distributions inside of the cross section of a circular unit cell belonging to the composite structures Aluminum matrix/Carbon fiber and Aluminum matrix/Glass fiber, respectively.*

Further, the investigation of the curved quasistatic thermal crack growth in the inhomogeneous and linearly elastic microcomponents has been performed by using the concepts of linear elastic fracture mechanics. Therefore applying the basic equations

$$\sigma_{ij} = 2\mu\{\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\} \tag{7}$$

$$\sigma_{ij,i} = 0 \tag{8}$$

$$\varepsilon_{ijk}\varepsilon_{lmn}\varepsilon_{km,jn} = -\alpha\varepsilon_{ijk}\varepsilon_{lmn}T_{,jn}\delta_{km} \tag{9}$$

and by assuming the existence of a plane strain state in the two-phase composite structures the following mixed boundary-value problems of the plane thermoelasticity have to be solved

$$\sigma^m_{rr}(r_m,\phi,t) = \sigma^m_{r\phi}(r_m,\phi,t) = 0 \quad ; \quad (|\phi| \leqq \pi) \tag{10}$$

$$\sigma^i_{rr}(r_f,\phi,t) = \sigma^i_{r\phi}(r_f,\phi,t) = 0 \quad ; \quad (i=f,m) \;,\; (|\phi| < \phi_o) \tag{11}$$

$$[\sigma_{rr}(r,\phi,t)]_{r=r_f} = [\sigma_{r\phi}(r,\phi,t)]_{r=r_f} = 0 \quad ; \quad (\phi_o \leqq |\phi| \leqq \pi) \tag{12}$$

$$[u_r(r,\phi,t)]_{r=r_f} = [u_\phi(r,\phi,t)]_{r=r_f} = 0 \quad ; \quad (\phi_o \leqq |\phi| \leqq \pi) \tag{13}$$

with the definition of the jump relations at the fiber-matrix interface S_{fm}

$$[\tau(r,\phi,t)]_{r=r_f} = \tau^f(r_f,\phi,t) - \tau^m(r_f,\phi,t) \quad ; \quad (\phi_o \leqq |\phi| \leqq \pi) \tag{14}$$

The boundary-value problems (10)-(13) for the two composite structures have been solved under the assumption of plane strain conditions using the standard finite element computer program ASKA together with the mesh-work generator program FEMGEN. Further, the finite element mesh-work for one-half of the cross section of the composite microcomponents consists of about 240 triangular elements focused essentially along the prospective crack line and by using quadratic displacement functions. Because of the symmetry of the microcomponents with respect to the x-axis only the upper halves of the bimaterial specimens have to be considered. Thus, the following boundary conditions have to be added on the symmetry line

$$\sigma_{xy}(x,0,t) = 0 \;,\; u_y(x,0,t) = 0 \quad ; \quad (|x| \leqq r_m) \tag{15}$$

*Thereby the graphs 4-12 should be turned clockwise at a 90 degree angle.

Calculation of the Strain Energy Release Rate and Discussions

The determination of the strain energy release rate at the tip of a quasi-static extending thermal interface crack has been performed applying a method originated by Rybicki and Kanninen (7). The method, originally formulated for a straight crack in a homogeneous material, bases of the calculation of a crack closure integral modified by applying the finite element method. This procedure is carried over here to a curved interface crack growing in a fiber reinforced compound material under the influence of an instationary temperature distribution. Then, for small crack extensions $\Delta a \ll a$ the desired strain energy release rates G_j (j=I,II) are given by the formulae

$$G_I = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} F_{rc}(u_{rc}-u_{rd}) \tag{16}$$

$$G_{II} = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} F_{\theta c}(u_{\theta c}-u_{\theta d}) \tag{17}$$

Fig. 8 shows the finite element mesh-work in the vicinity of the crack tip used in the numerical calculations which were performed on the computer PRIME 550 at the University of Paderborn.

Finally, the figures 9 - 12 give the strain energy release rates G_j(j=I,II) and $G = G_I + G_{II}$, respectively, in dependence on time and with the crack opening angle ϕ_o as a parameter. Therein Fig. 9 shows the existence of pure Mode I-loading for an interface crack with a crack opening angle of $\phi_o = 3.75^o$. Further, the corresponding G_I-curve increases monotonously with increasing time, the stationary value is reached at time $t = 5 \cdot 10^{-1}$s. A similar behaviour can be stated for the slope of the other G_I-curves as shown in the corresponding graphs with different parameter values ϕ_o. In addition, the curves belonging to the energy release rate G_I at the tip of an interface crack with different crack opening angles ϕ_o in the interval $60^o < \phi_o < 120^o$ have their maximum values in the time interval $6 \cdot 10^{-2}\,s < t < 7 \cdot 10^{-2}\,s$ whereas the stationary values are lower than those maximum values. Moreover, the graphs also show an increase of the G_{II}-values with increasing parameter values ϕ_o. Further, the numerical calculations give dominating G_{II}-values for crack opening angles $\phi_o > 120^o$.

Finally, the figures 10 and 11, respectively, show a comparison of the strain energy release rates for a crack opening angle of $\phi_o = 112{,}5^o$ for the two different composite structures Aluminum matrix/Carbon fiber and Aluminum matrix/Glass fiber. It can be seen that the latter composite exhibits remarkable dynamic peaks in the values of the strain energy release rate. A physical explanation of this phenomenon can be given by the larger difference of the thermal material properties of the Aluminum matrix/Glass fiber composite structure in comparison with the Aluminum matrix/Carbon fiber compound material. Besides, Fig. 12 gives the stationary values of the strain energy release rate for the composite structure Aluminum matrix/Carbon fiber in dependence on the crack opening angle ϕ_o. These results agree very well with calculations given in reference (5) for an interface crack in an Aluminum matrix/Silicon-carbide fiber unit cell under the influence of a stationary temperature distribution in the cross section of the composite microcomponent.

Acknowledgement

The financial support of the Stiftung Volkswagenwerk, Hannover is gratefully acknowledged.

SYMBOLS USED

a = crack length (mm)

α_i = linear coefficients of thermal expansion ($10^{-6} K^{-1}$)

E_i = Young's moduli (Nmm^{-2})

ε_{ij} = components of the strain tensor

ε_{ijk} = Levi-Civita tensor

ν_i = Poisson's ratios

F_{rc}, $F_{\theta,c}$ = nodal point forces (Nmm^{-1})

G_j = strain energy release rates (Nmm^{-1})

κ_i = thermal diffusivities ($mm^2 s^{-1}$)

λ_i = thermal conductivities (10^{-3} cal $mm^{-1} s^{-1} K^{-1}$)

σ_{ij} = components of the stress tensor (Nmm^{-2})

u_{rc}, u_{rd} = crack surface displacements (mm)

REFERENCES

1. Piva, A., and Viola, E., 1980, Engng. Fract. Mech. 13, 143.
2. Herrmann, K., 1977, Conference Proceedings "Continuum Models of Discrete Systems", Quebec, Canada.
3. Herrmann, K., Braun, H., and Kemeny, P., 1979, Int. J. Fract. Mech. 15, 187.
4. Herrmann, K., 1981, Mechanics Research Communications 8, 97.
5. Buchholz, F.-G., Herrmann, K., and Strathmeier, U., 1981, DVM-Mitteilungen, 12. Sitzung Arbeitskreis Bruchvorgänge, Freiburg 1980, 215.
6. Marsal, D., 1976, "Die numerische Lösung partieller Differentialgleichungen", B.I.-Wissenschaftsverlag, Zürich, Schweiz.
7. Rybicki, E. F., and Kanninen, M. F., 1977, Engng. Fract. Mech. 9, 931.

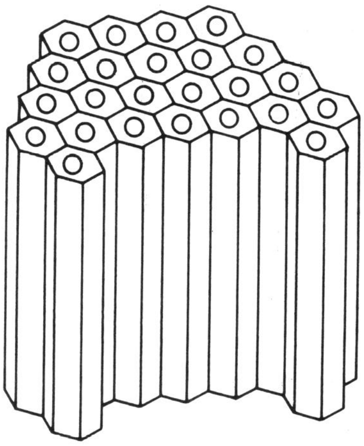

Figure 1 Fiber reinforced composite structure

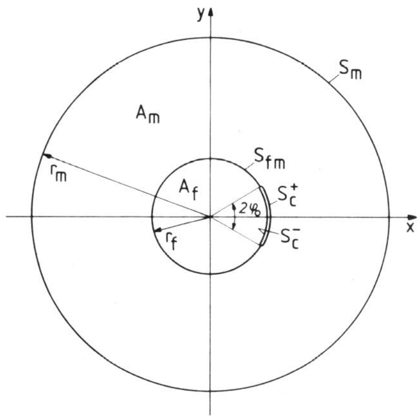

Figure 2 Cross section of a cracked circular unit cell

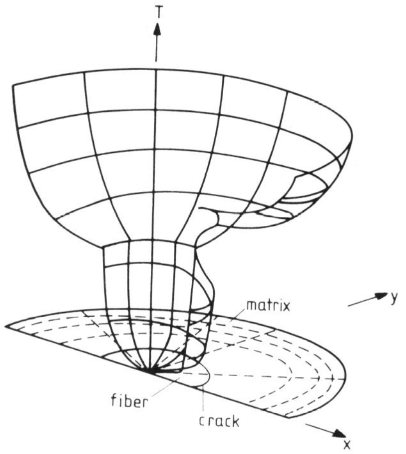

Figure 3 Temperature distribution in a cracked microcomponent

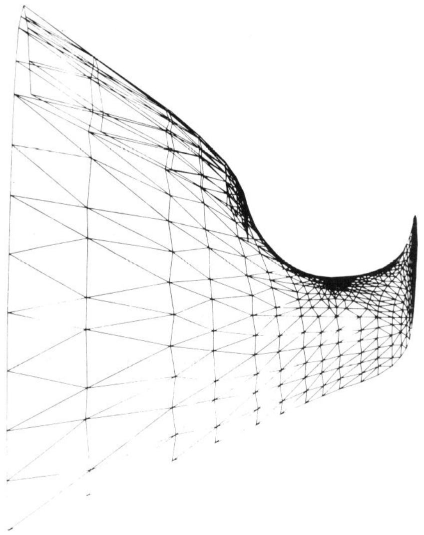

Figure 4 Temperature distribution in an Al-matrix/C-fiber compound
$\phi_o = 112.5^o$; $t = 0.0103$ sec

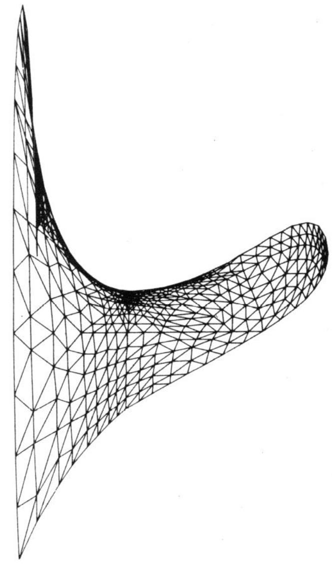

Figure 5 Temperature distribution, Al-matrix/C-fiber composite structure $\phi_o = 112.5^o$; $t = 0.100$ sec

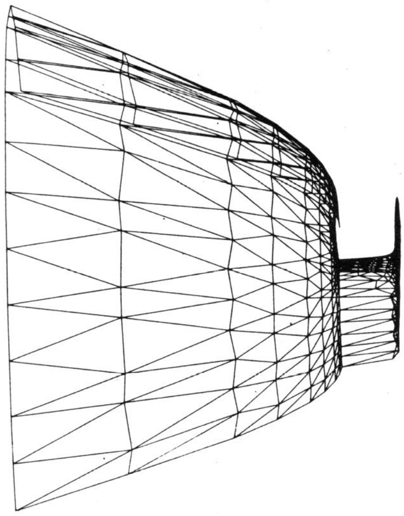

Figure 6 Temperature distribution, Al-matrix/Glass-fiber composite structure $\phi_o = 112.5^o$; $t = 0.0103$ sec

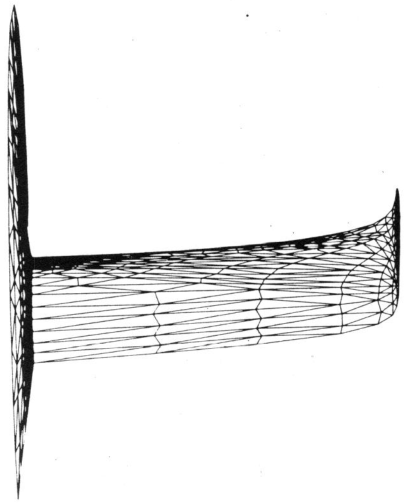

Figure 7 Temperature distribution, Al-matrix/Glass-fiber composite structure $\phi_o = 112.5^o$; $t = 0.2087$ sec

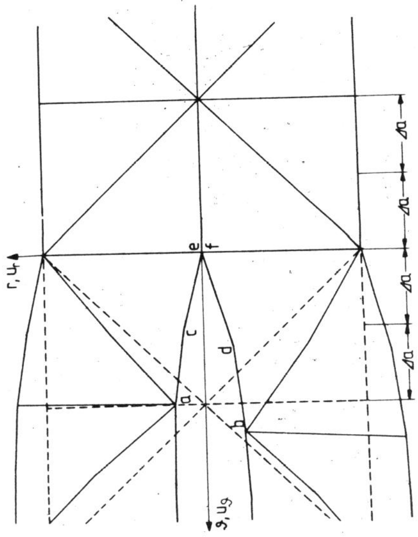

Figure 8 Finite element mesh-work in the vicinity of the crack tip

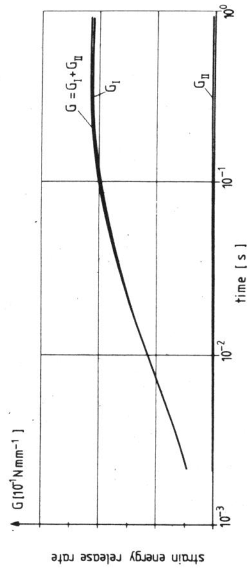

Figure 9 Strain energy release rates Al-matrix/C-fiber composite structure crack opening angle $\phi_o = 3.75^o$

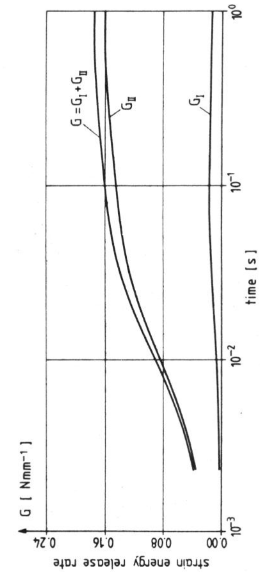

Figure 10 Strain energy release rates Al-matrix/C-fiber composite structure crack opening angle $\phi_o = 112.50^o$

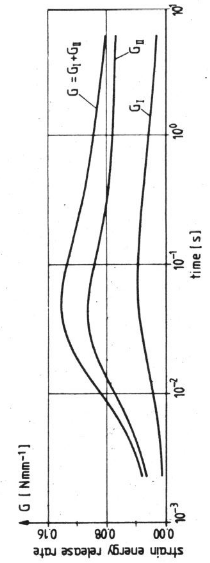

Figure 11 Strain energy release rates Al-matrix/Glass-fiber composite structure crack opening angle $\phi_o = 112.50^o$

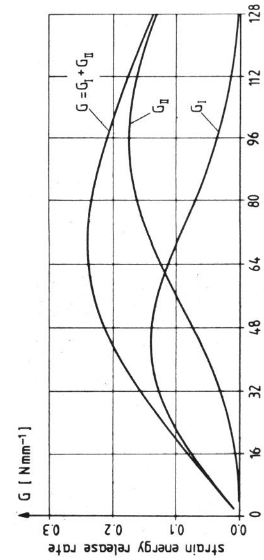

Figure 12 Strain energy release rates Al-matrix/C-fiber composite structure stationary values